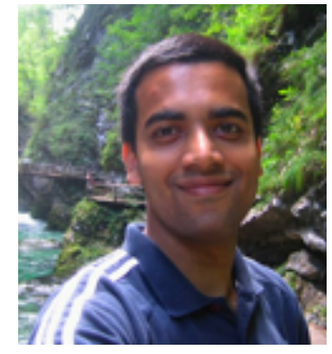
A Tutorial on

Graph-based Semi-Supervised Learning Algorithms for Speech and Spoken Language Processing



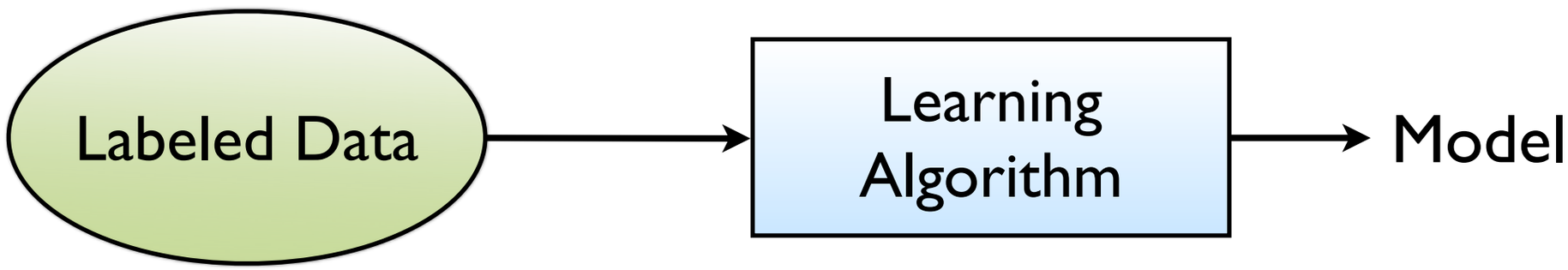
Amarnag Subramanya
(Google Research)

Partha Pratim Talukdar
(Carnegie Mellon University)

ICASSP 2013, Vancouver

http://graph-ssl.wikidot.com/

Supervised Learning



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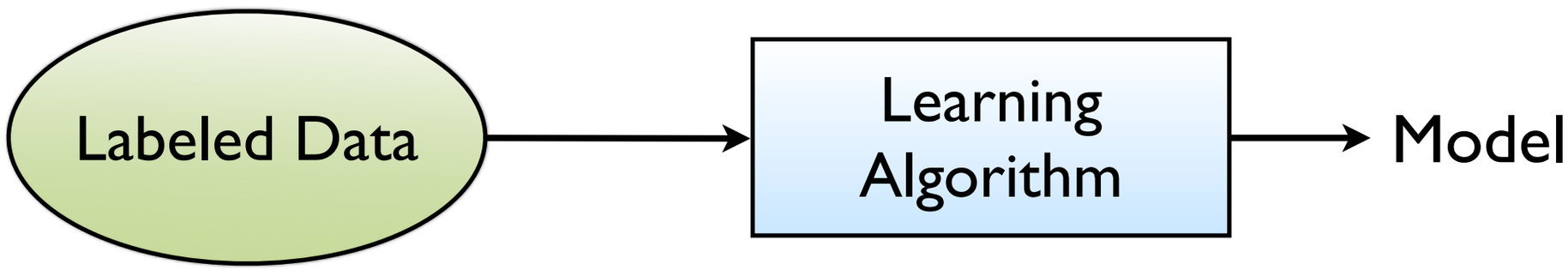
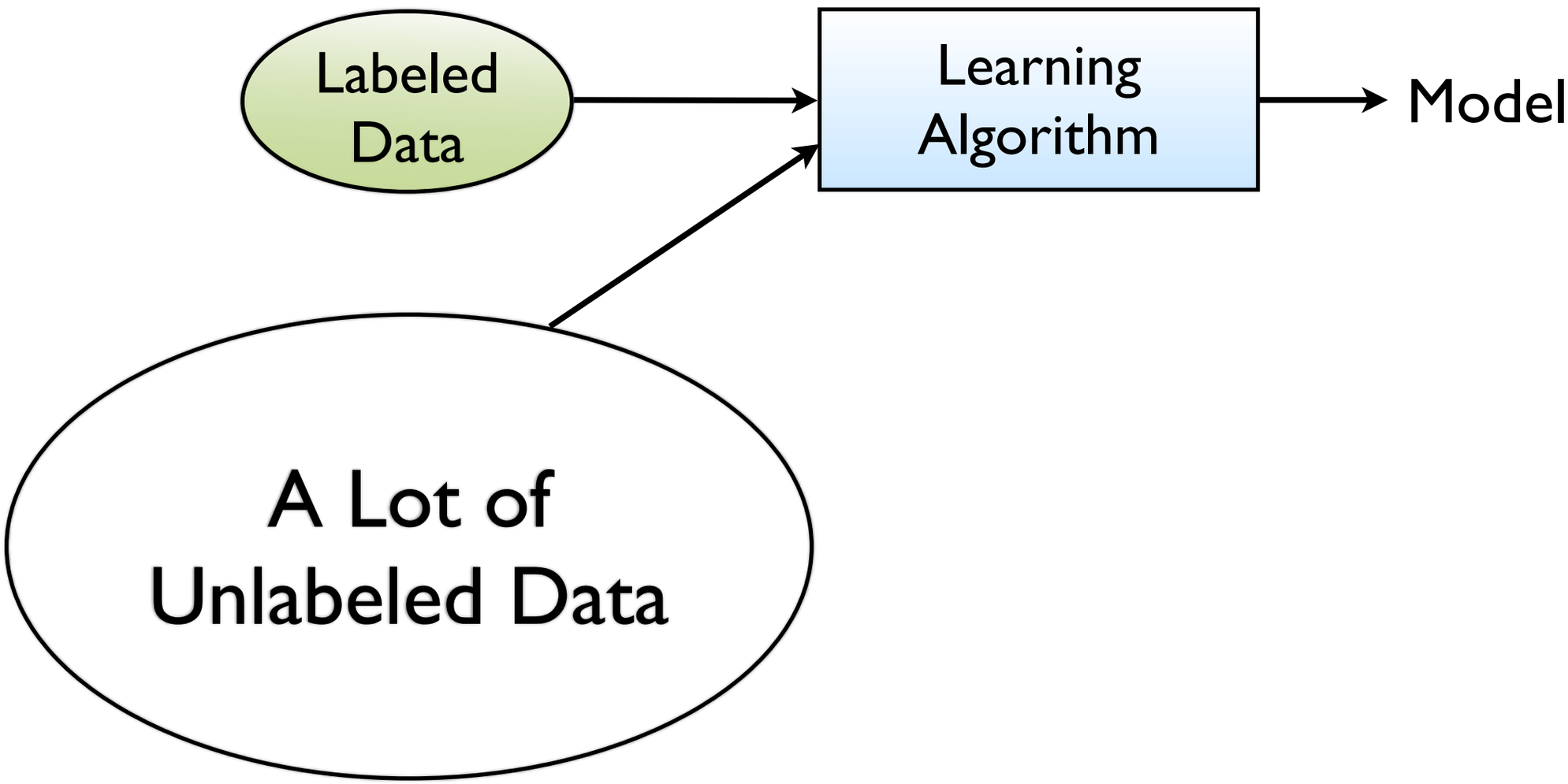


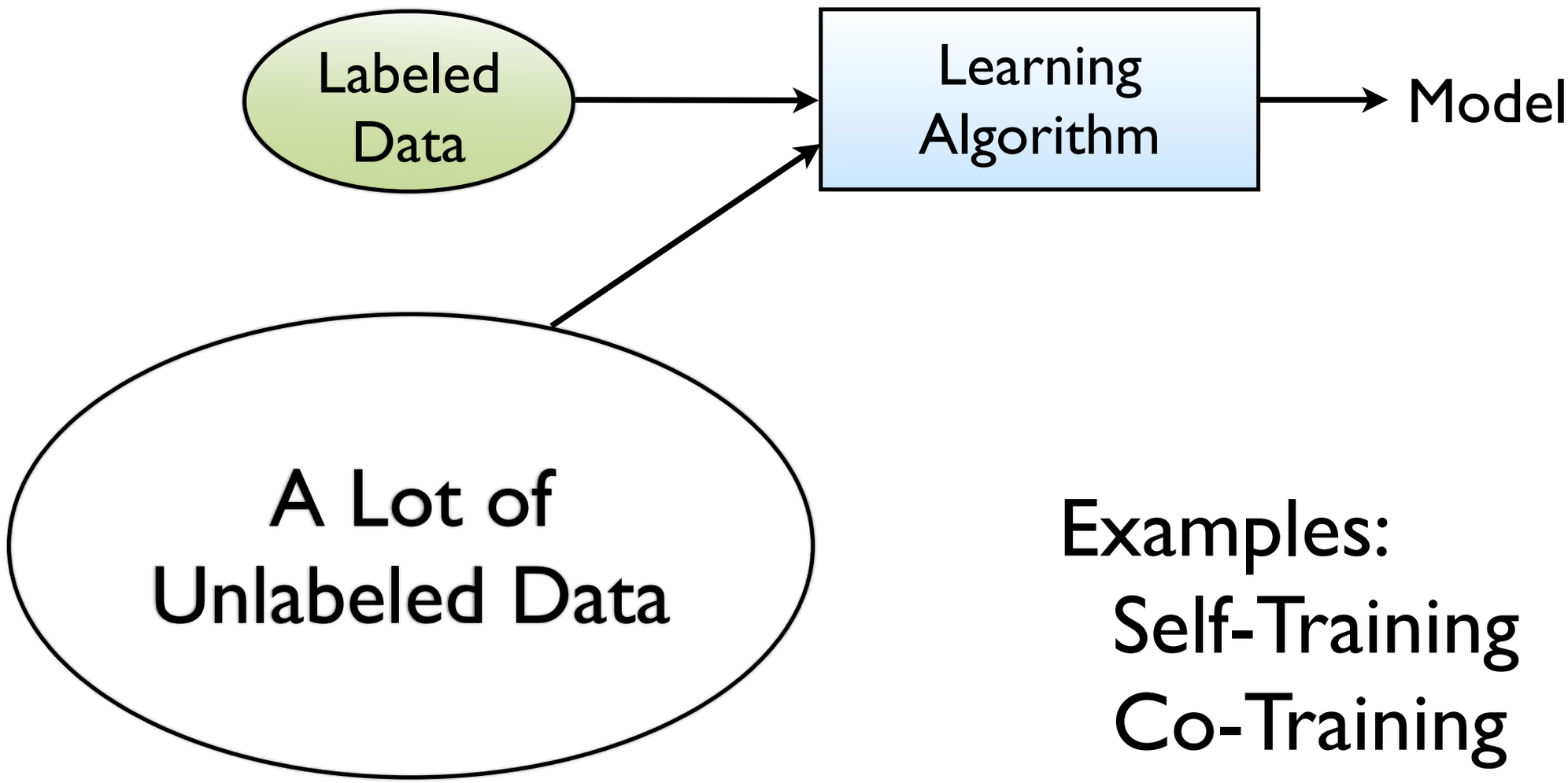
Examples:

- Decision Trees
- Support Vector Machine (SVM)
- Maximum Entropy (MaxEnt)

Semi-Supervised Learning (SSL)



Semi-Supervised Learning (SSL)



Examples:
- Self-Training
- Co-Training

Why SSL?

How can unlabeled data be helpful?

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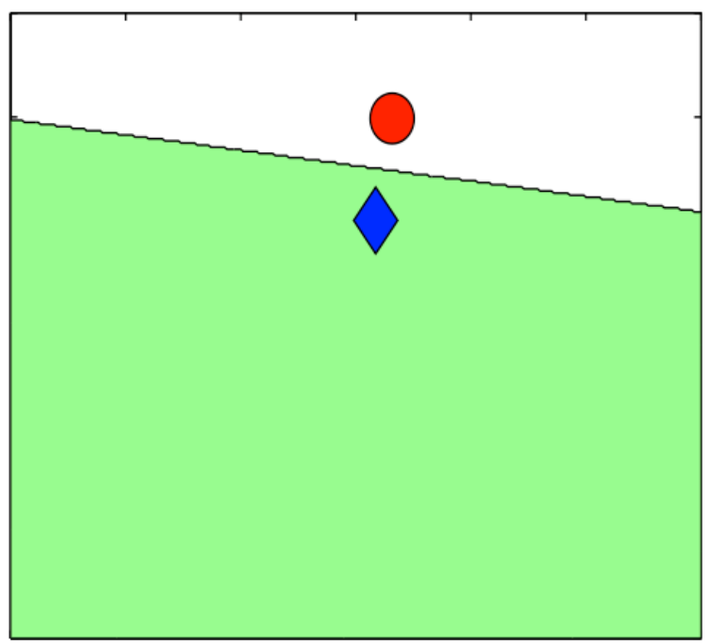

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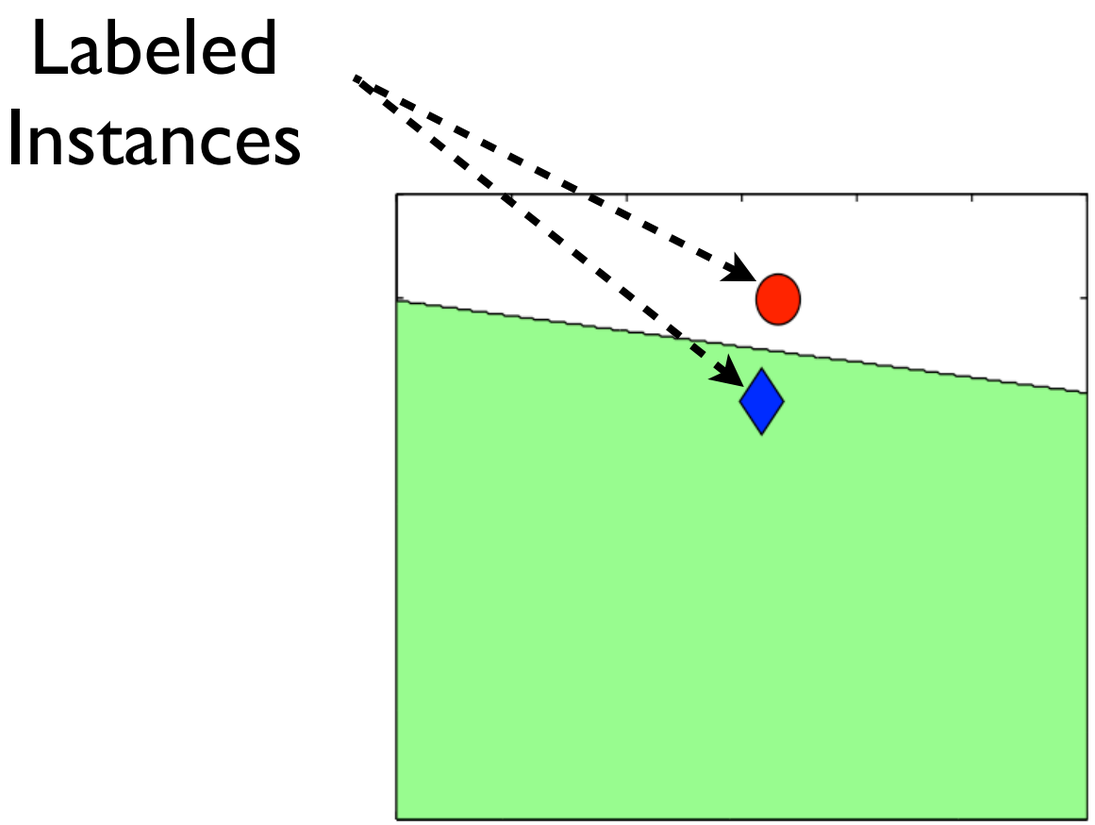

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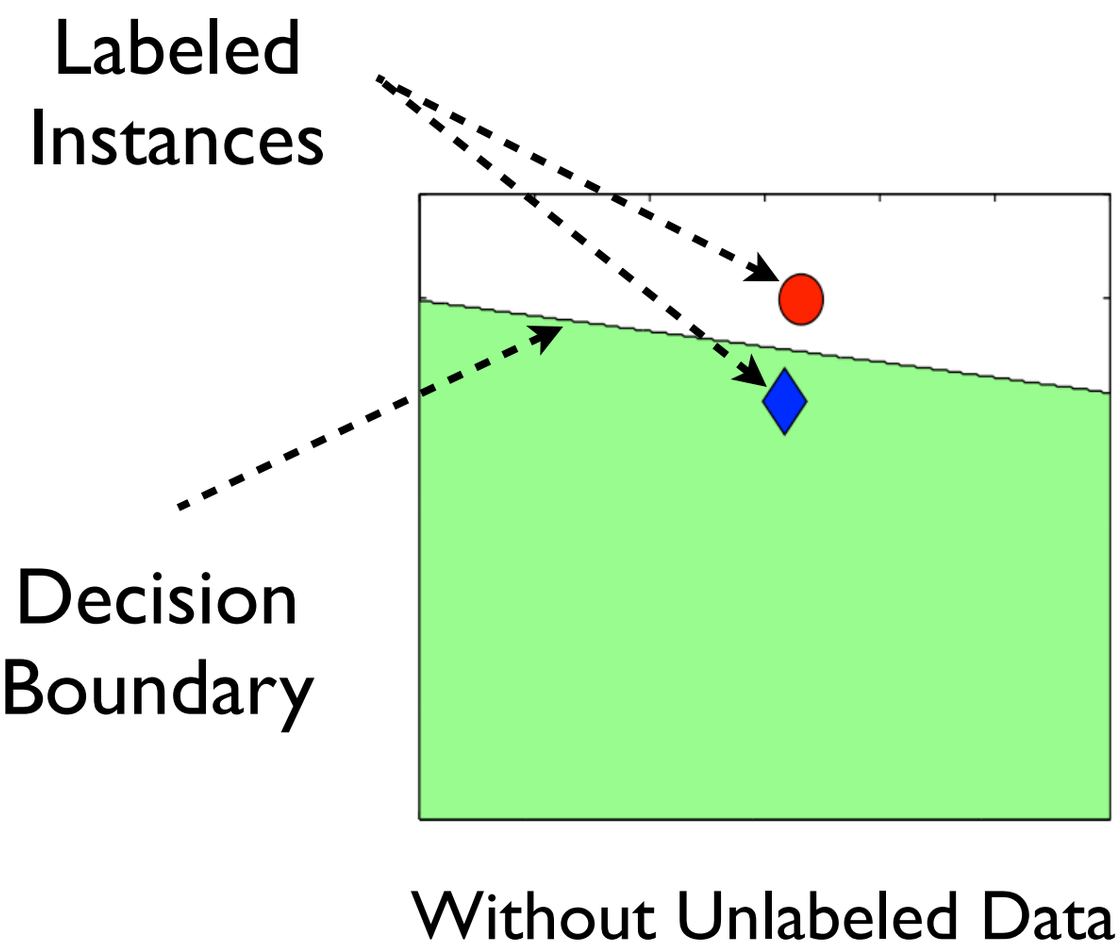

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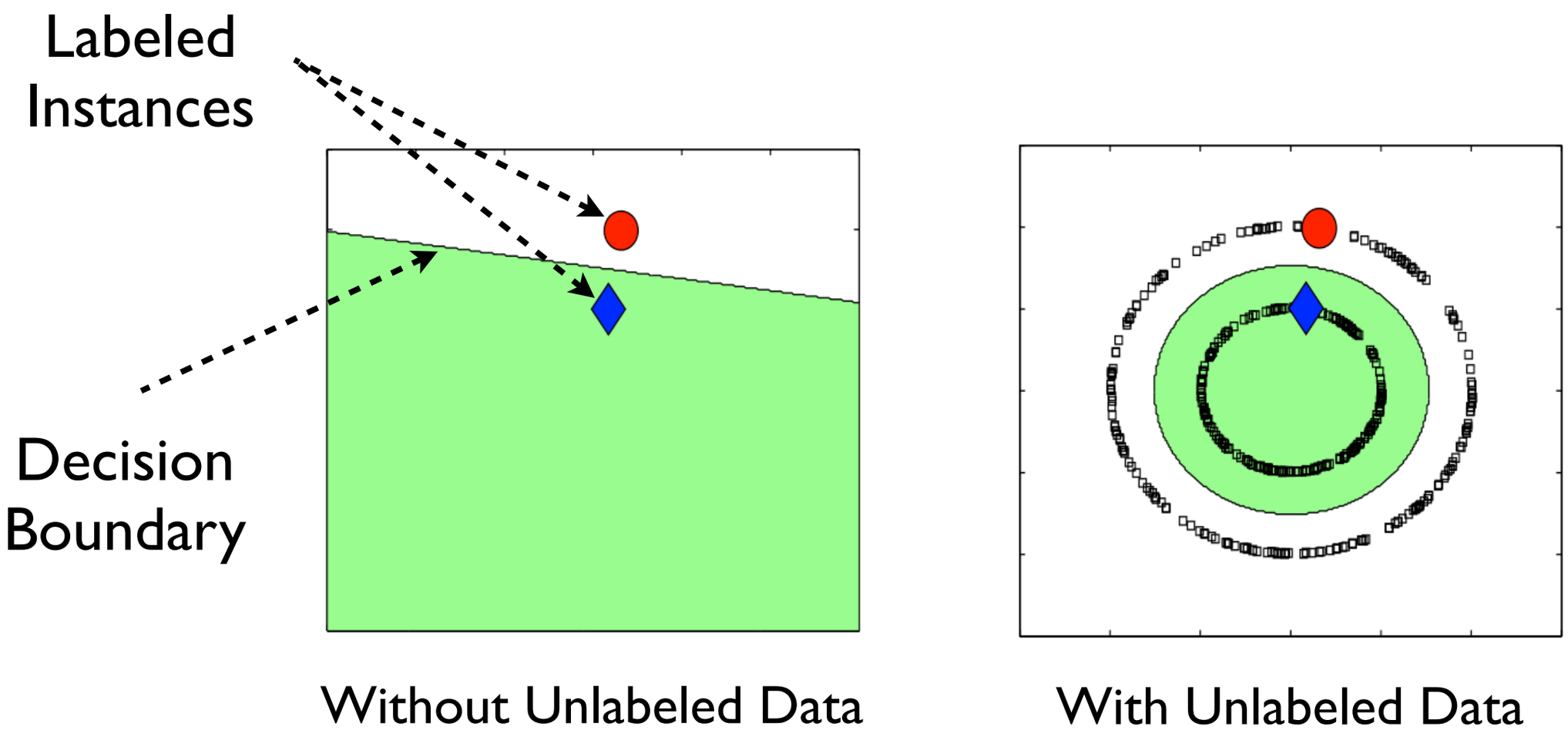

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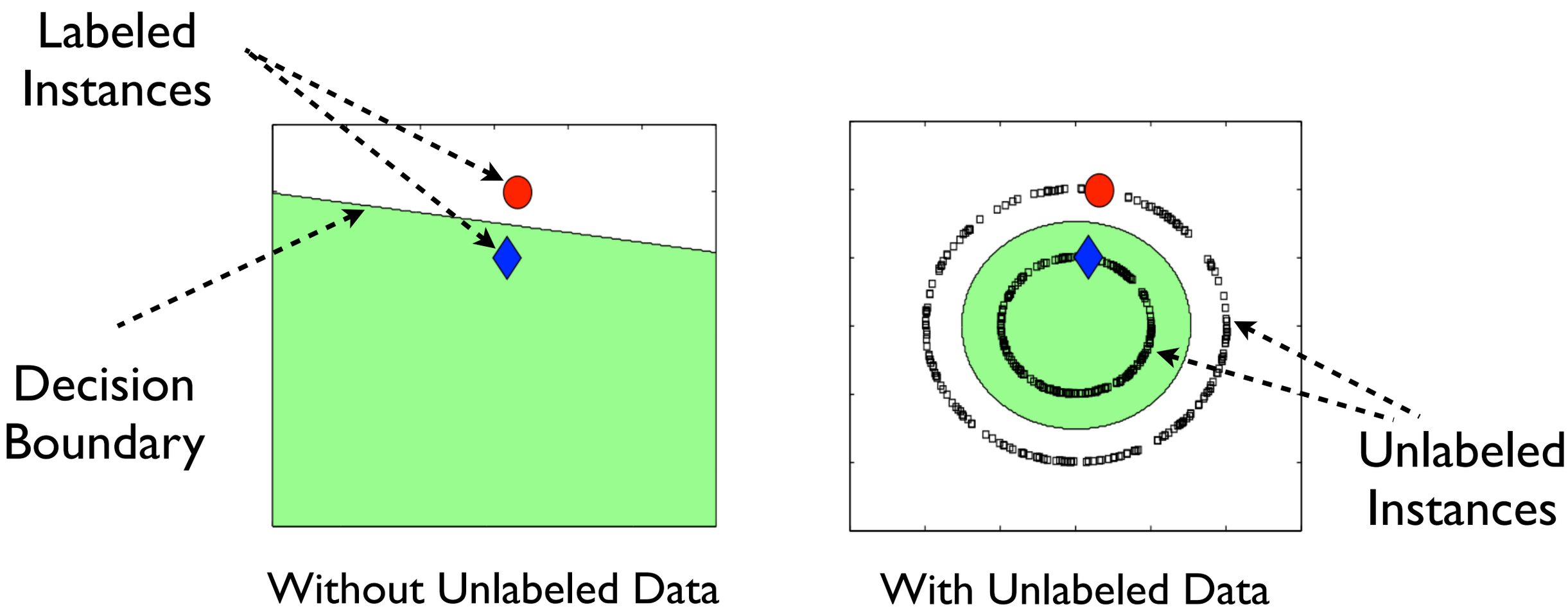

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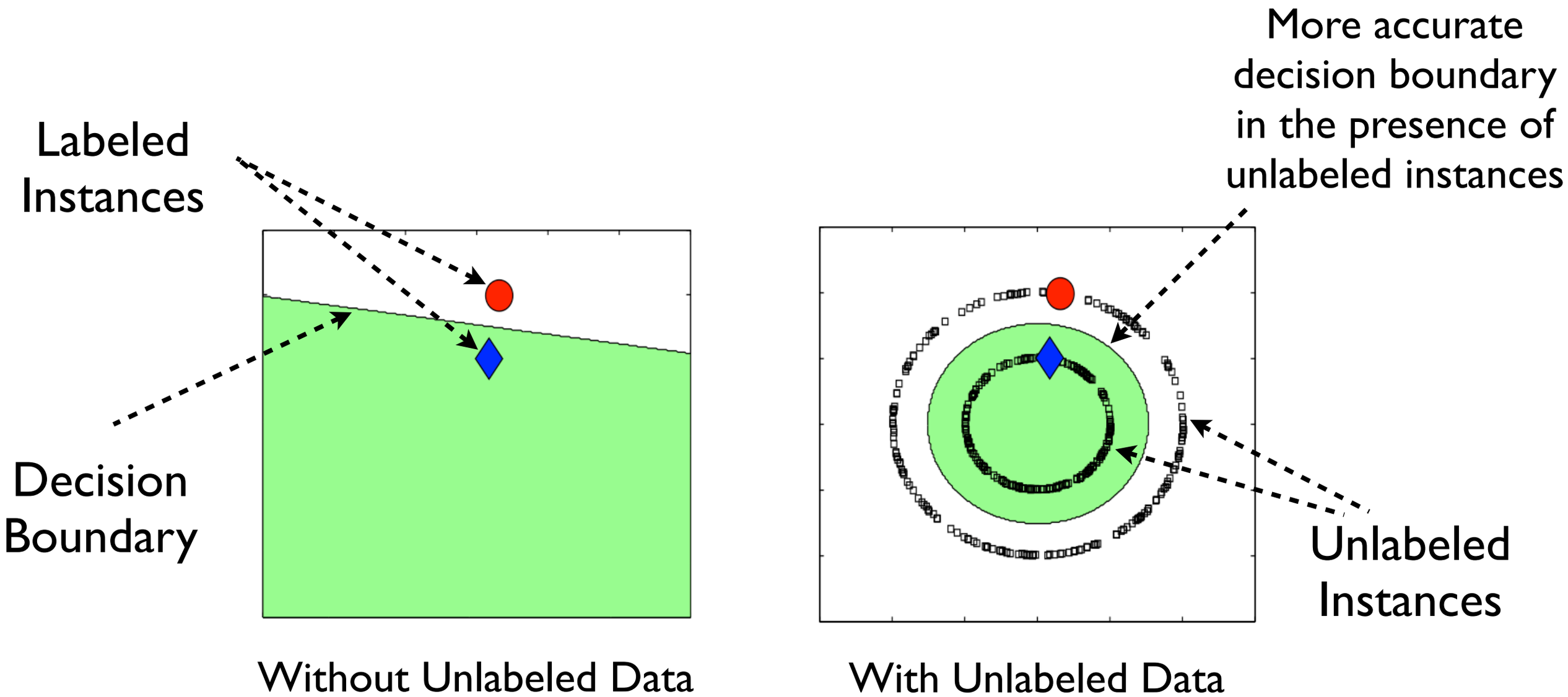

Example from [Belkin et al., JMLR 2006]

Inductive vs Transductive

Inductive vs Transductive

Supervised
(Labeled)

Semi-supervised
(Labeled + Unlabeled)

Inductive vs Transductive

	Inductive (Generalize to Unseen Data)	Transductive (Doesn't Generalize to Unseen Data)
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See Chapter 25 of SSL Book: http://olivier.chapelle.cc/ssl-book/discussion.pdf

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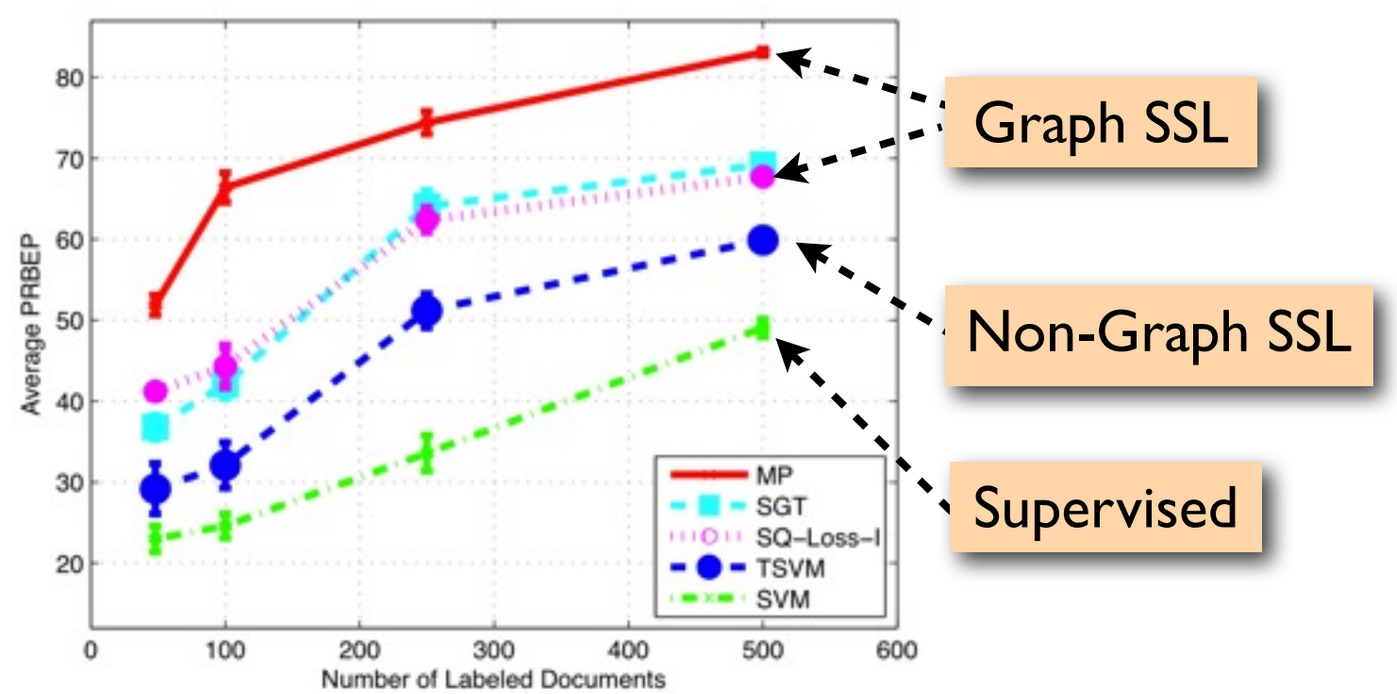
Text Classification



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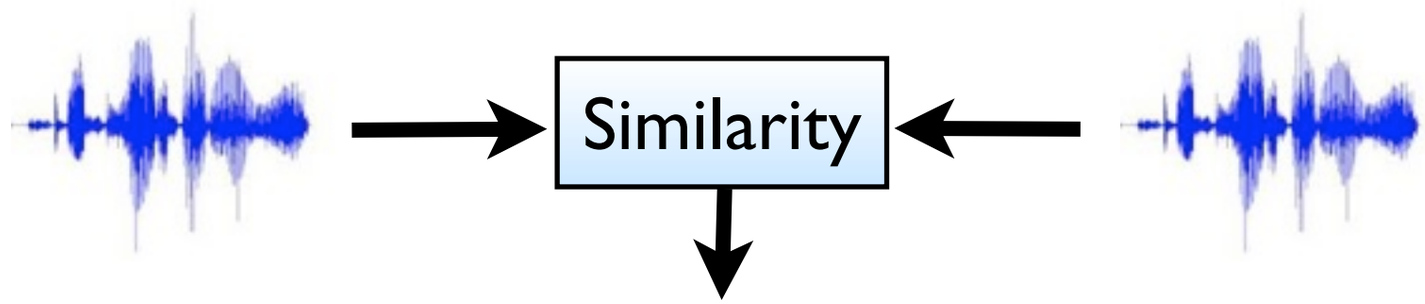

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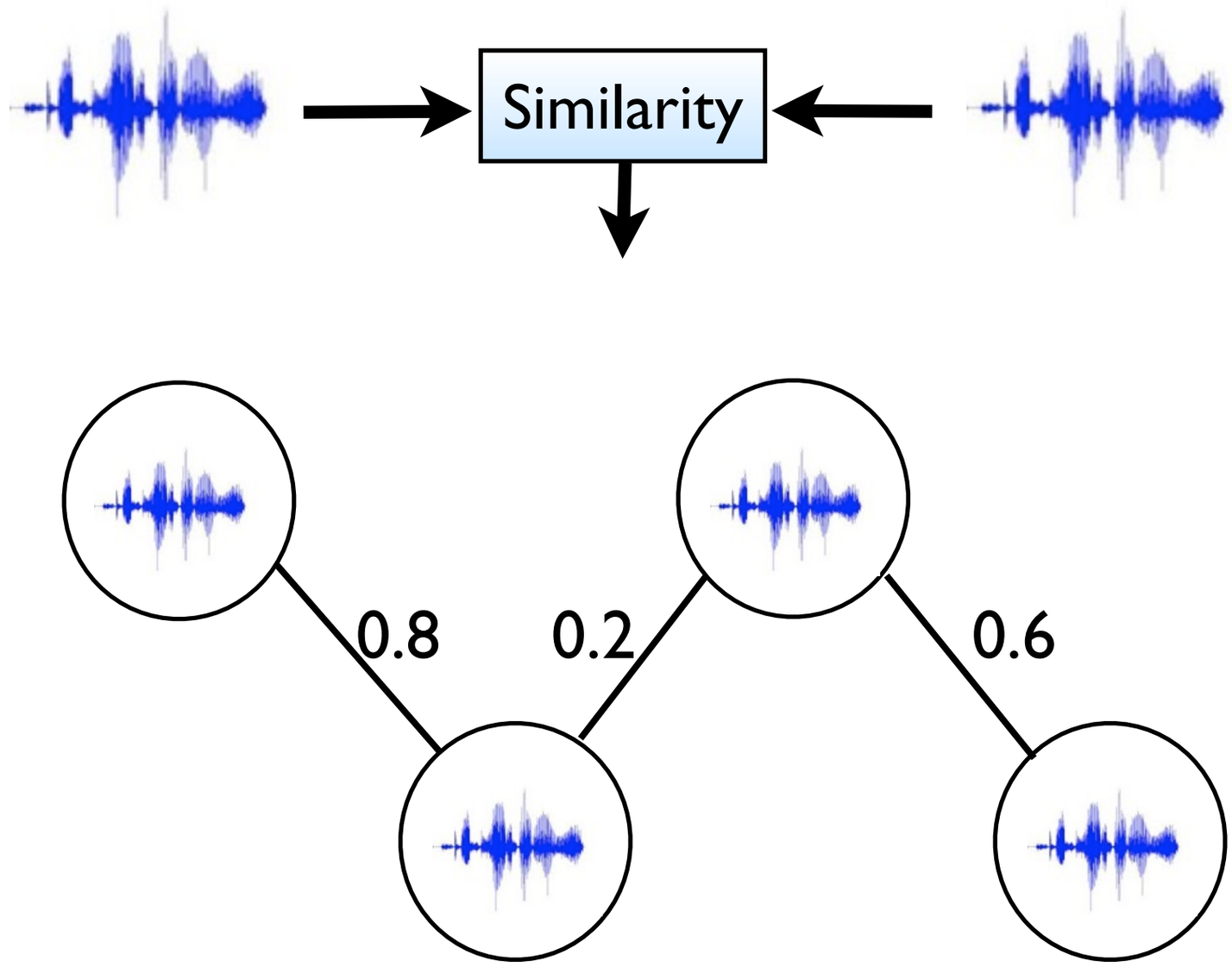

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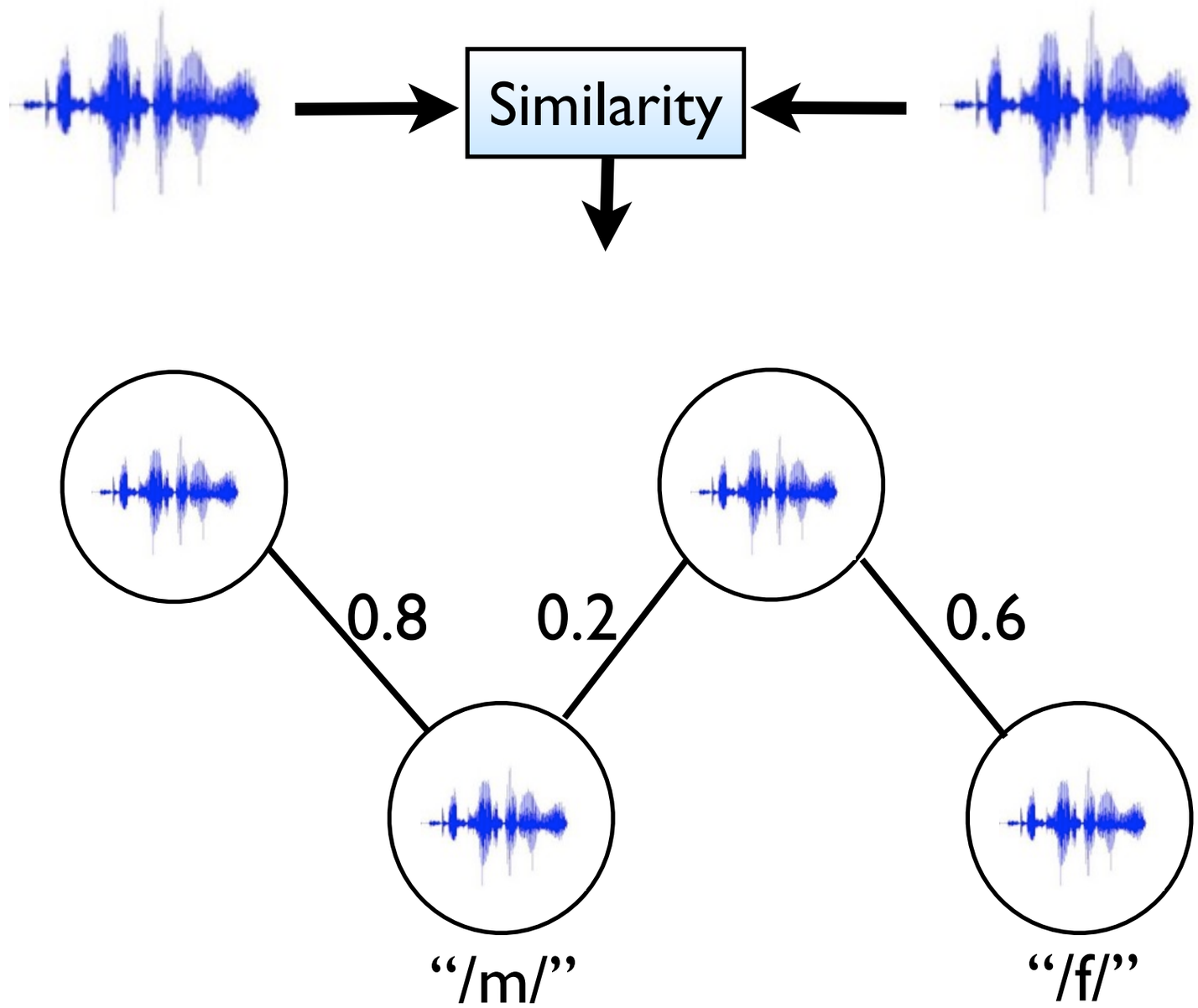

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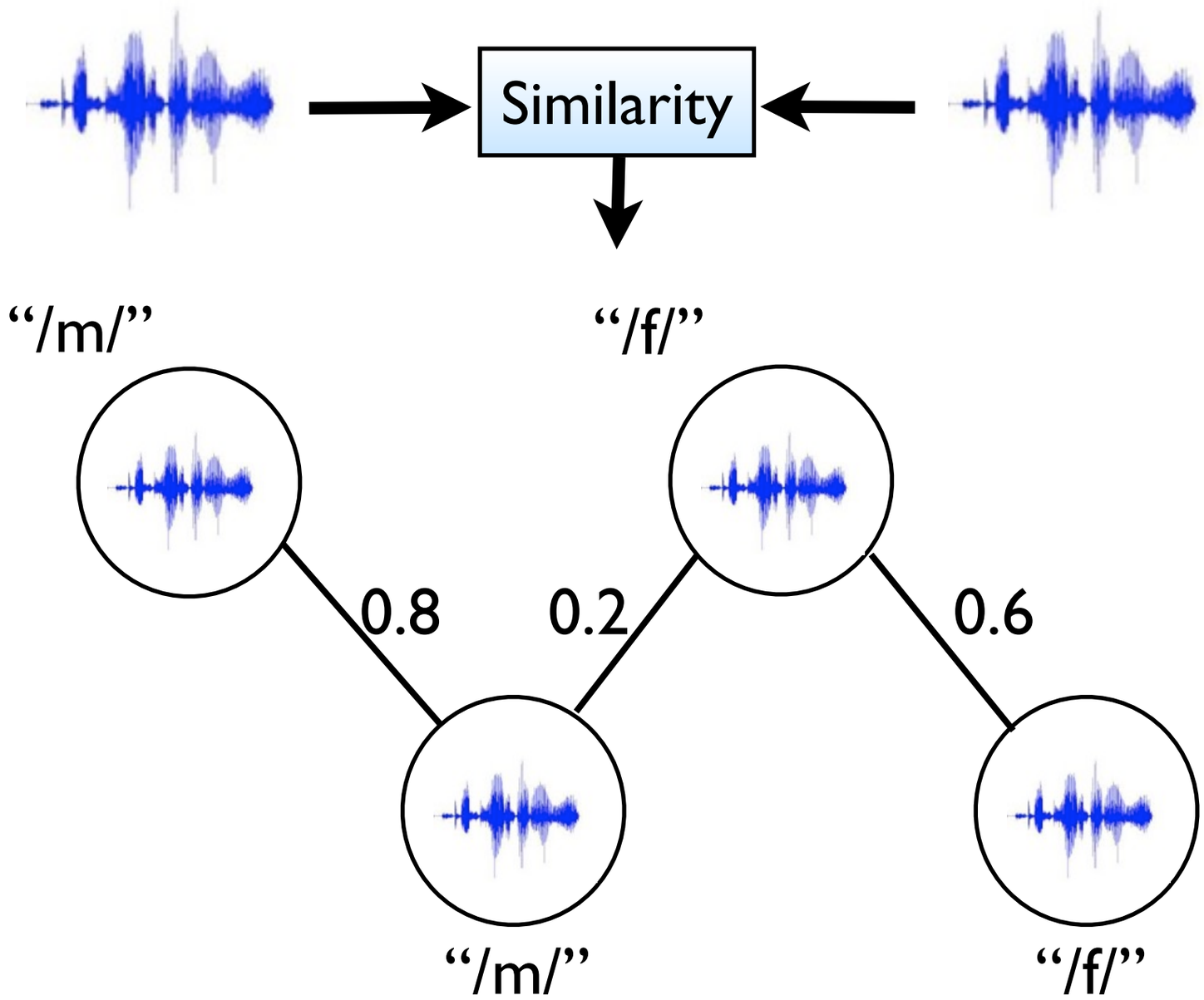

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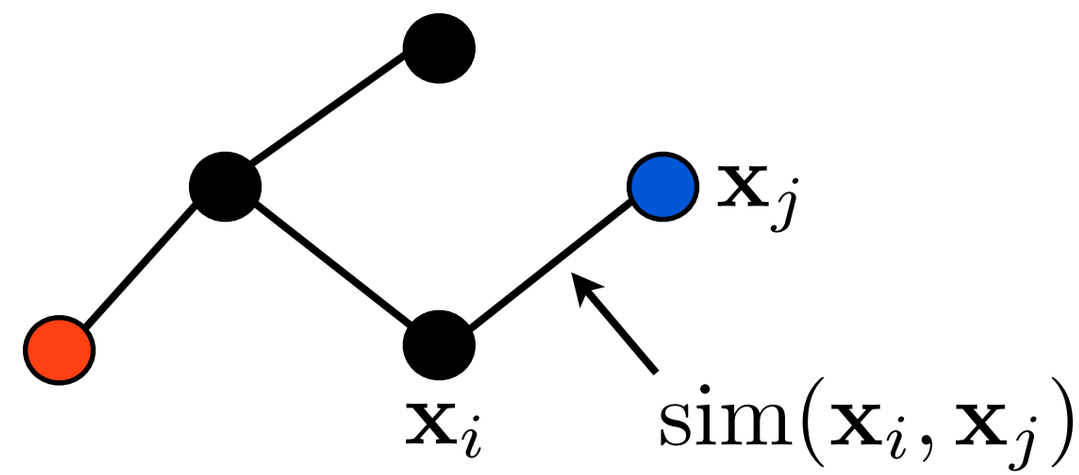

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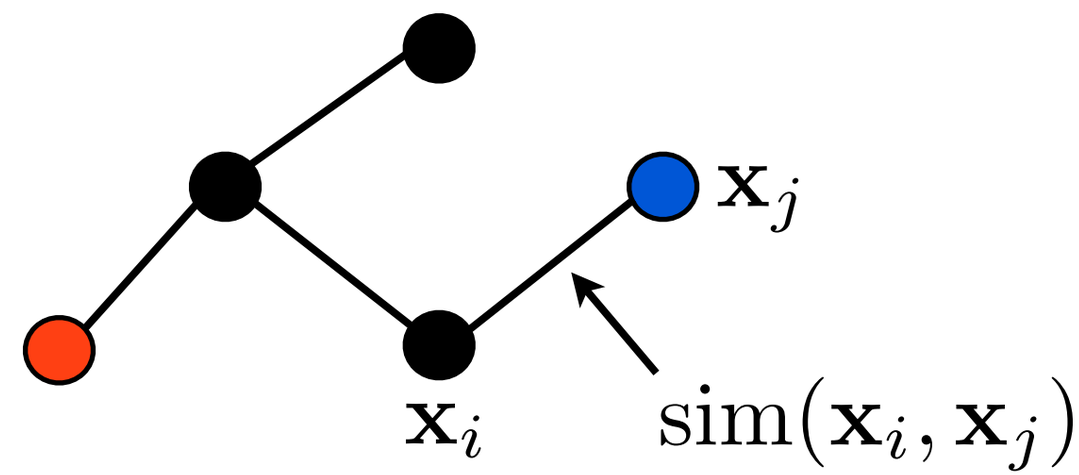

- Two stages
  - Graph construction (if not already present)
  - Label Inference

Outline

- Motivation
- Graph Construction
- Inference Methods
- Scalability
- Applications
- Conclusion & Future Work

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Graph Construction

- Neighborhood Methods
  - k-NN Graph Construction (k-NNG)
  - e-Neighborhood Method
- Metric Learning
- Other approaches

Neighborhood Methods

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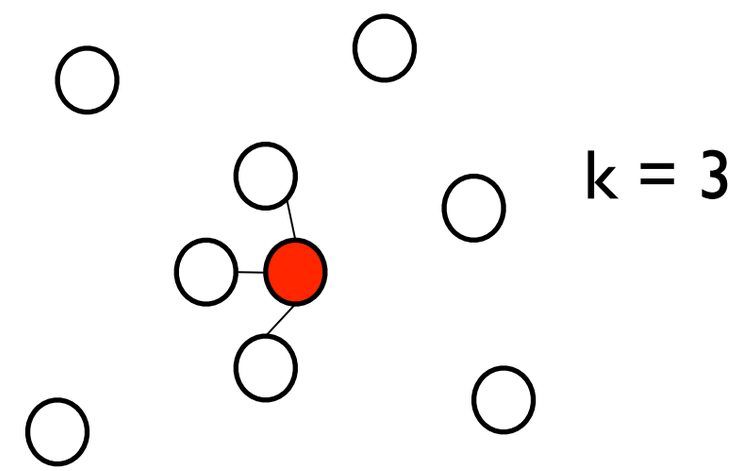

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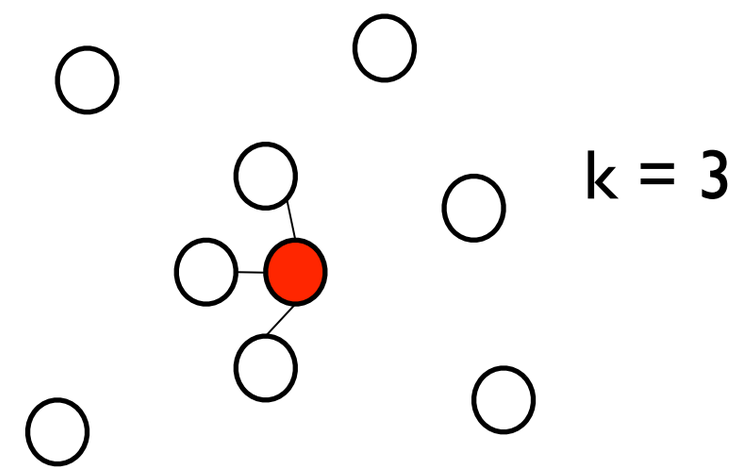

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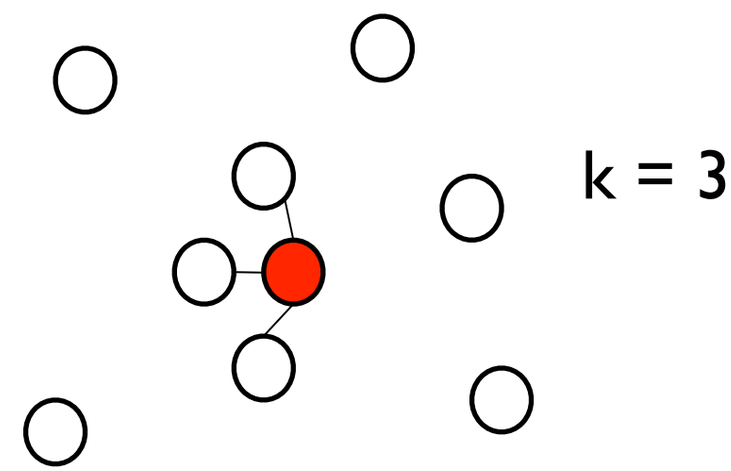

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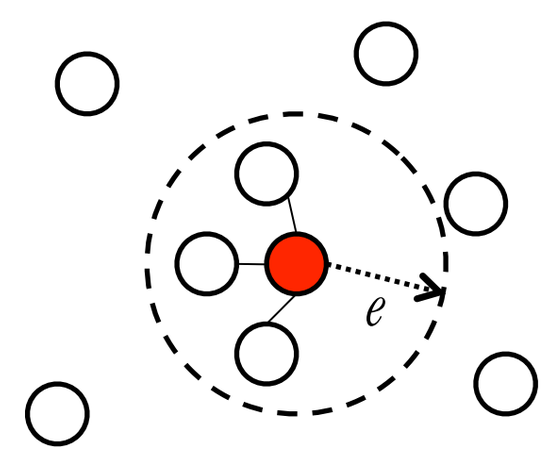

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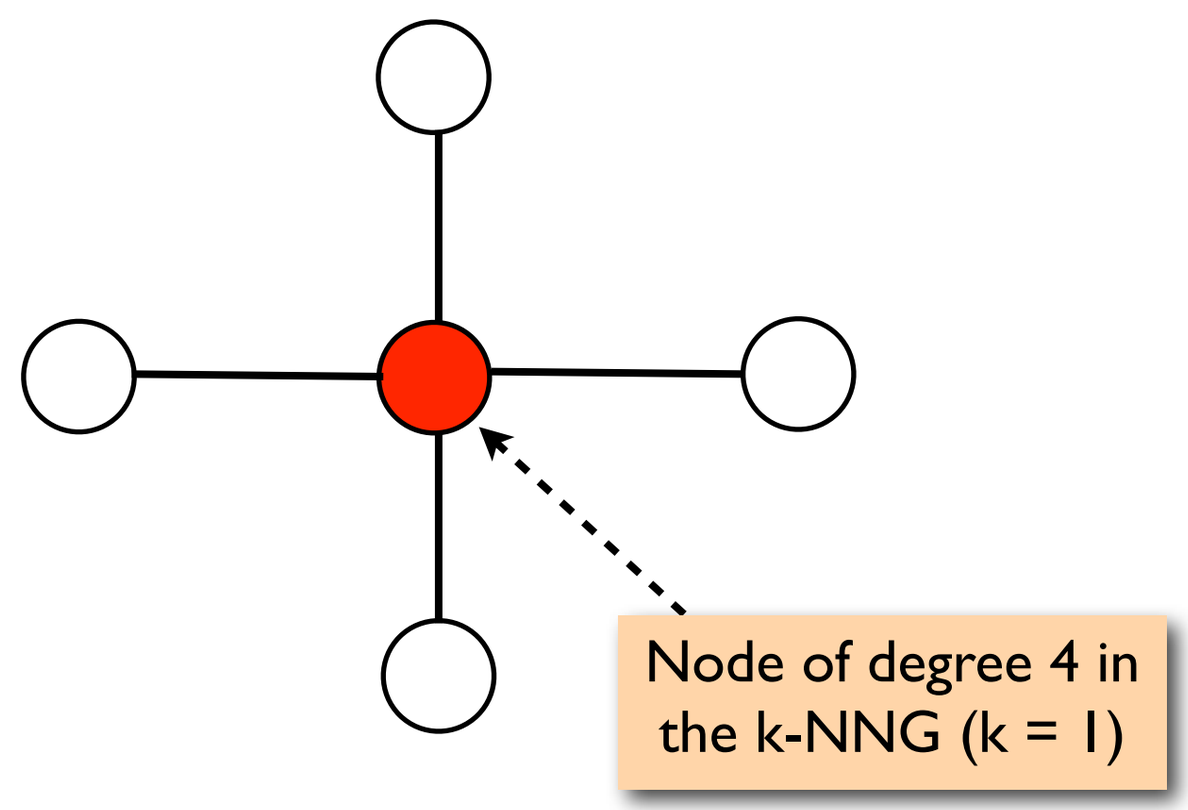

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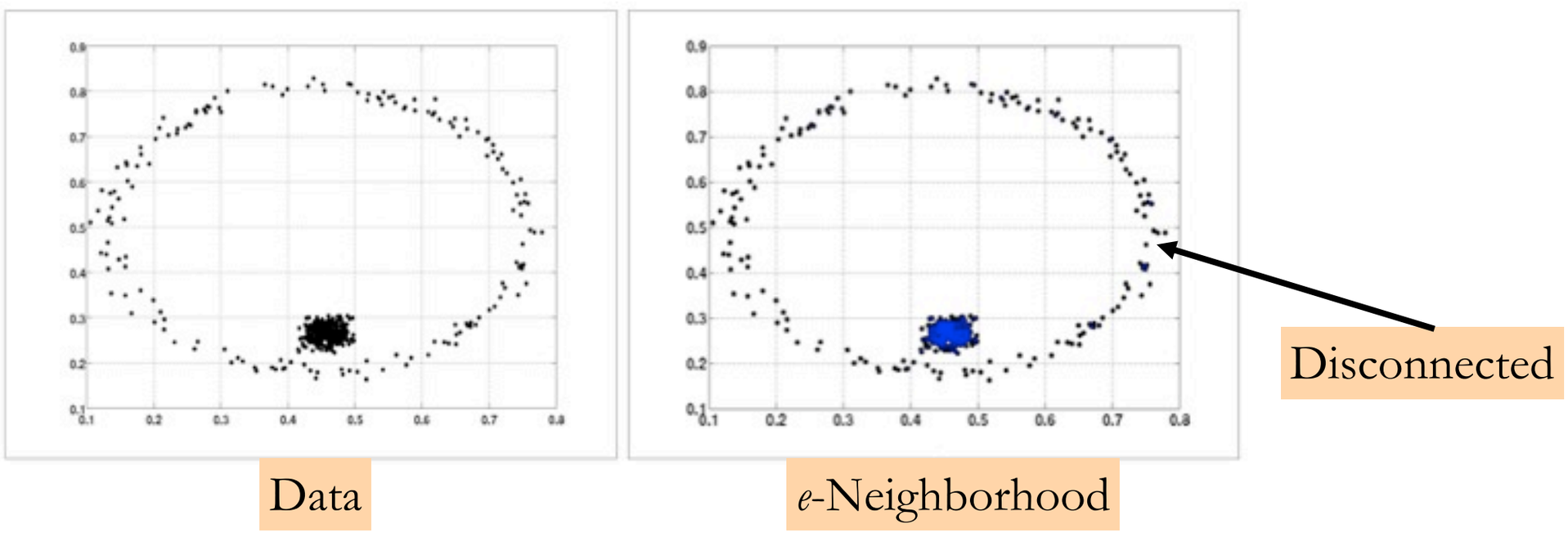

Figure from [Jebara et al., ICML 2009]

Graph Construction using Metric Learning

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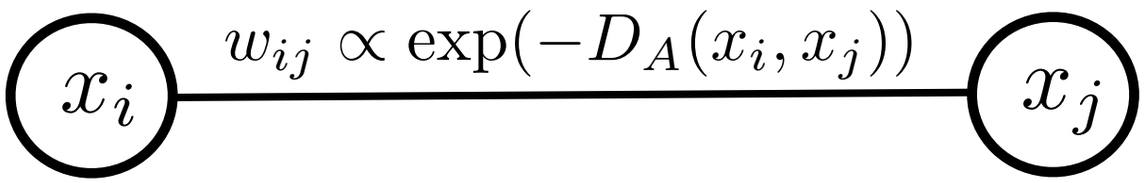

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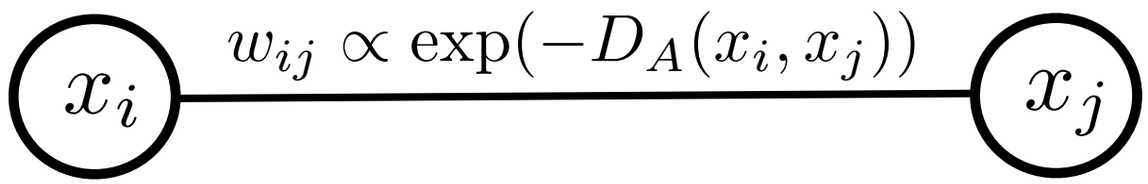

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Estimated using Mahalanobis metric learning algorithms

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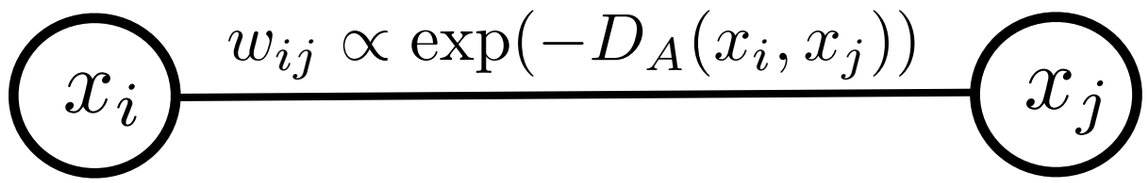

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Estimated using Mahalanobis metric learning algorithms

- Supervised Metric Learning
  - ITML [Kulis et al., ICML 2007]
  - LMNN [Weinberger and Saul, JMLR 2009]
- Semi-supervised Metric Learning
  - IDML [Dhillon et al., UPenn TR 2010]

Benefits of Metric Learning for Graph Construction

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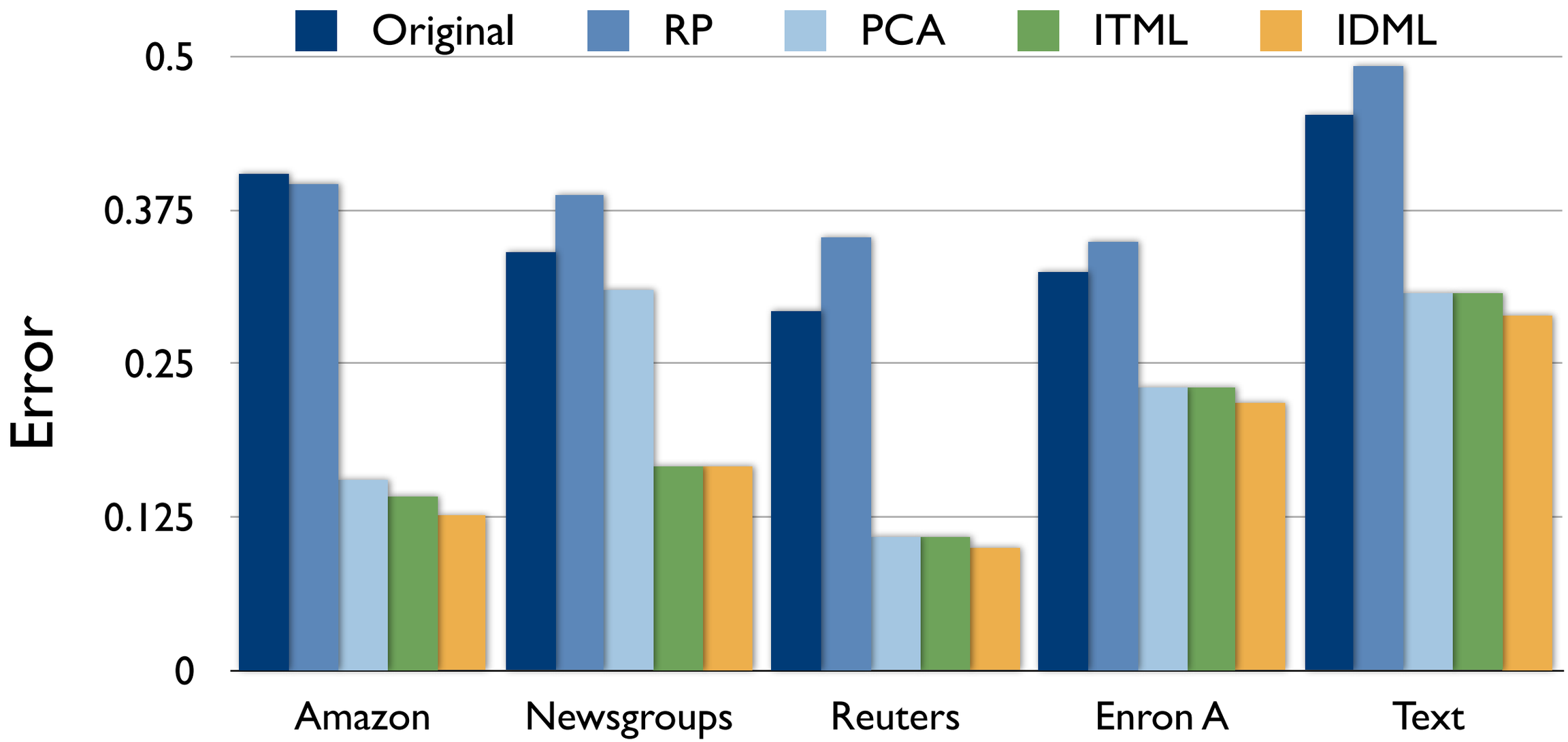

100 seed and1400 test instances, all inferences using LP

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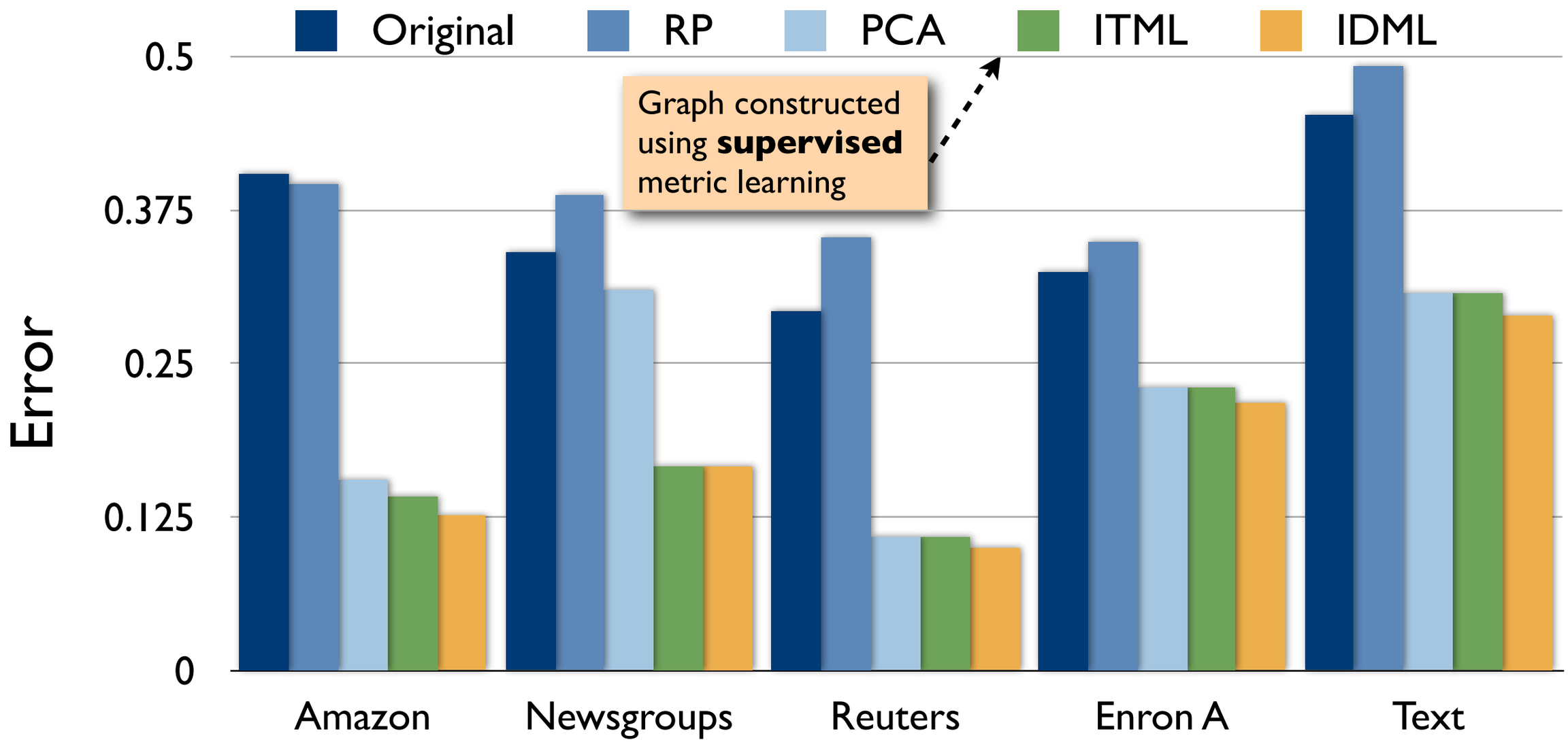

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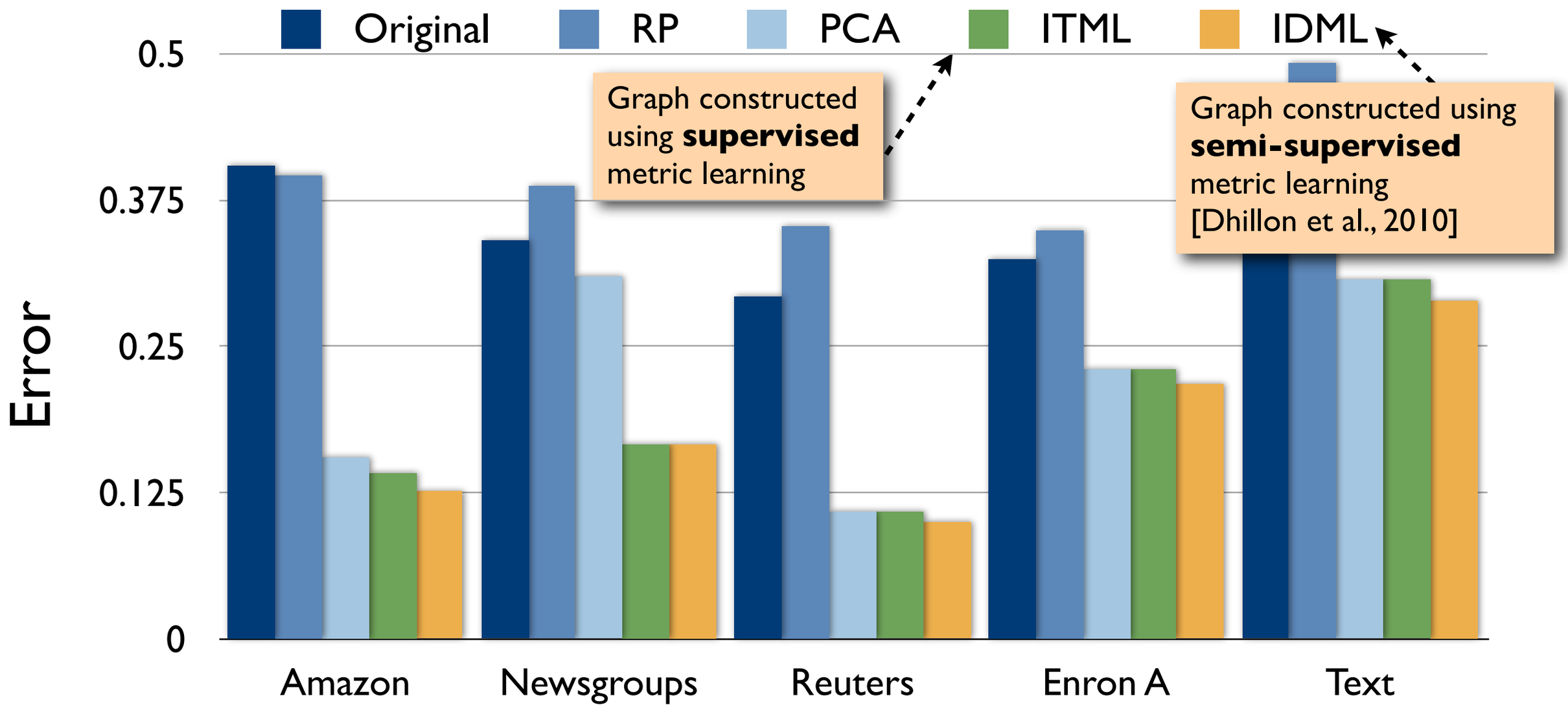

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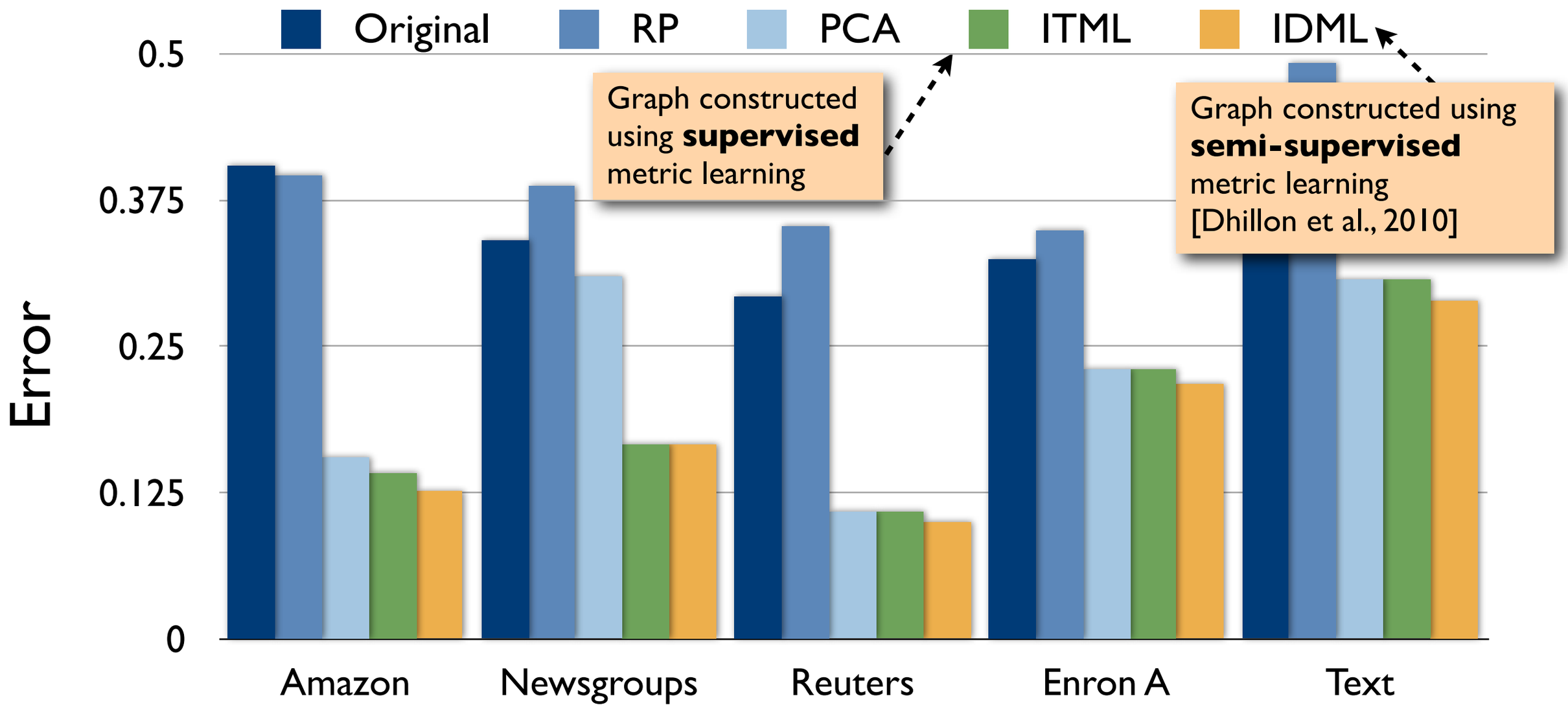

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Careful graph construction is critical!

[Dhillon et al., UPenn TR 2010]

Other Graph Construction Approaches

- Local Reconstruction
  - Linear Neighborhood [Wang and Zhang, ICML 2005]
  - Regular Graph: b-matching [Jebara et al., ICML 2008]
  - Fitting Graph to Vector Data [Daitch et al., ICML 2009]
- Graph Kernels
  - [Zhu et al., NIPS 2005]

Outline

- Motivation
- Graph Construction
- Inference Methods
  - Label Propagation
  - Modified Adsorption
  - Transduction with Confidence
  - Manifold Regularization
  - Measure Propagation
  - Sparse Label Propagation
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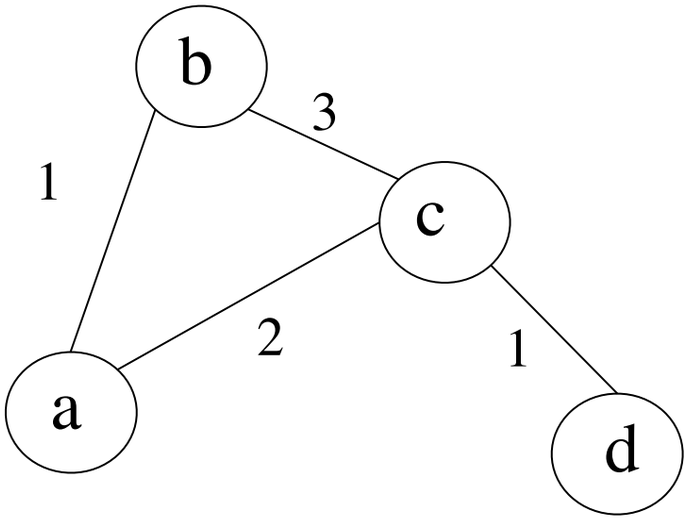

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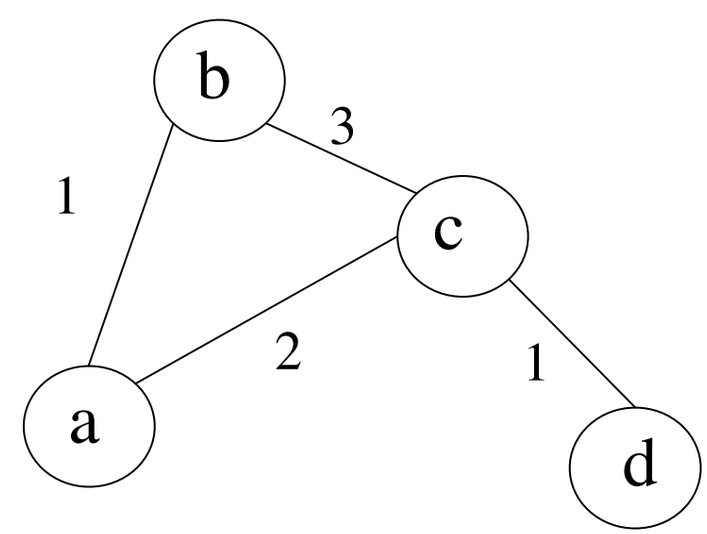

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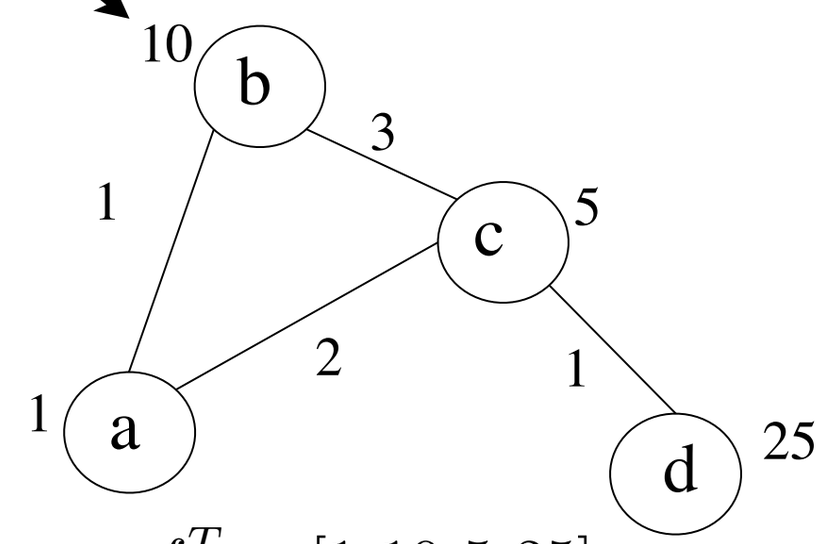

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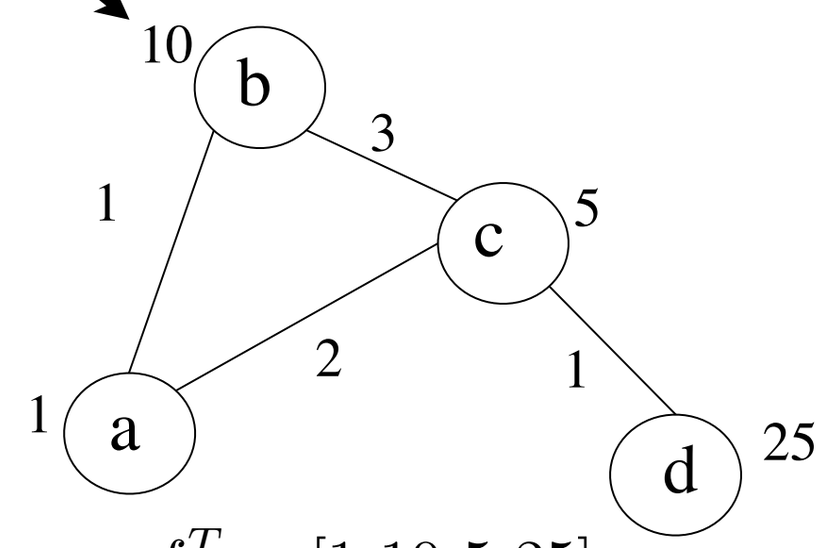

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$f^T L f = 588$

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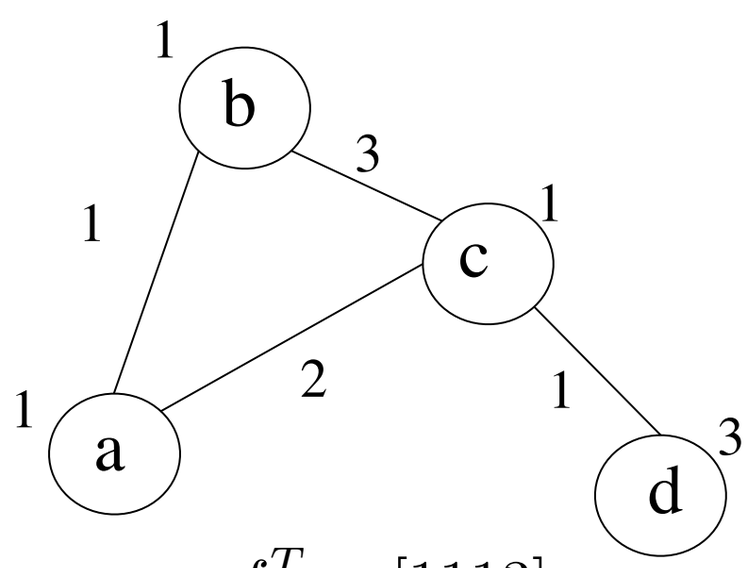
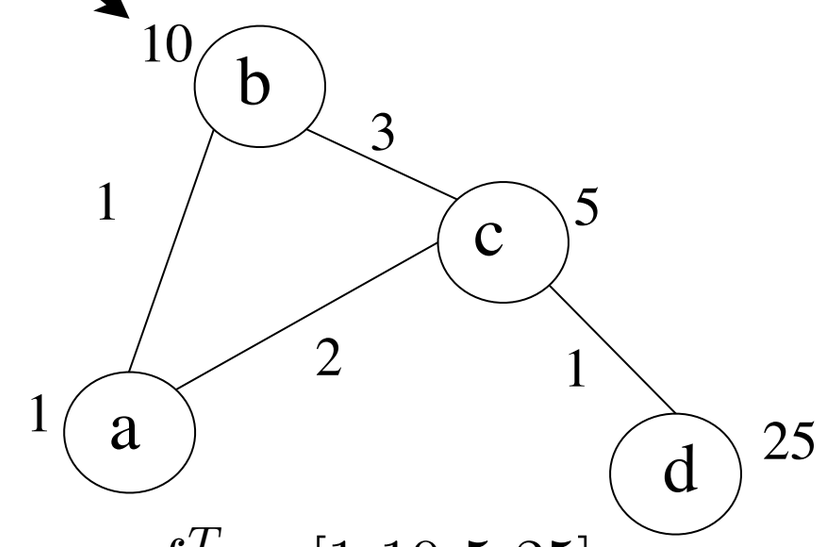


$f^T = [1113]$
$f^T L f = 4$



$f^T = [1\ 10\ 5\ 25]$
$f^T L f = 588$

Not Smooth

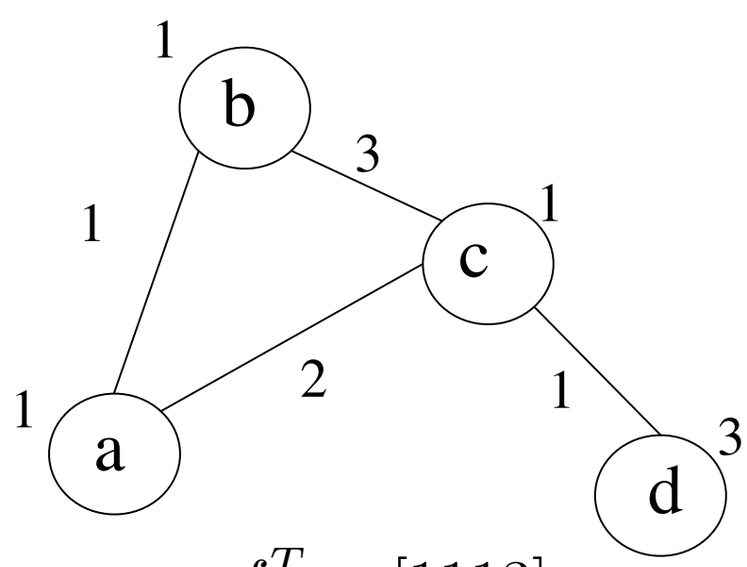
Graph Laplacian (contd.)

- L is positive semi-definite (assuming non-negative weights)
- Smoothness of prediction f over the graph in terms of the Laplacian:

Vector of scores for single label on nodes

Measure of Non-Smoothness

$$f^T L f = \sum_{i,j} W_{ij}(f_i - f_j)^2$$



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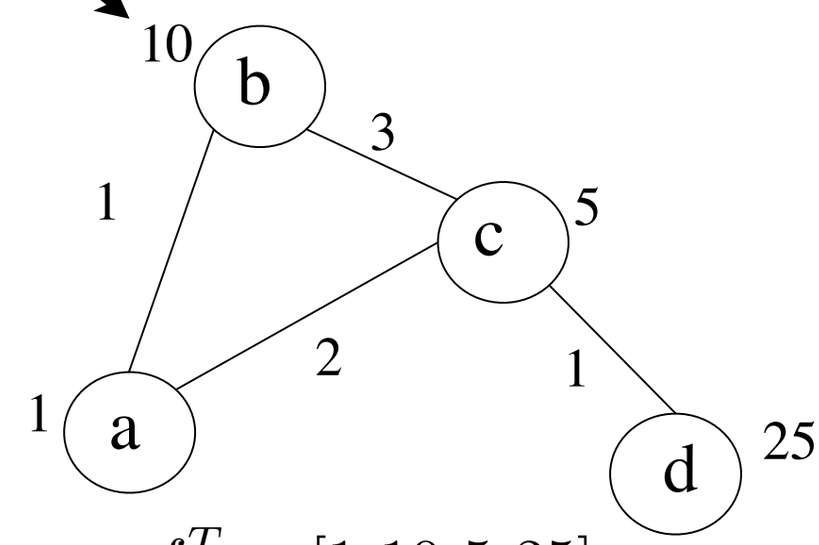

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Not Smooth

Relationship between Eigenvalues of the Laplacian and Smoothness

$$Lg = \lambda g$$

$$g^T L g = \lambda g^T g$$

$$g^T L g = \lambda$$

Relationship between Eigenvalues of the Laplacian and Smoothness

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Measure of Non-Smoothness (previous slide)

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Measure of Non-Smoothness (previous slide)

If an eigenvector is used to classify nodes, then the corresponding eigenvalue gives the measure of non-smoothness

Spectrum of the Graph Laplacian

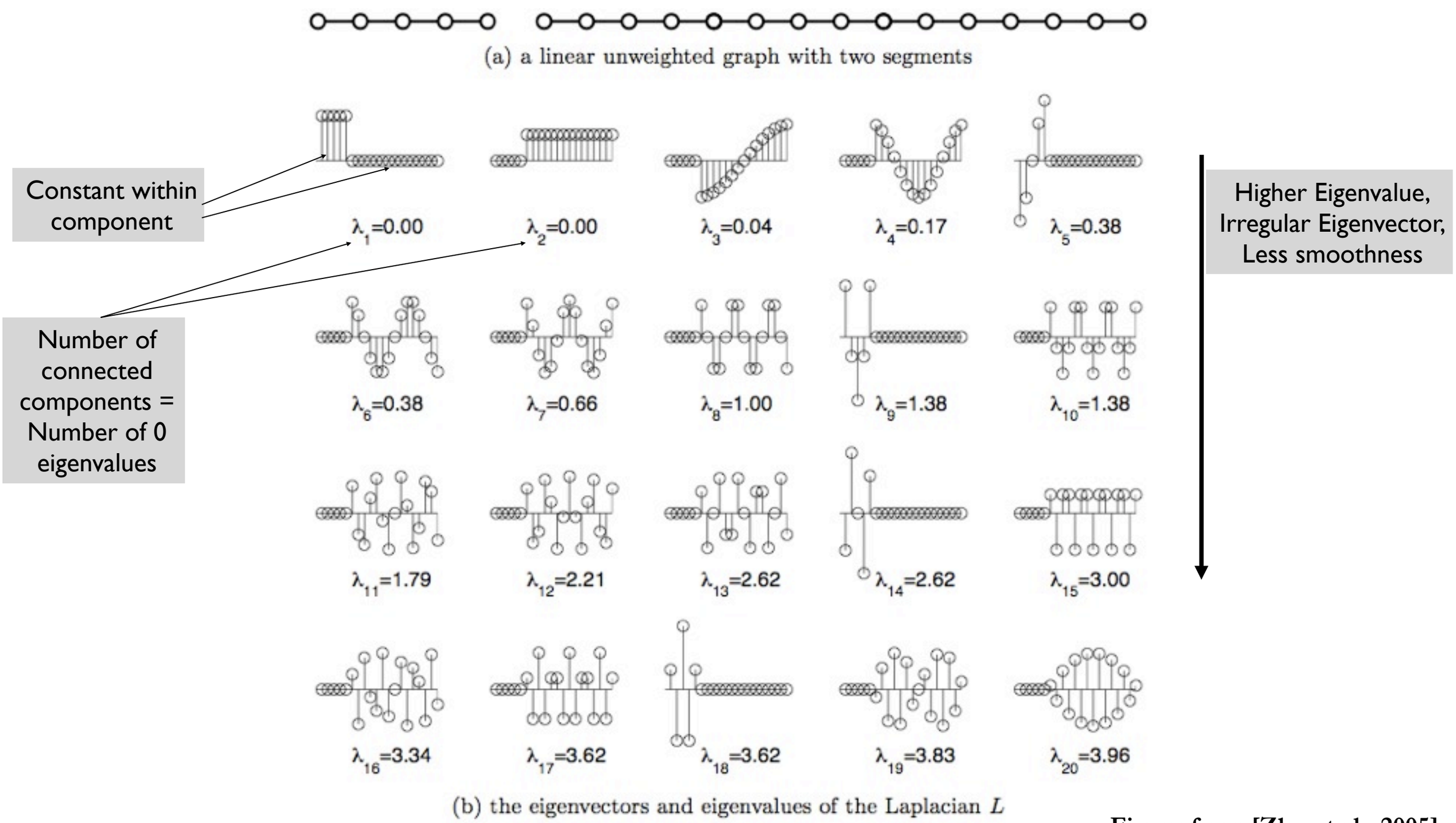
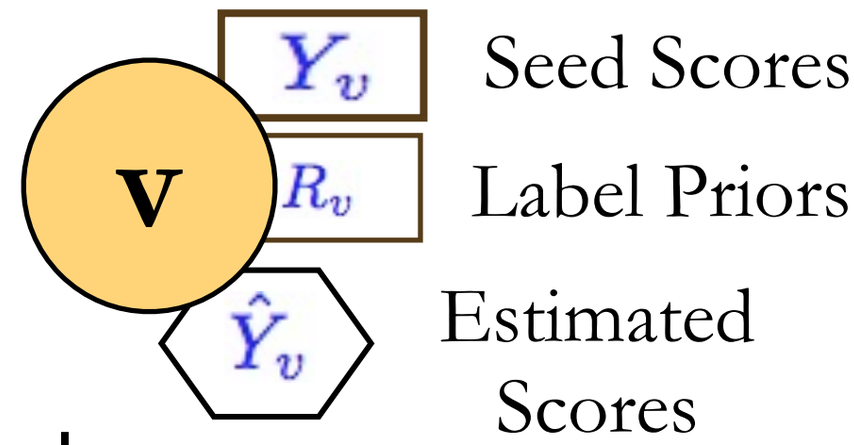


Figure from [Zhu et al., 2005]

Notations

$\hat{Y}_{v,l}$: score of estimated label l on node v



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$R_{v,l}$: regularization target for label l on node v

S : seed node indicator (diagonal matrix)

W_{uv} : weight of edge (u, v) in the graph

Outline

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- Graph Construction
- Inference Methods
  - Label Propagation
  - Modified Adsorption
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  - Manifold Regularization
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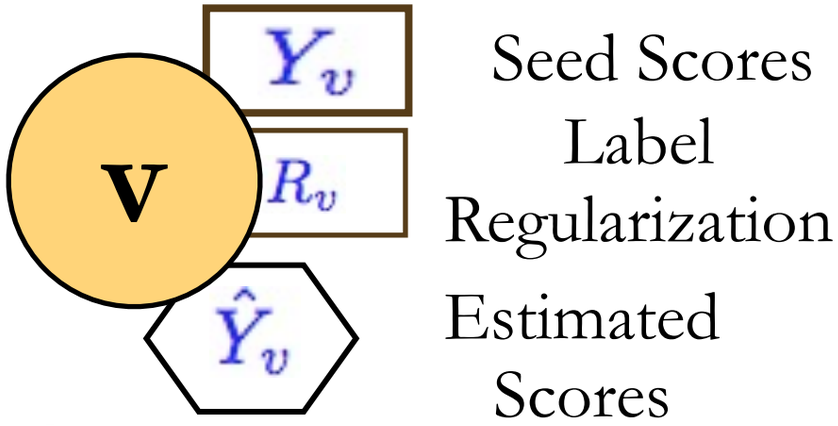

LP-ZGL [Zhu et al., ICML 2003]

$$\arg\min_{\hat{Y}} \sum_{l=1}^{m} W_{uv}(\hat{Y}_{ul} - \hat{Y}_{vl})^2 = \sum_{l=1}^{m} \hat{Y}_l^T L \hat{Y}_l$$

Graph Laplacian

such that $Y_{ul} = \hat{Y}_{ul}, \ \forall S_{uu} = 1$

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Match Seeds (hard)

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  - two nodes connected by an edge with high weight should be assigned similar labels
- Solution satisfies harmonic property

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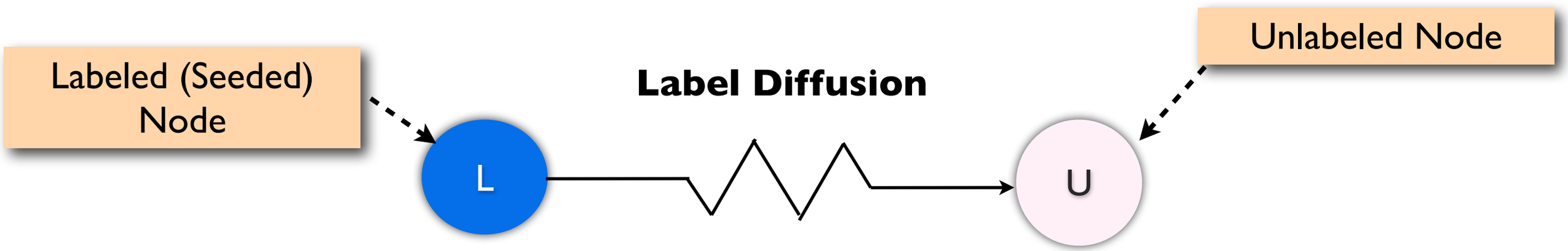
Two Related Views



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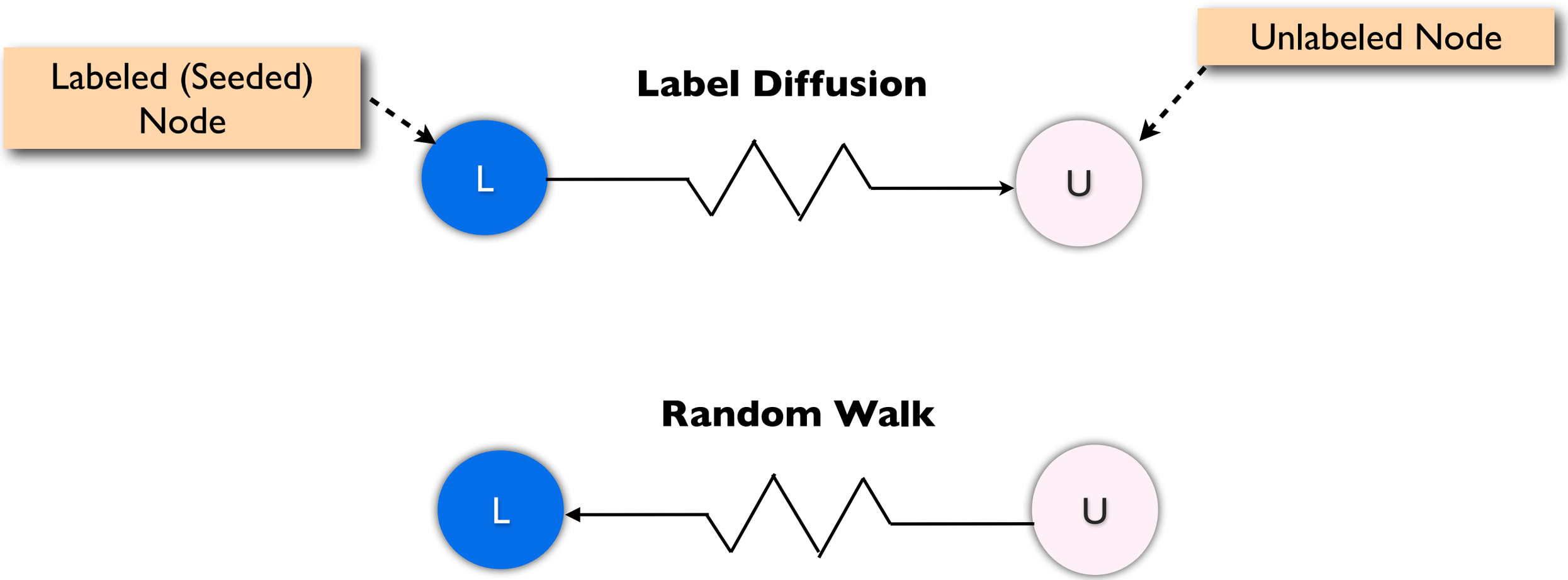
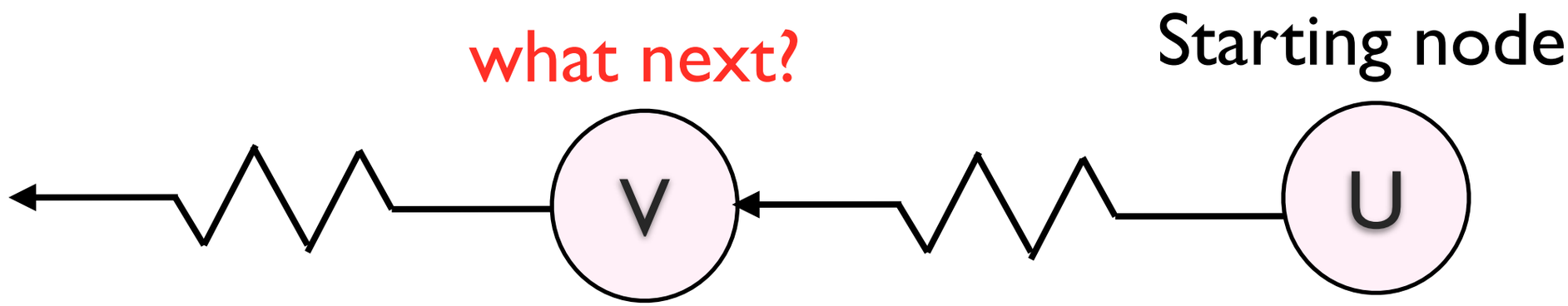


Random Walk View

Random Walk View



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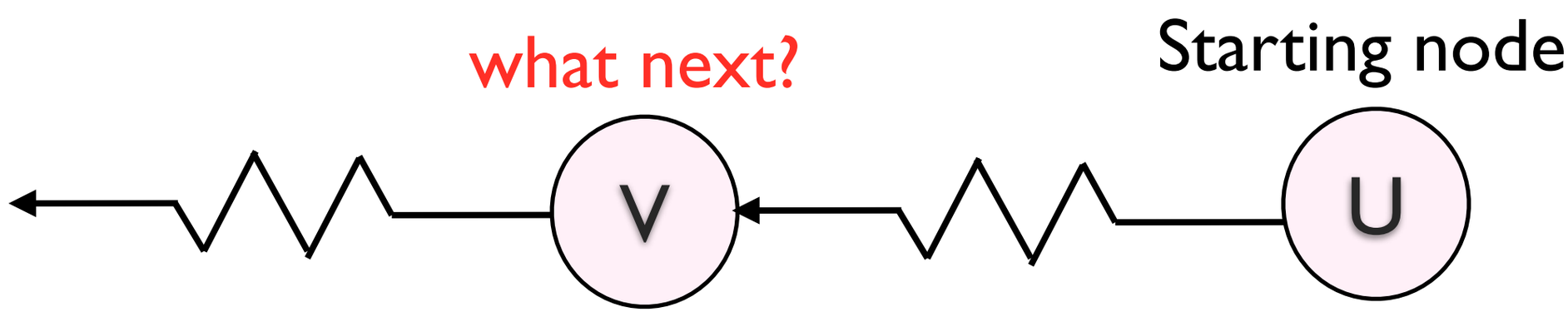

- Continue walk with probability $\mathbf{p_v^{cont}}$
- Assign V's seed label to U with probability $\mathbf{p_v^{inj}}$
- Abandon random walk with probability $\mathbf{p_v^{abnd}}$
  - assign U a dummy label

Discounting Nodes

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- Certain nodes can be unreliable (e.g., high degree nodes)
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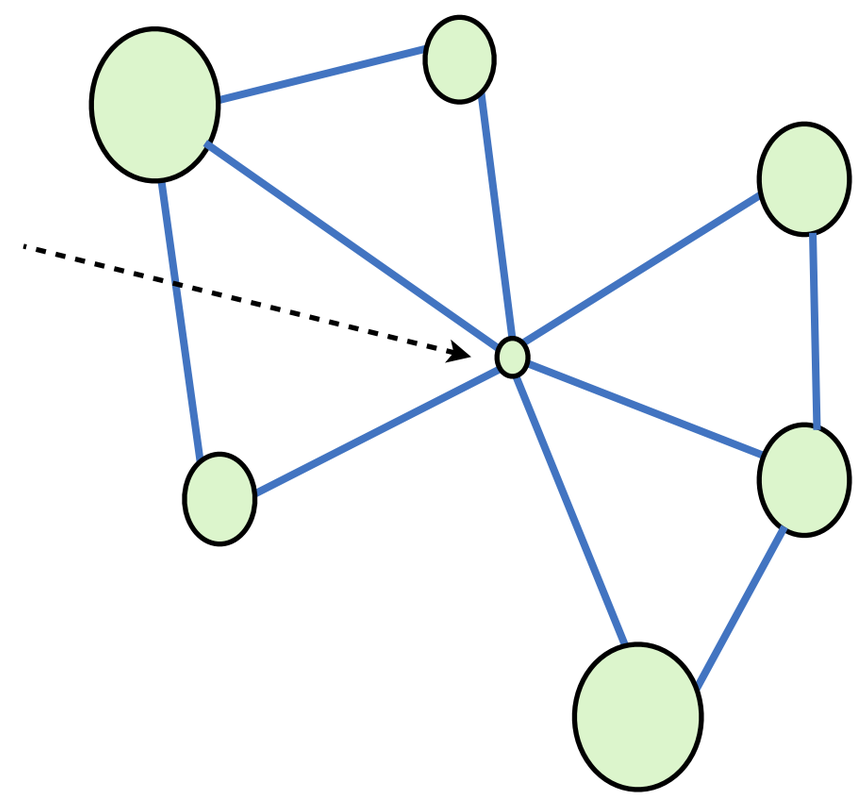
$$\mathbf{p_v^{abnd}} \propto \mathrm{degree(v)}$$



Redefining Matrices

$$W'_{uv} = p_u^{cont} \times W_{uv}$$

New Edge Weight

$$S_{uu} = \sqrt{p_u^{inj}}$$

$$R_{u\top} = p_u^{abnd}$$, and 0 for non-dummy labels

Dummy Label

Modified Adsorption (MAD)

[Talukdar and Crammer, ECML 2009]

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$$\arg\min_{\hat{\boldsymbol{Y}}} \sum_{l=1}^{m+1} \left[\|\boldsymbol{S}\hat{\boldsymbol{Y}}_l - \boldsymbol{S}\boldsymbol{Y}_l\|^2 + \mu_1 \sum_{u,v} \boldsymbol{M}_{uv}(\hat{\boldsymbol{Y}}_{ul} - \hat{\boldsymbol{Y}}_{vl})^2 + \mu_2 \|\hat{\boldsymbol{Y}}_l - \boldsymbol{R}_l\|^2 \right]$$

- m labels, $+1$ dummy label
- $\boldsymbol{M} = \boldsymbol{W}'^{\top} + \boldsymbol{W}'$ is the symmetrized weight matrix
- $\hat{\boldsymbol{Y}}_{vl}$: weight of label l on node v
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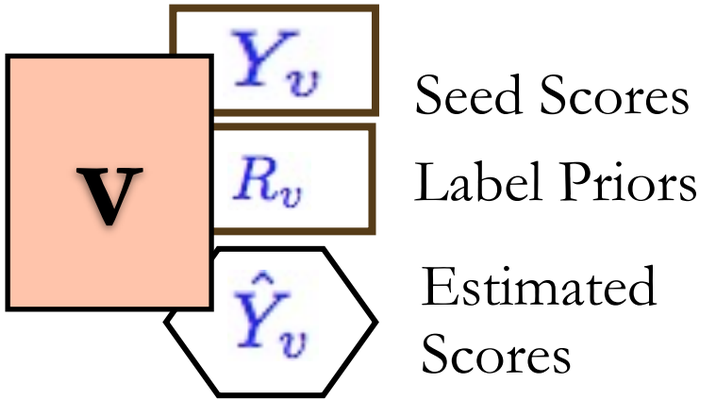

Modified Adsorption (MAD)

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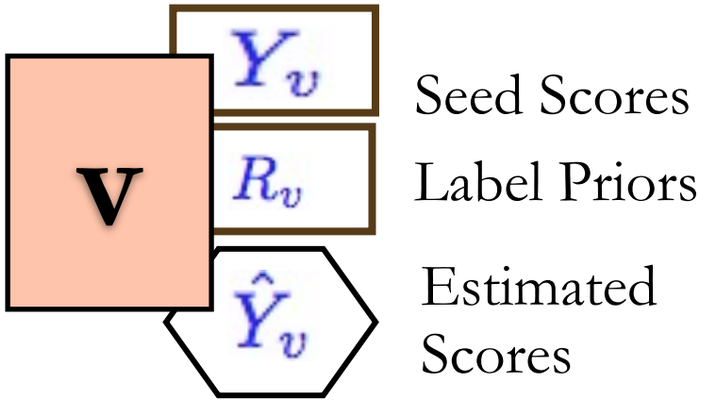

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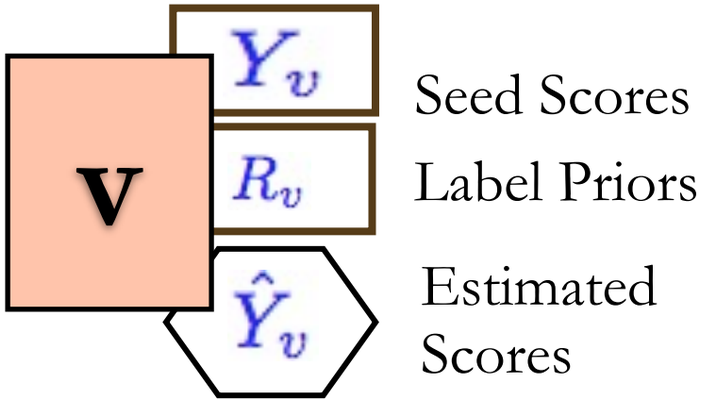

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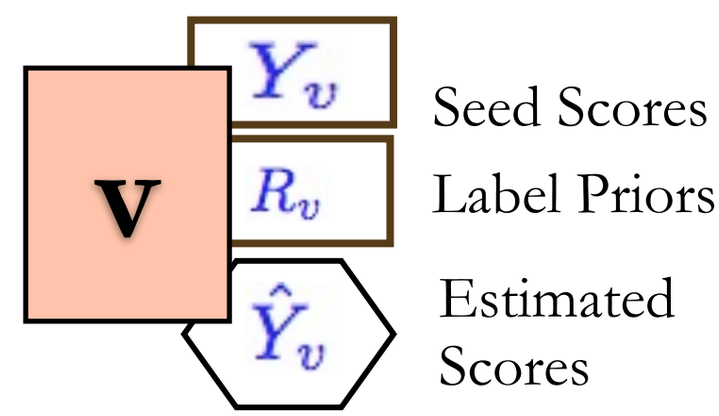

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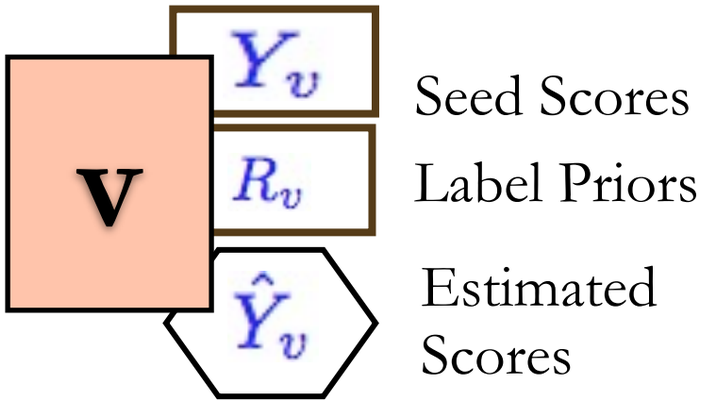

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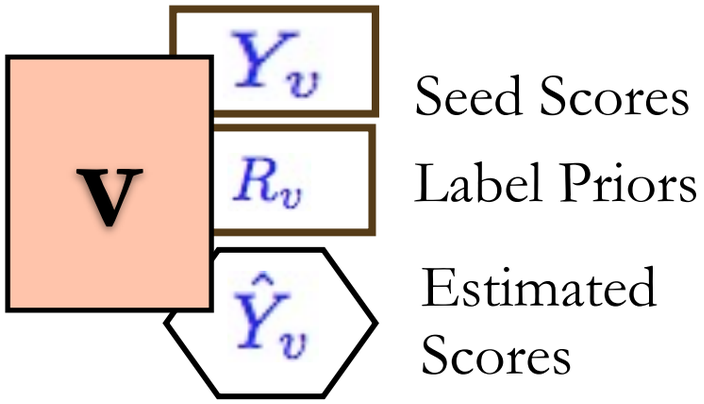

MAD has extra regularization compared to LP-ZGL [Zhu et al, ICML 03]; similar to QC [Bengio et al, 2006]

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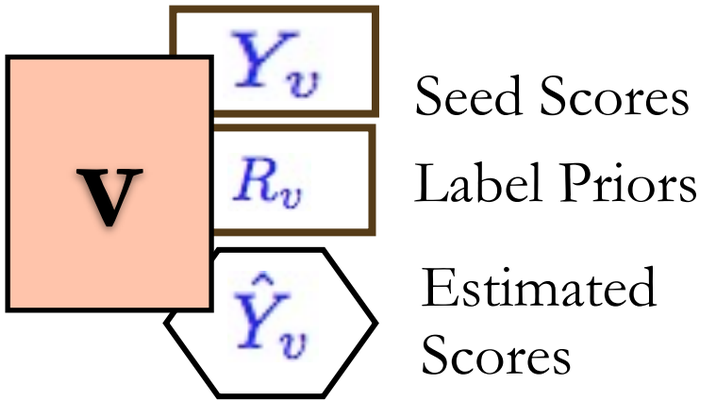

MAD's Objective is Convex

MAD has extra regularization compared to LP-ZGL [Zhu et al, ICML 03]; similar to QC [Bengio et al, 2006]

Solving MAD Objective

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- Can be solved using matrix inversion (like in LP)
  - but matrix inversion is expensive

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- Can be solved using matrix inversion (like in LP)
  - but matrix inversion is expensive
- Instead solved exactly using a system of linear equations (Ax = b)
  - solved using Jacobi iterations
  - results in iterative updates
  - guaranteed convergence
  - see [Bengio et al., 2006] and [Talukdar and Crammer, ECML 2009] for details

Solving MAD using Iterative Updates

Inputs $\boldsymbol{Y}, \boldsymbol{R} : |V| \times (|L|+1)$, $\boldsymbol{W} : |V| \times |V|$, $\boldsymbol{S} : |V| \times |V|$ diagonal

$\hat{\boldsymbol{Y}} \leftarrow \boldsymbol{Y}$

$\boldsymbol{M} = \boldsymbol{W}' + \boldsymbol{W}^{\dagger}$

$Z_v \leftarrow \boldsymbol{S}_{vv} + \mu_1 \sum_{u \neq v} \boldsymbol{M}_{vu} + \mu_2 \quad \forall v \in V$

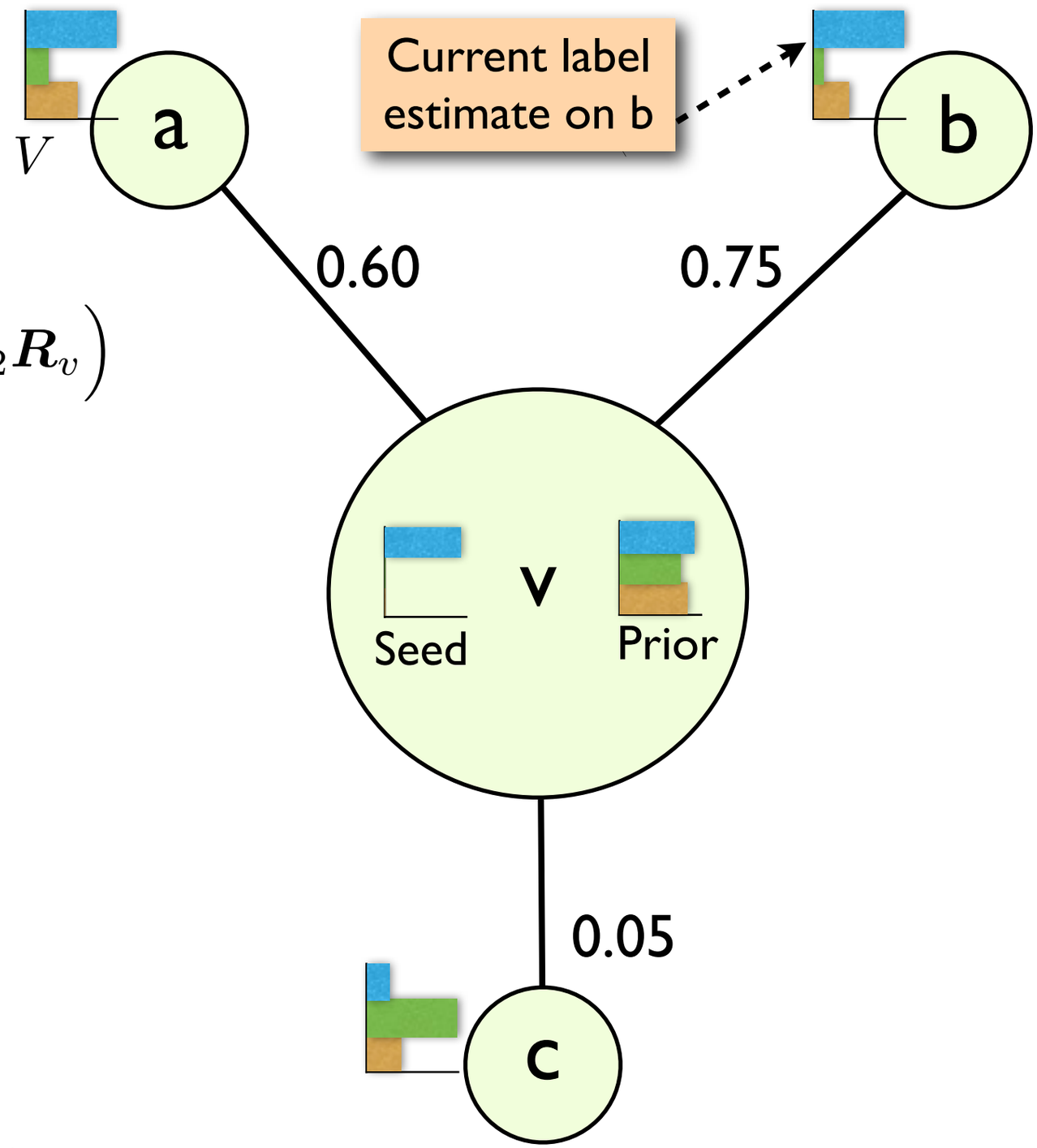
repeat

  for all $v \in V$ **do**

    $\hat{\boldsymbol{Y}}_v \leftarrow \frac{1}{Z_v} \left((\boldsymbol{S}\boldsymbol{Y})_v + \mu_1 \boldsymbol{M}_{v\cdot} \hat{\boldsymbol{Y}} + \mu_2 \boldsymbol{R}_v \right)$

  end for

until convergence



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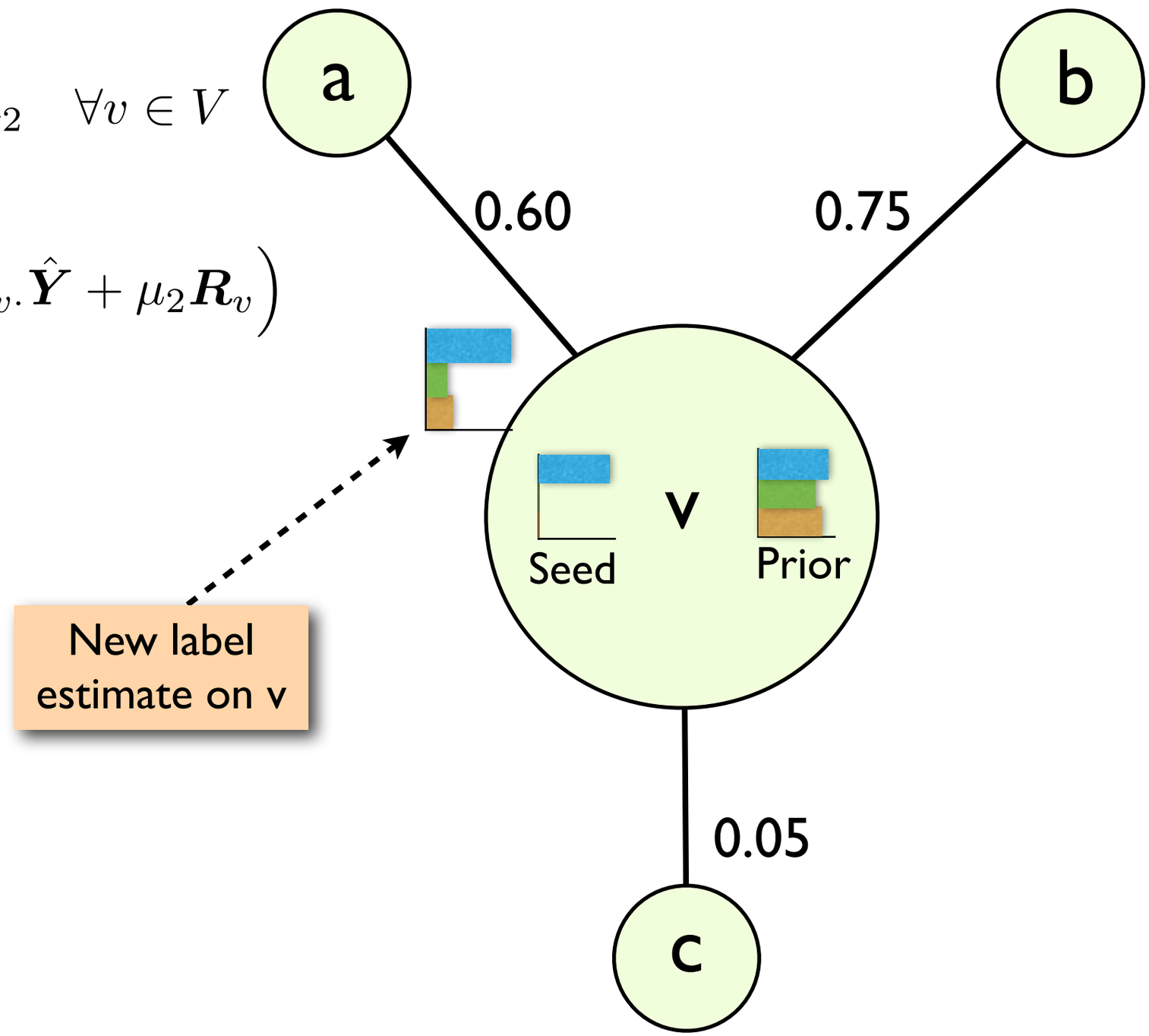

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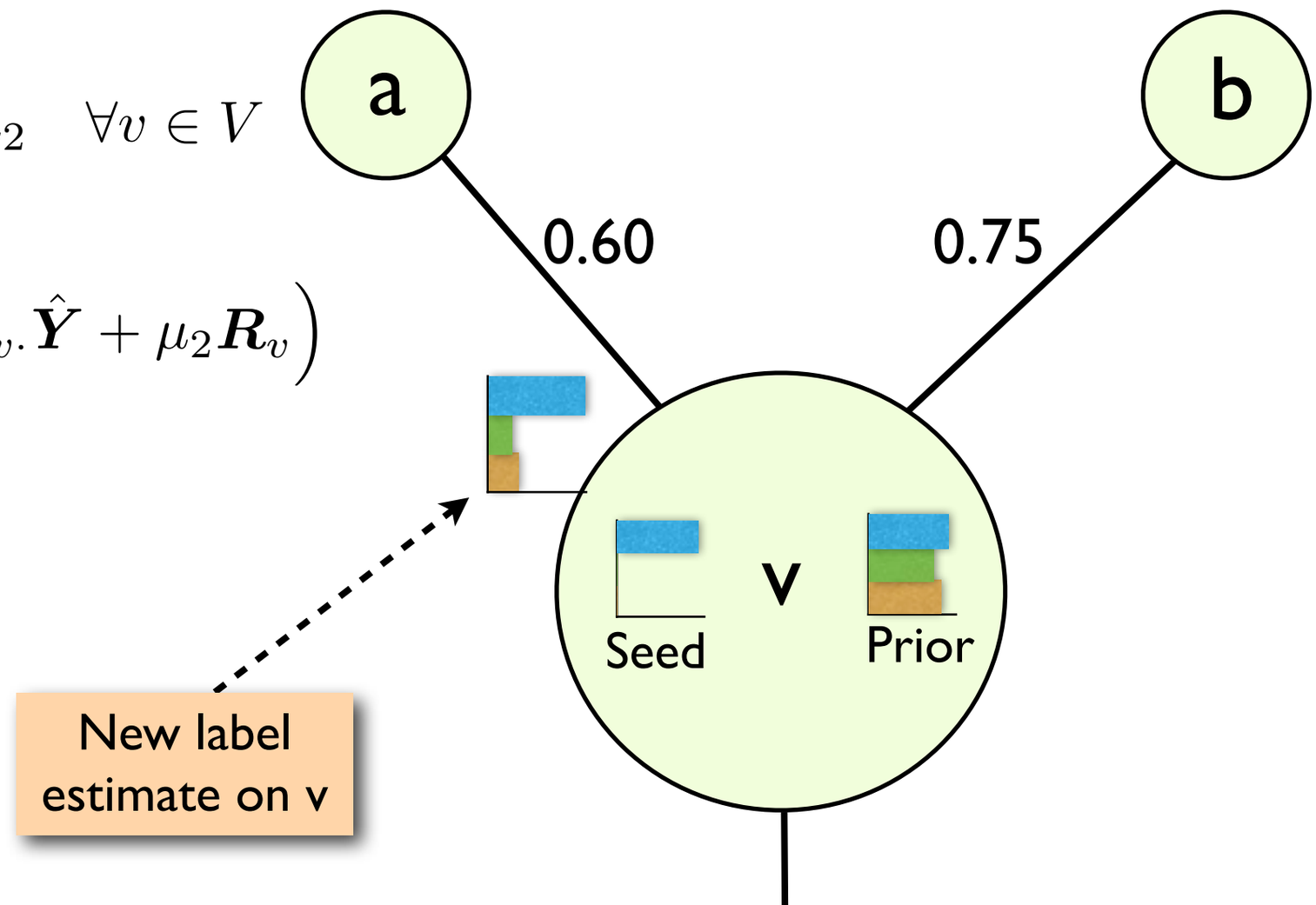

- Importance of a node can be discounted
- Easily Parallelizable: Scalable (more later)

When is MAD most effective?

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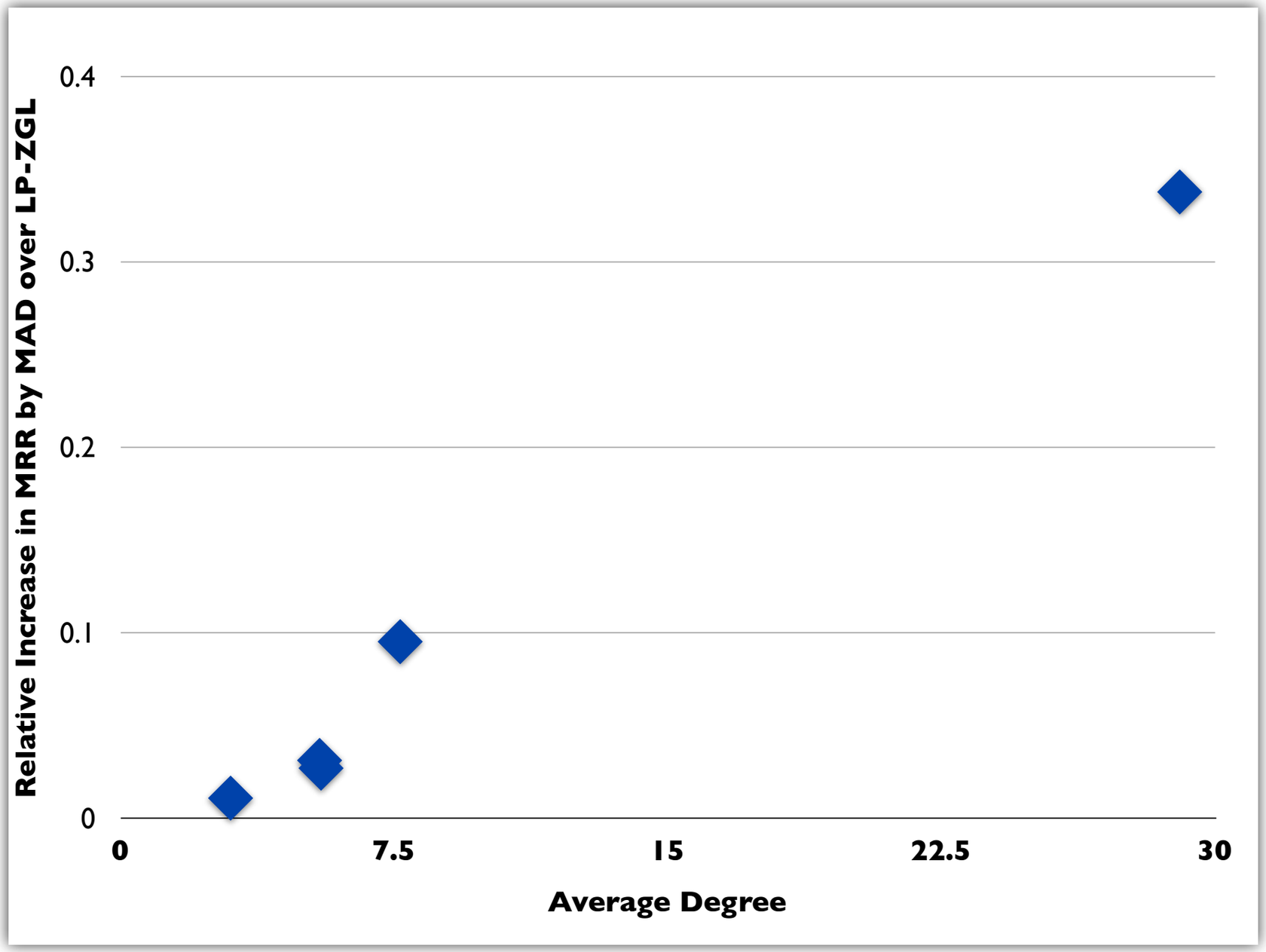
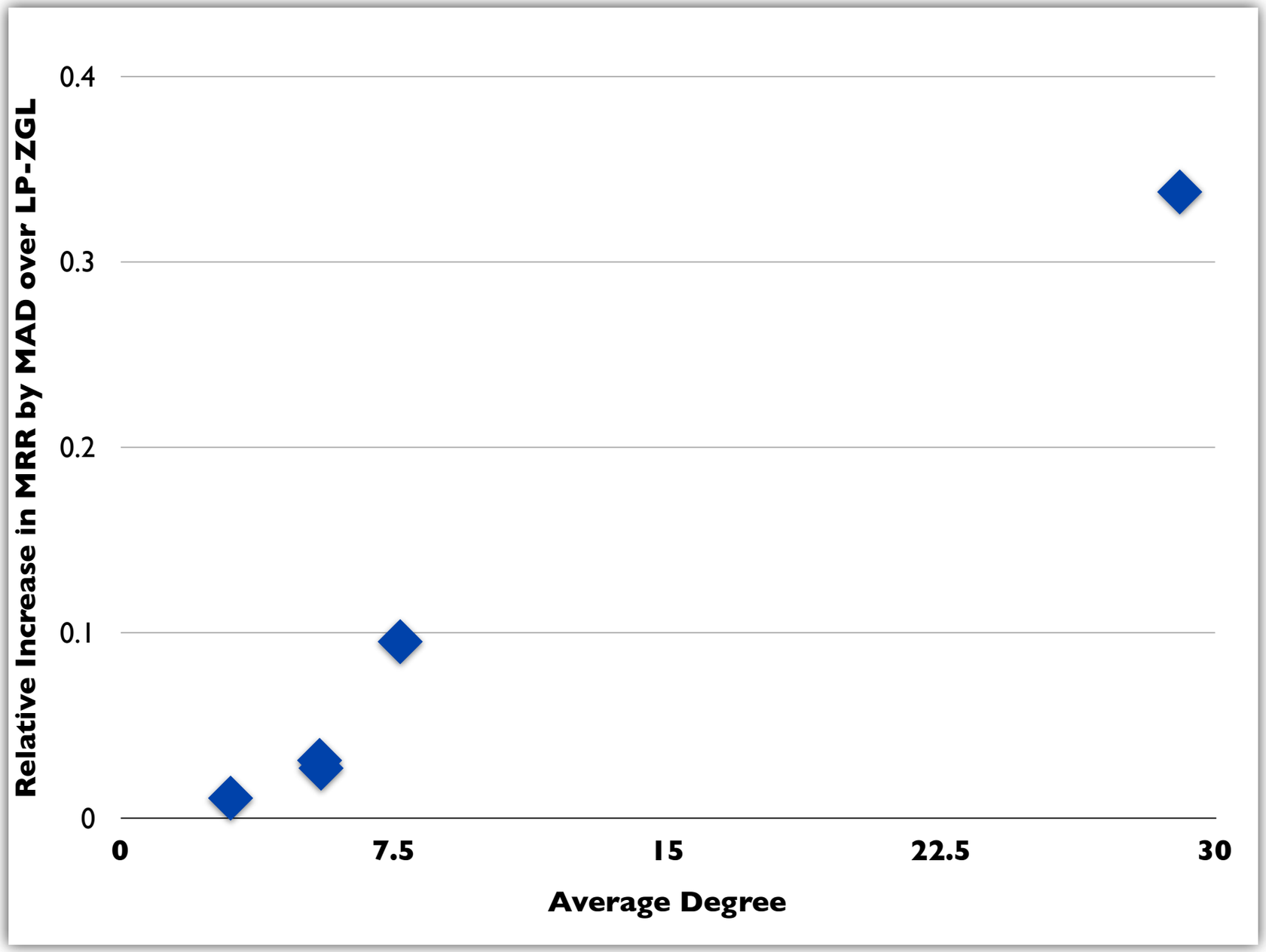


When is MAD most effective?



MAD is particularly effective in denser graphs, where there is greater need for regularization.

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Transduction Algorithm with Confidence (TACO) [Orbach and Crammer, ECML 2012]

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- Main lesson from MAD: discount *bad* (high degree) nodes

Transduction Algorithm with Confidence (TACO) [Orbach and Crammer, ECML 2012]

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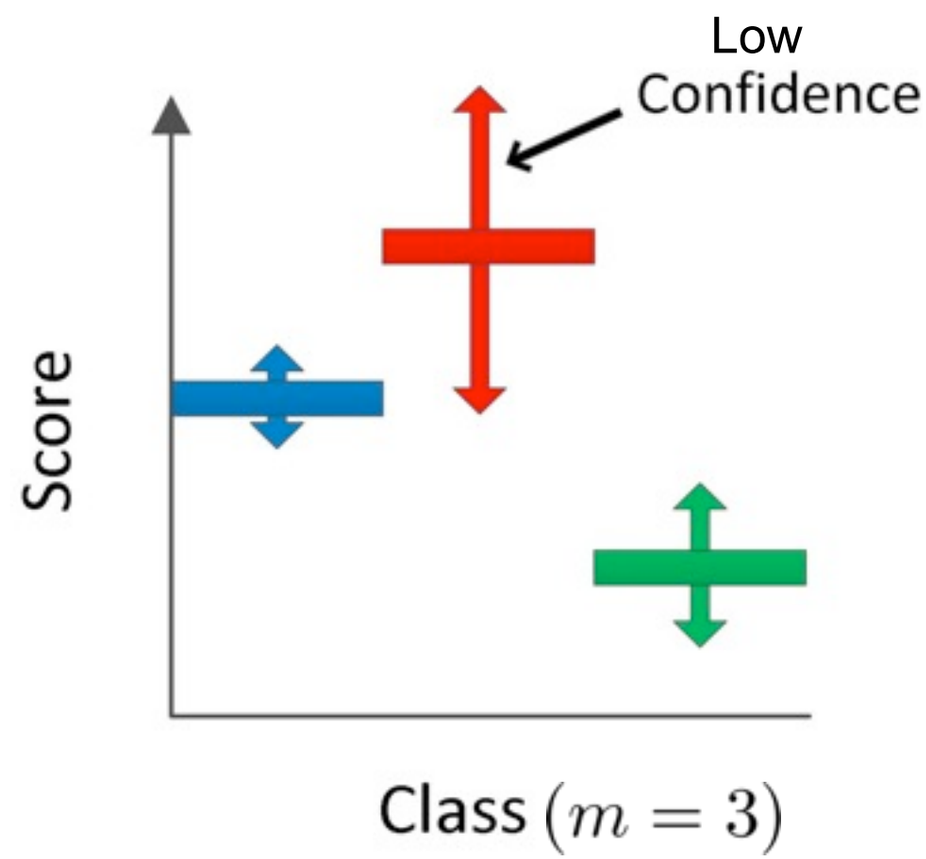

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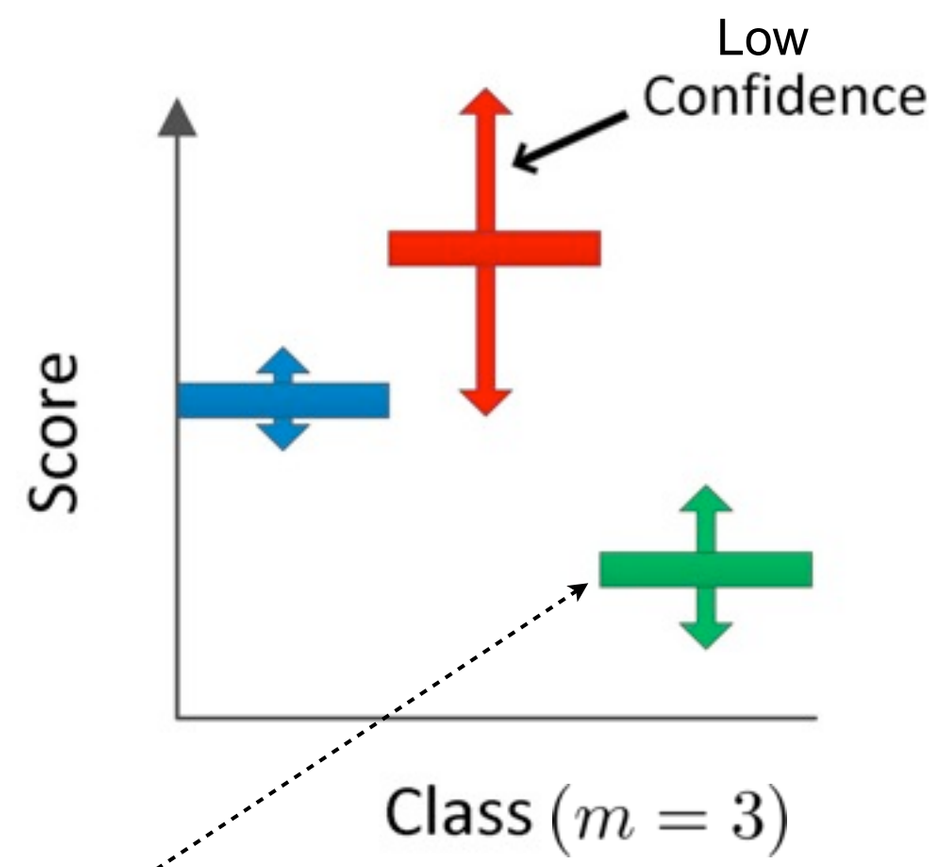
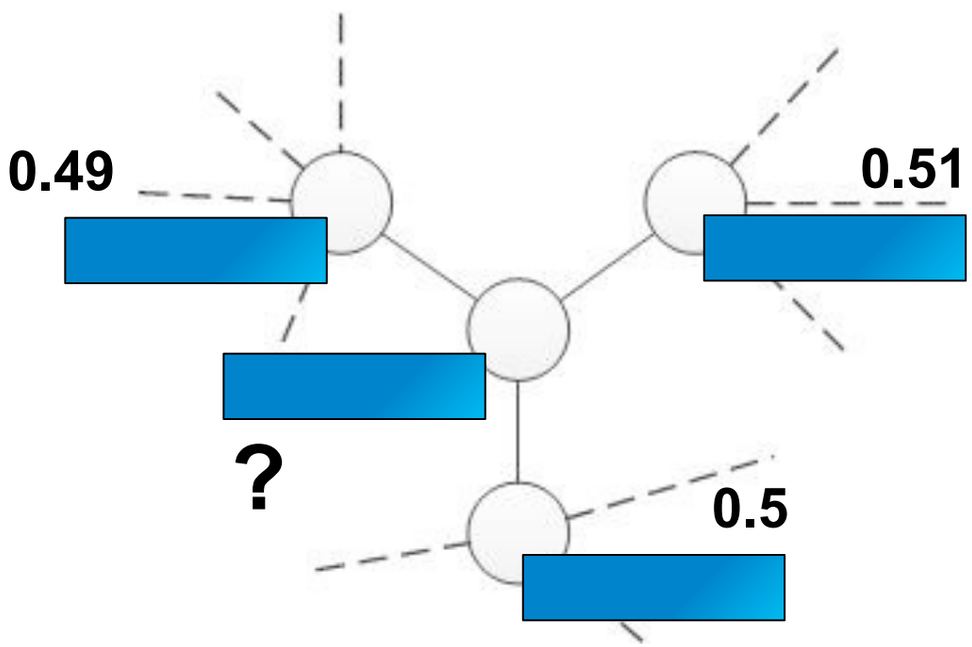
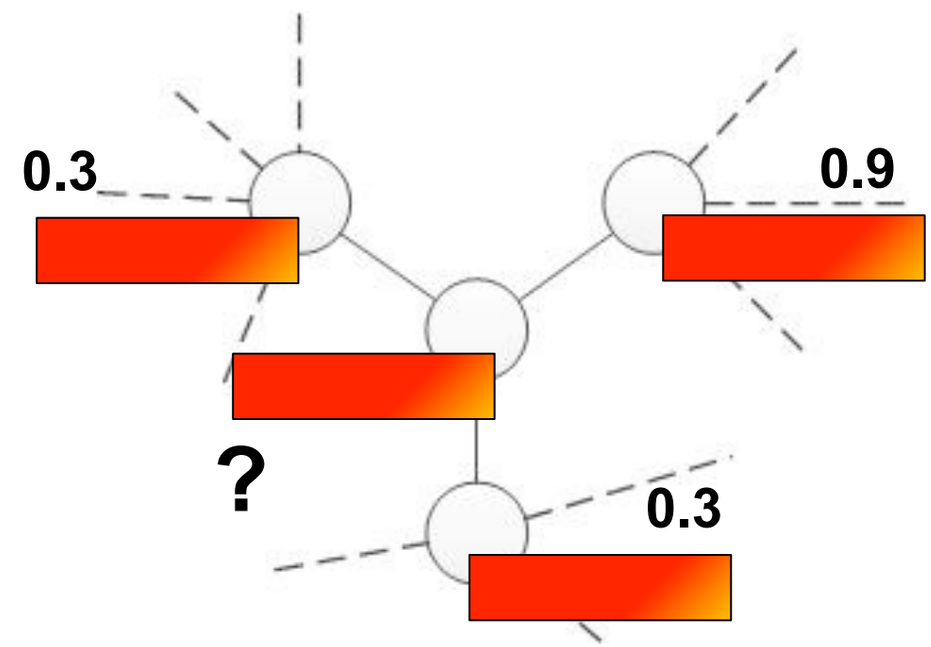


Label Scores: $\boldsymbol{\mu}_i = \begin{bmatrix} \mu_{i,1} & \dots & \mu_{i,m} \end{bmatrix} \in \mathbb{R}^m$

Confidence: $\boldsymbol{\sigma}_i = \begin{bmatrix} \sigma_{i,1} & \dots & \sigma_{i,m} \end{bmatrix} \in \mathbb{R}^m$

Introducing Confidence





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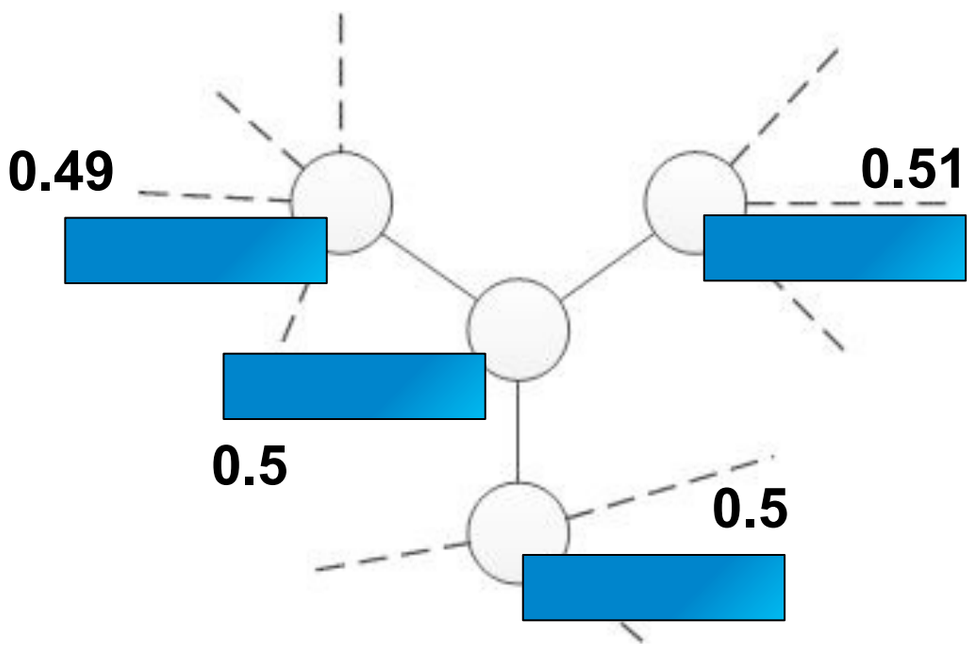

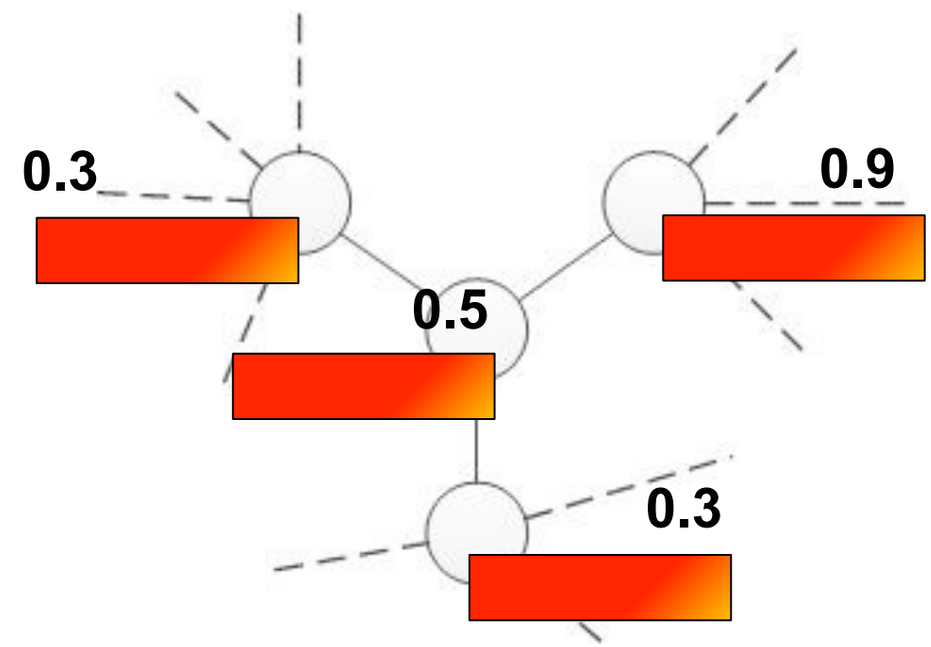

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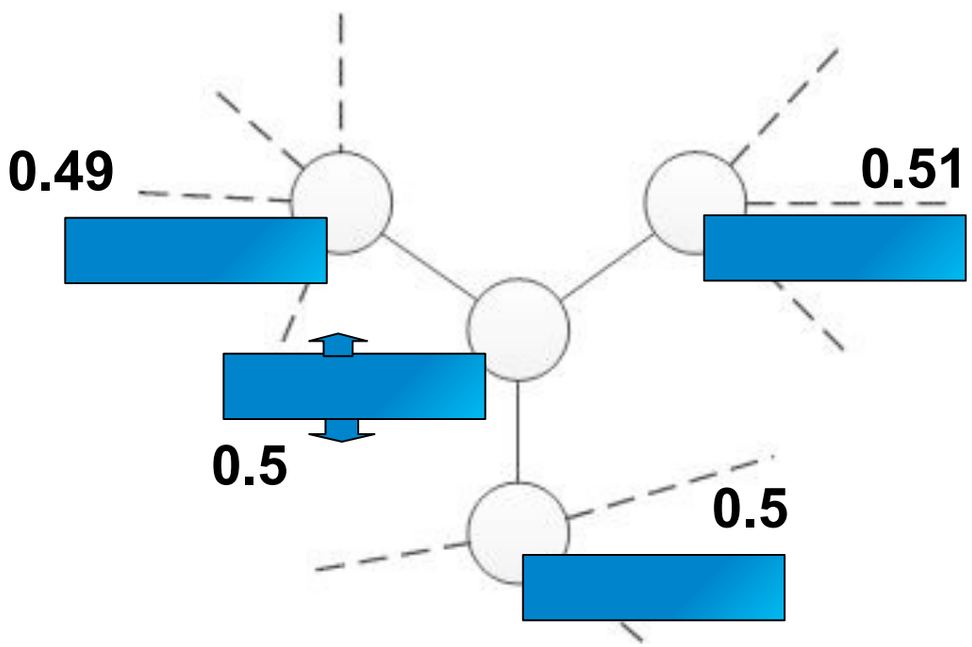

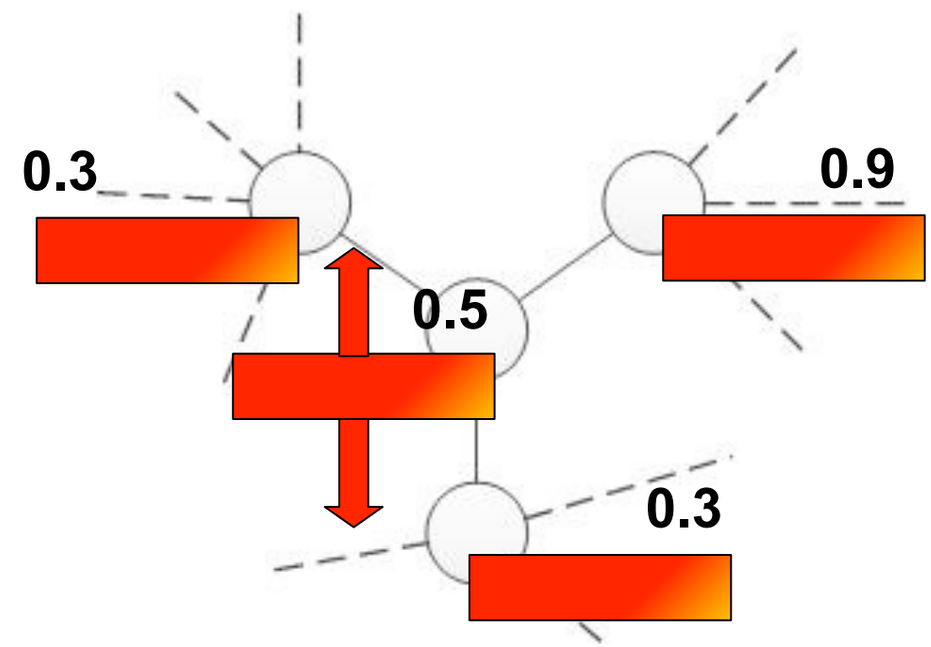

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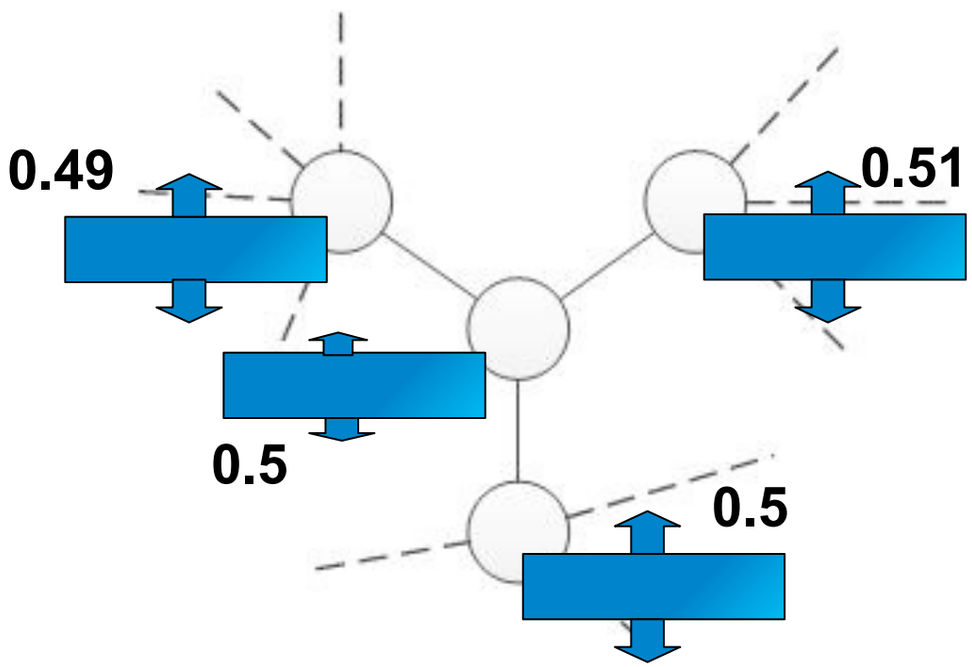

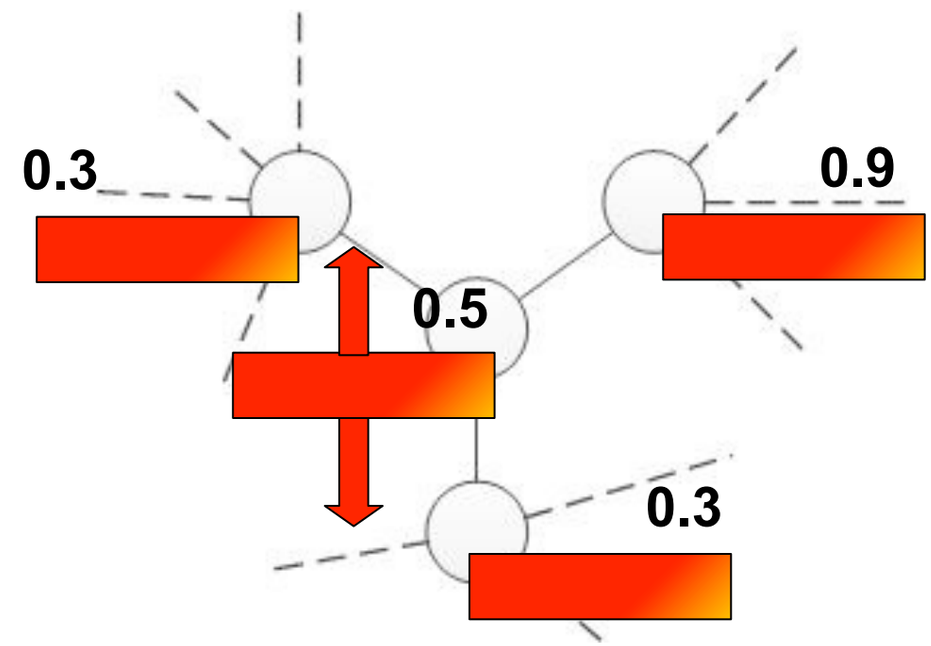

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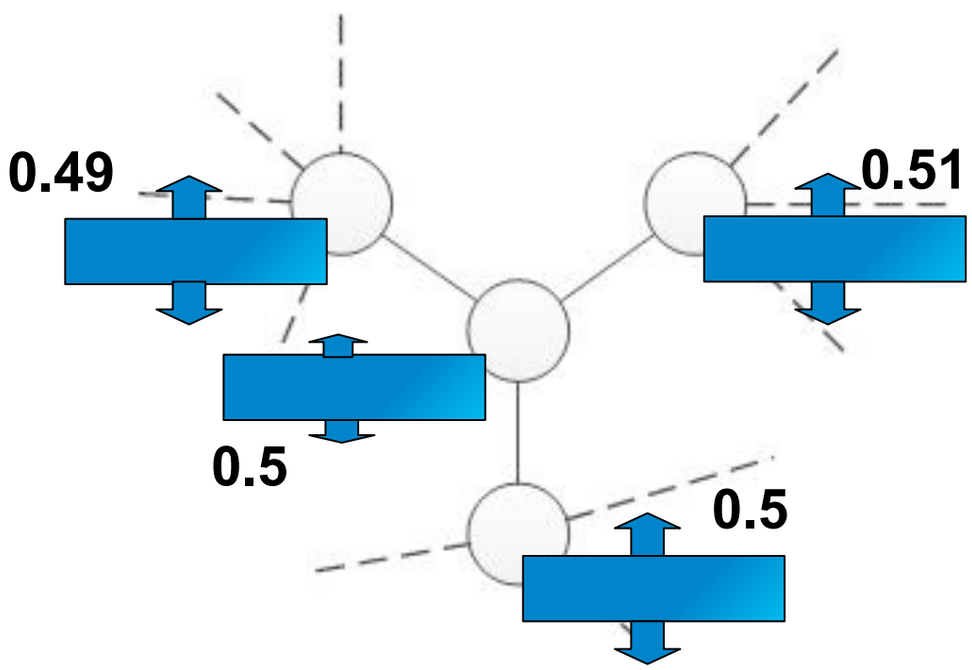

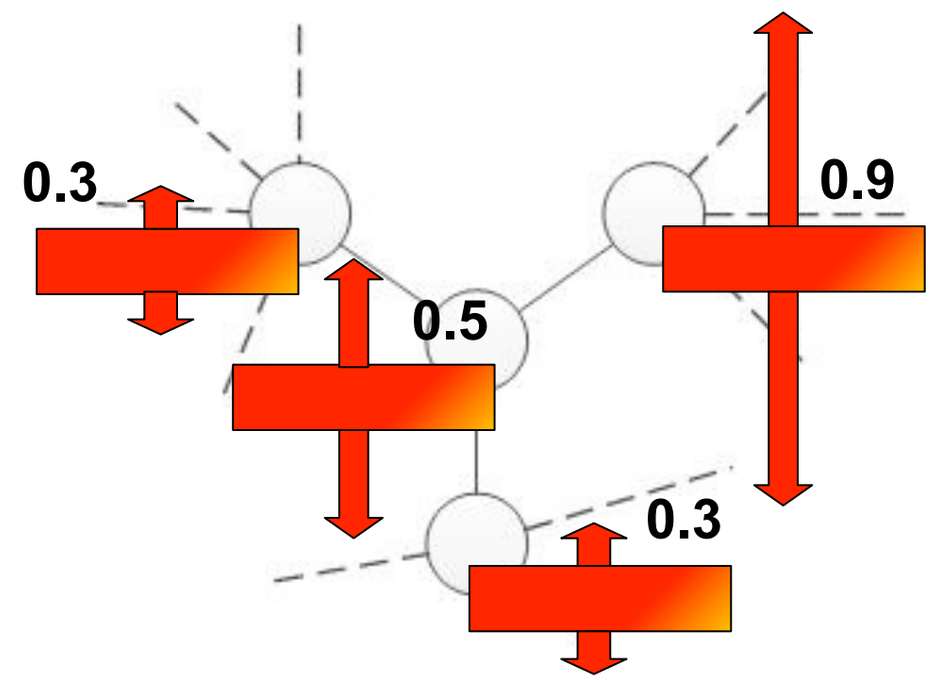

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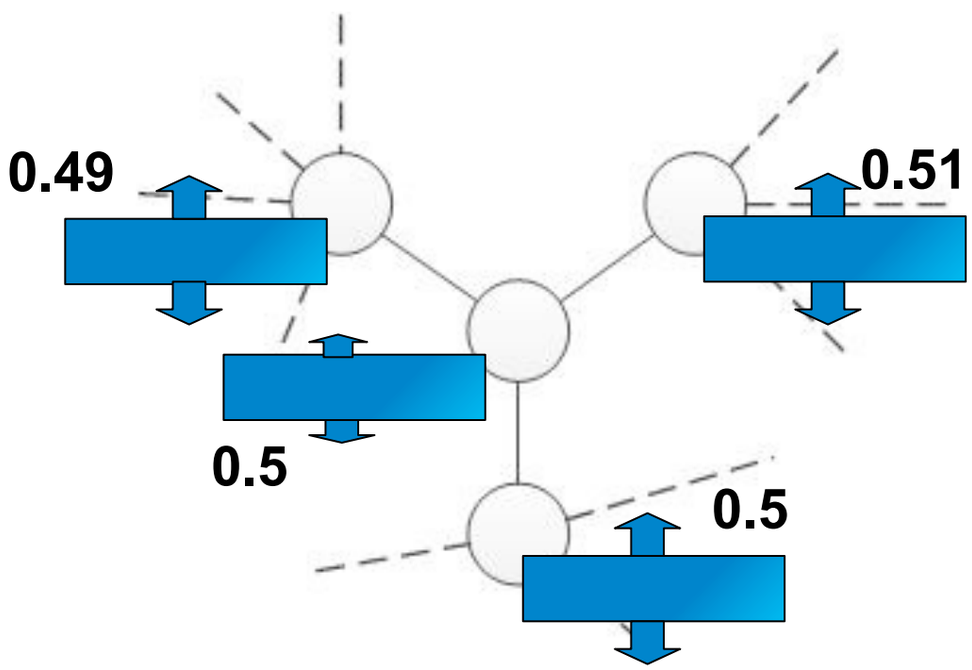

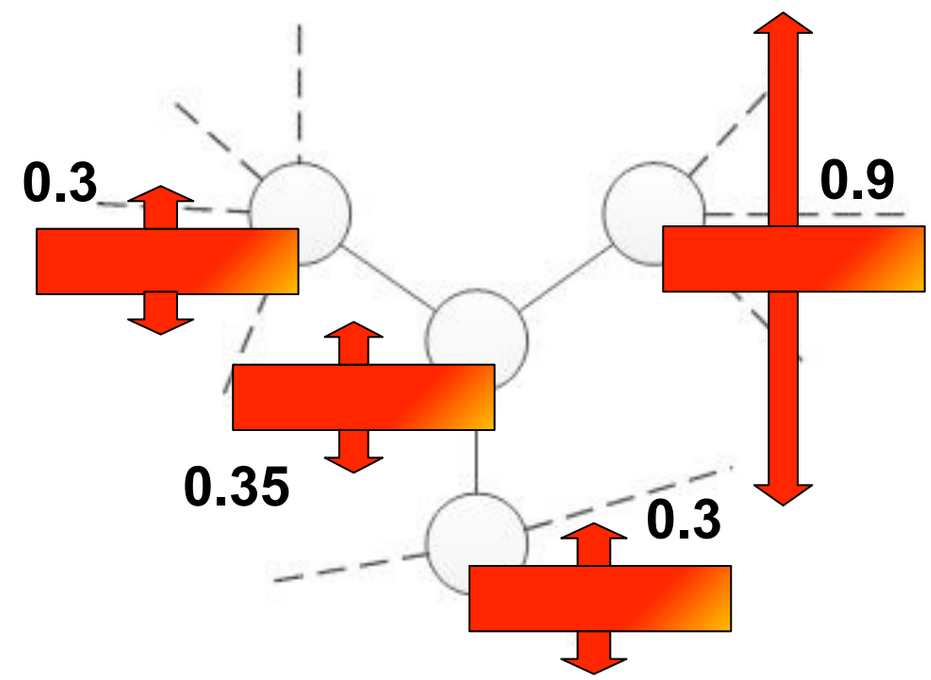

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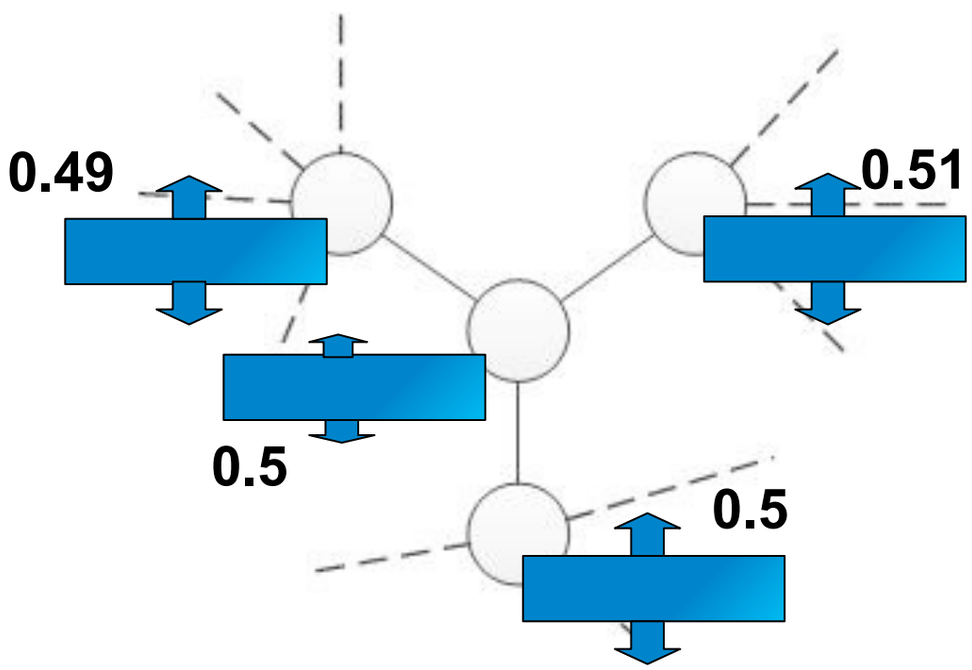
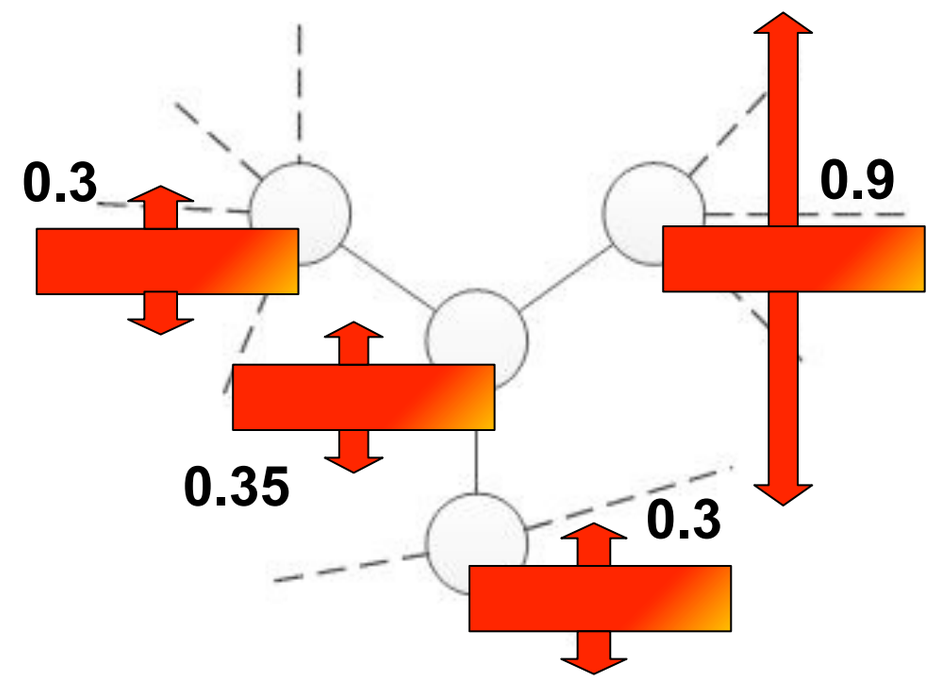
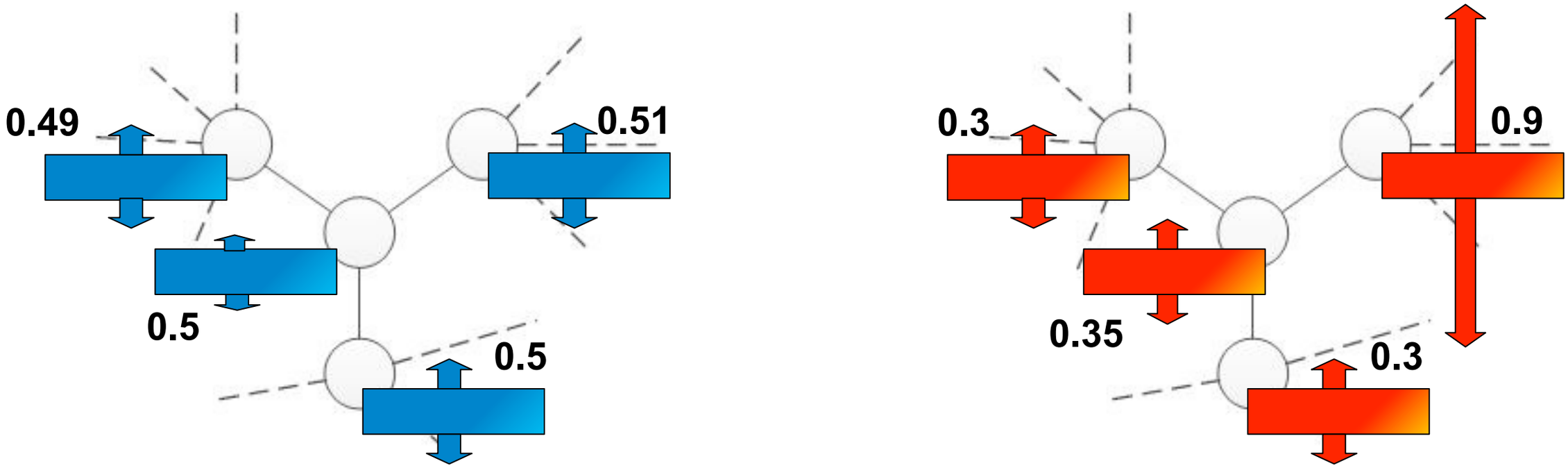




- Neighborhood disagreement => low confidence

Introducing Confidence



- Neighborhood disagreement => low confidence
- Lower the effect of poorly estimated (low confidence) scores

TACO Objective

$$\underset{\{\boldsymbol{\mu}_i\},\{\boldsymbol{\sigma}_i\}}{\arg\min} \overbrace{\sum_{i,j=1}^{n} w_{i,j} D\left(\boldsymbol{x}_i, \boldsymbol{x}_j\right)}^{\text{manifold term}} + \overbrace{\sum_{i=1}^{n_l} D\left(\boldsymbol{x}_i, \boldsymbol{y}_i\right)}^{\text{labeled term}} + \overbrace{\sum_{i=1}^{n} R\left(\boldsymbol{x}_i\right)}^{\text{regularization}}$$

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$$D\left(\boldsymbol{x}_i, \boldsymbol{x}_j\right) = \sum_{r=1}^{m} \left(\frac{1}{\sigma_{i,r}} + \frac{1}{\sigma_{j,r}}\right) \left(\mu_{i,r} - \mu_{j,r}\right)^2$$

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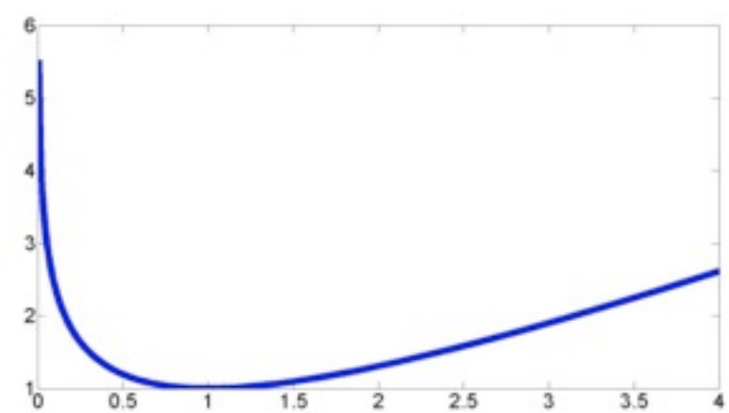
$$R\left(\boldsymbol{x}_i\right) = \alpha \sum_{r=1}^{m} \sigma_{i,r} - \beta \sum_{r=1}^{m} \log \sigma_{i,r}$$

$$\alpha x - \beta \log\left(x\right)$$



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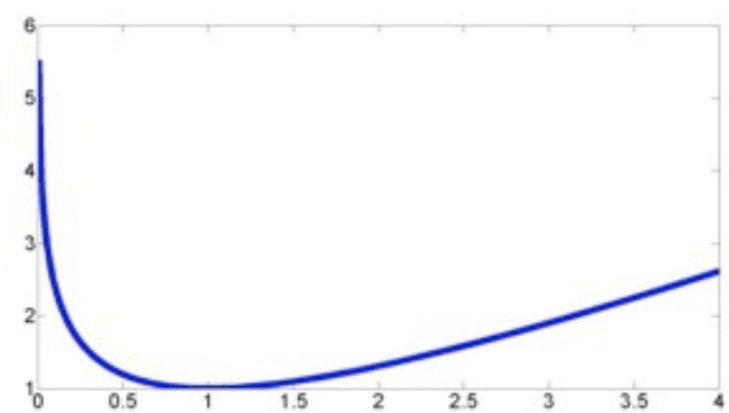

- Convex objective
- Iterative solution

TACO: Iterative Algorithm

Parameters: $\alpha, \beta > 0$ (controls regularization)
- $\gamma > 0$ (labeled confidence)

Input: Graph $G = (V, E, W)$ and prior labeling $\boldsymbol{y}_i$ for each $v_i \in V$

Initialize: $\boldsymbol{\mu}_i = \mathbf{0}$ and $\boldsymbol{\sigma}_i = \mathbf{1}$ for all $v_i \in V$

Repeat updates:

- For $v_i \in V$: [$C\left(G, \{\boldsymbol{\mu}_j\}, \{\boldsymbol{\sigma}_j\}\right)$ is the objective]

$$\frac{\partial C\left(G, \{\boldsymbol{\mu}_j\}, \{\boldsymbol{\sigma}_j\}\right)}{\partial \boldsymbol{\mu}_i} = 0 \quad \Rightarrow \quad \boldsymbol{\mu}_i = \ldots$$

$$\frac{\partial C\left(G, \{\boldsymbol{\mu}_j\}, \{\boldsymbol{\sigma}_j\}\right)}{\partial \boldsymbol{\sigma}_i} = 0 \quad \Rightarrow \quad \boldsymbol{\sigma}_i = \ldots$$

Until convergence

Output: Estimated scores $\{\boldsymbol{\mu}_i\}$

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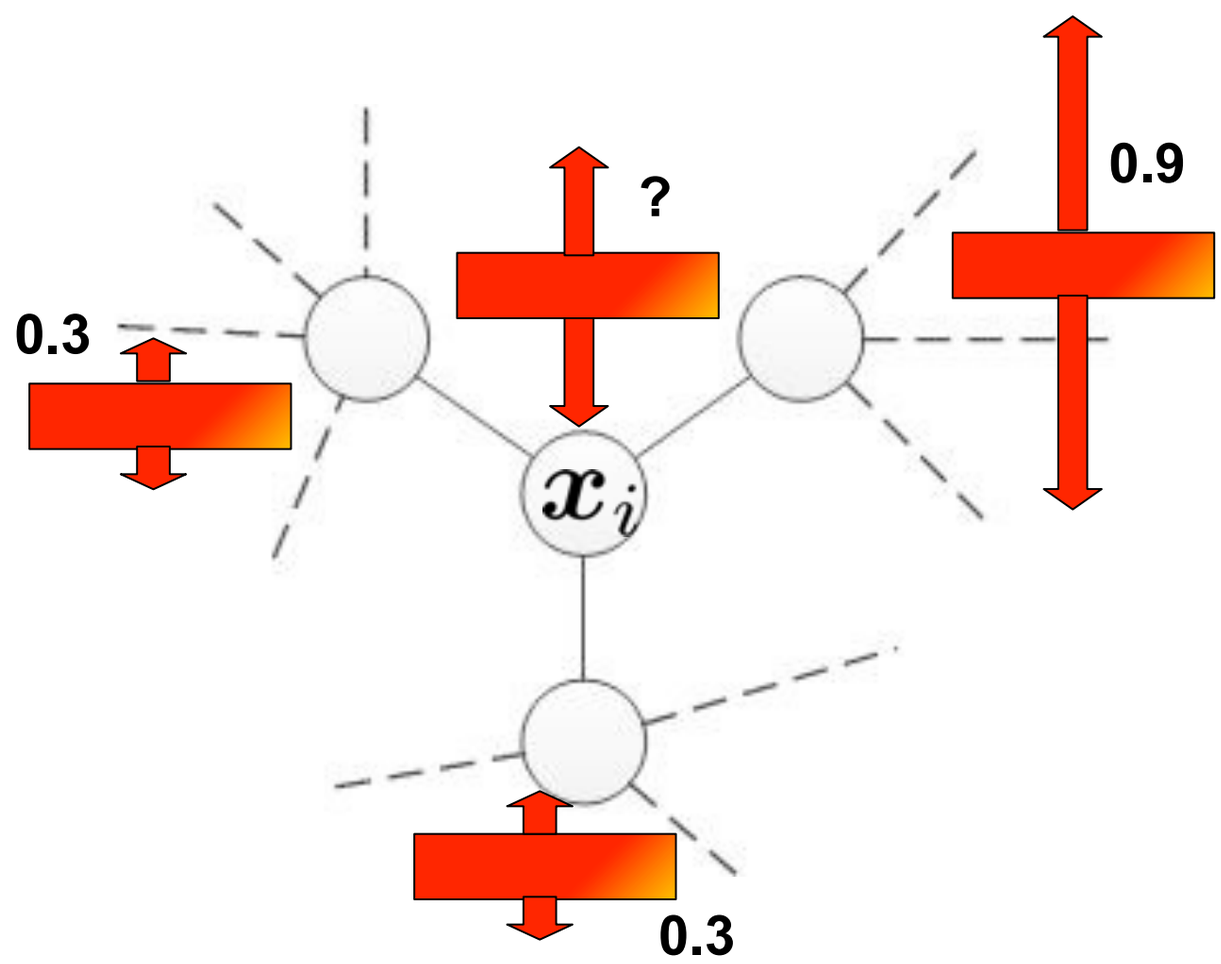
TACO: Score Update

$$\mu_{i,r}^{(t)} \leftarrow \frac{\sum_{\substack{j \neq i \\ j=1}}^{n} w_{i,j} \left(\frac{1}{\sigma_{i,r}^{(t-1)}} + \frac{1}{\sigma_{j,r}^{(t-1)}} \right) \mu_{j,r}^{(t-1)} + \delta_l(i) \left(\frac{1}{\sigma_{i,r}^{(t-1)}} + \frac{1}{\gamma} \right) y_{i,r}}{\sum_{\substack{j \neq i \\ j=1}}^{n} w_{i,j} \left(\frac{1}{\sigma_{i,r}^{(t-1)}} + \frac{1}{\sigma_{j,r}^{(t-1)}} \right) + \delta_l(i) \left(\frac{1}{\sigma_{i,r}^{(t-1)}} + \frac{1}{\gamma} \right)}$$



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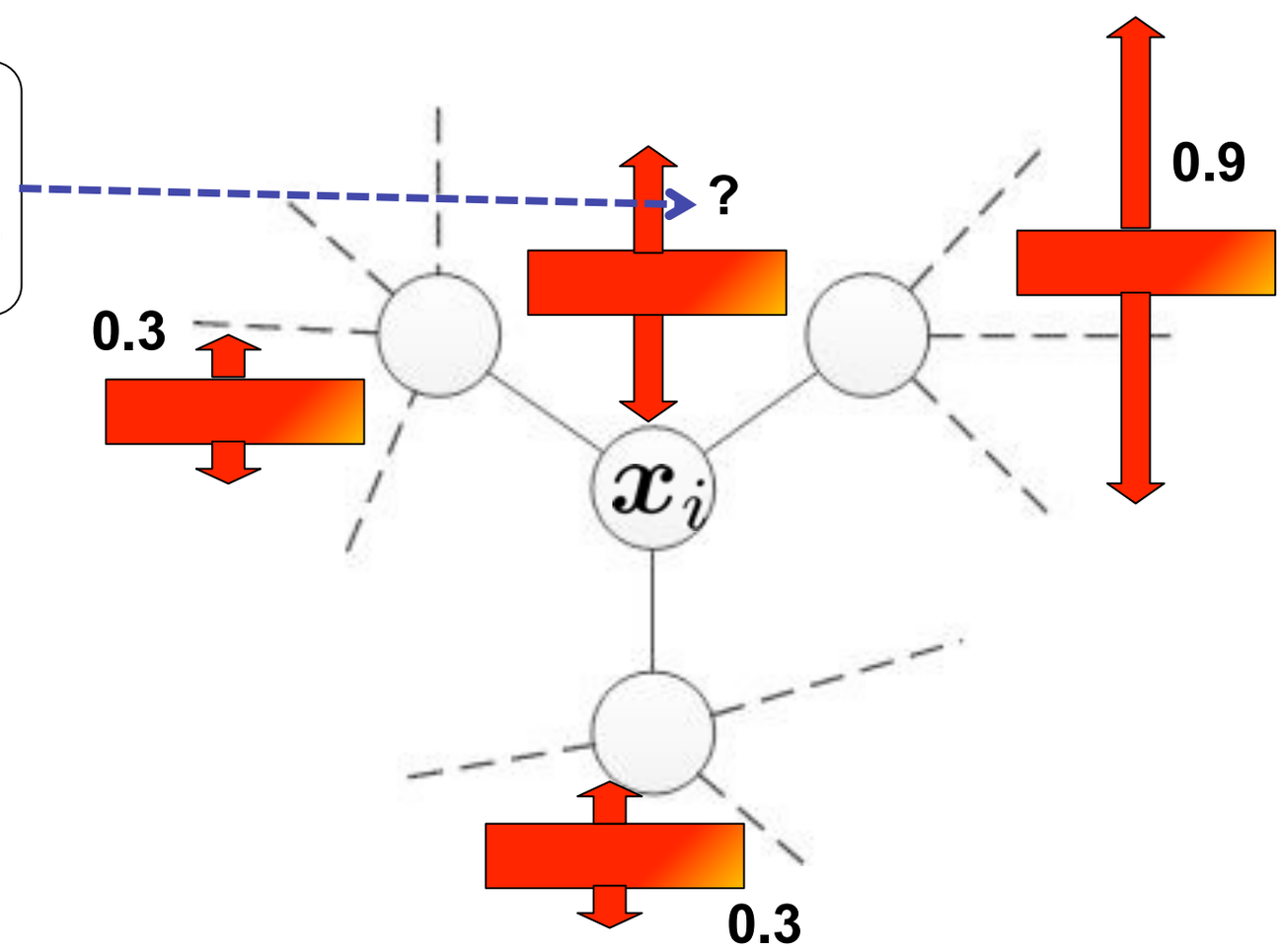
$\mu_{i,r}^{(t)}$
updated score



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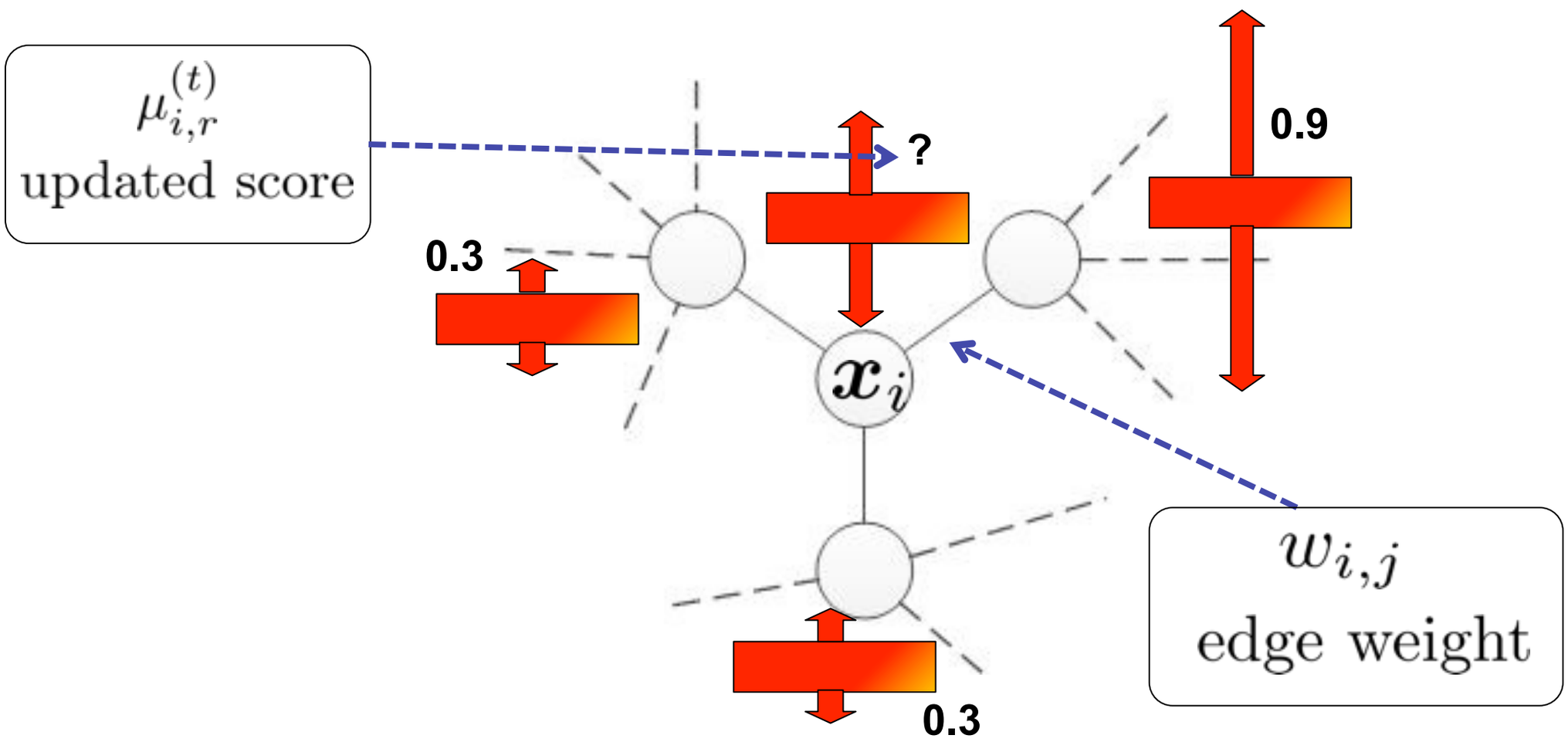

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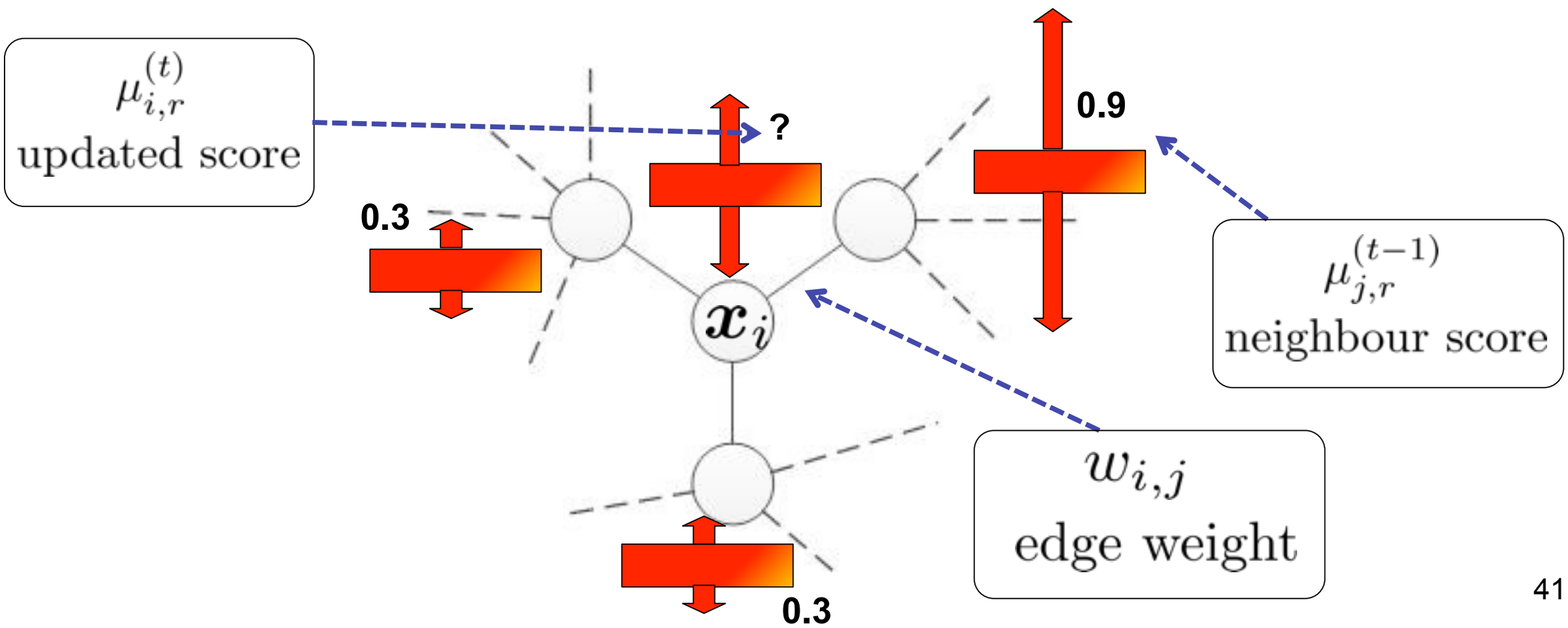

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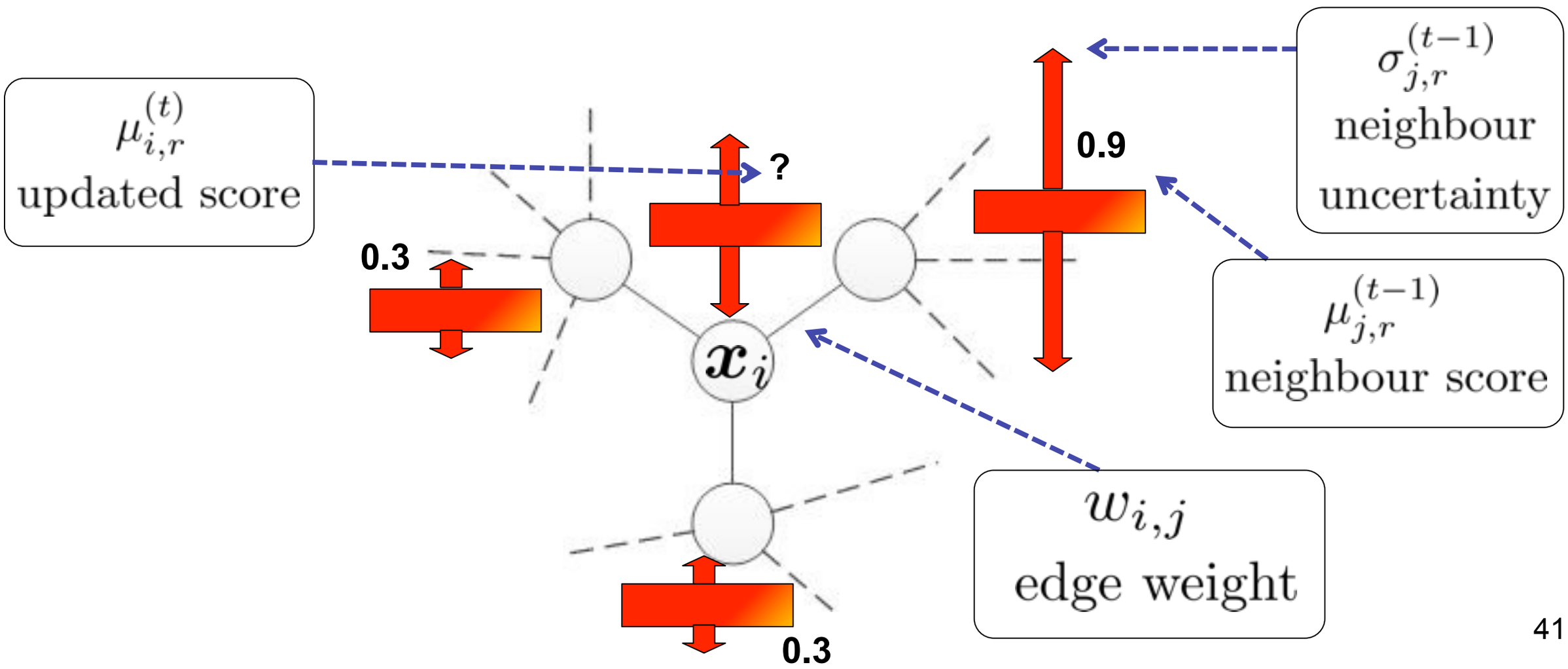

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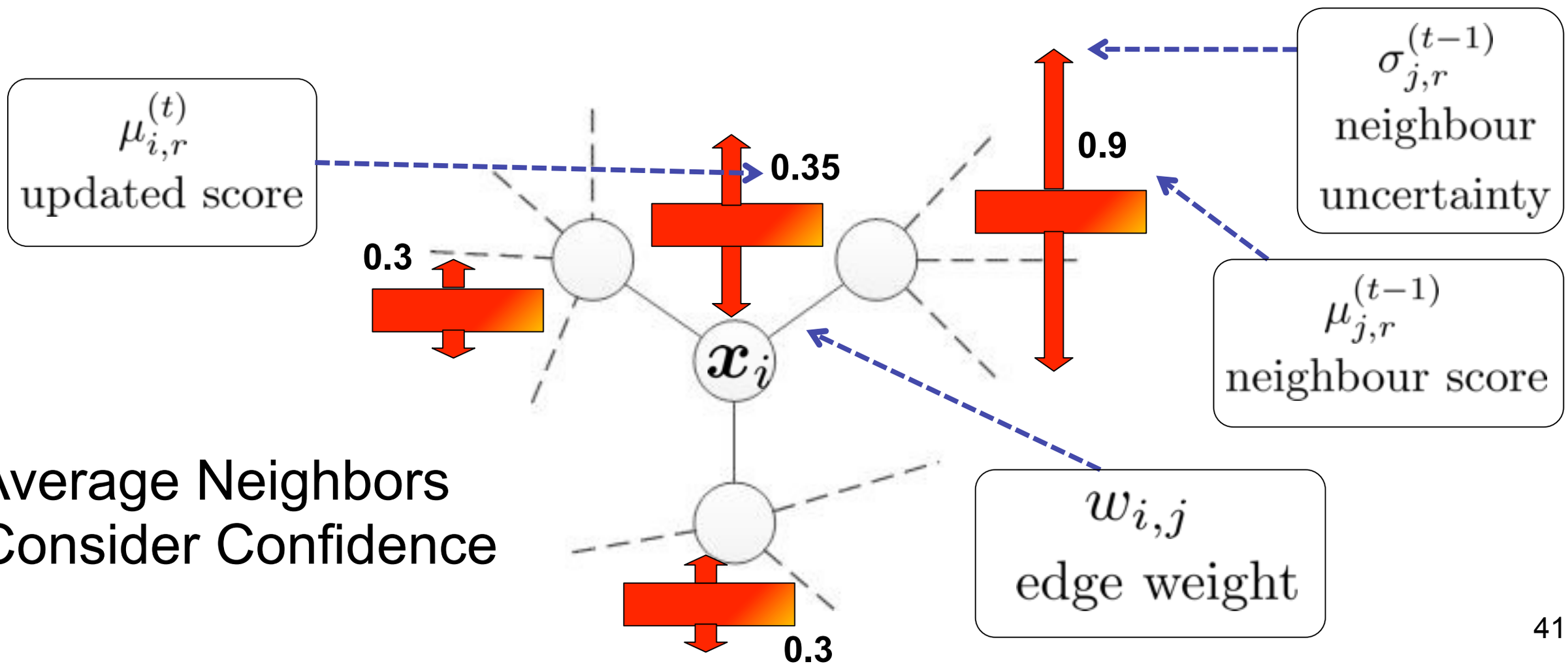
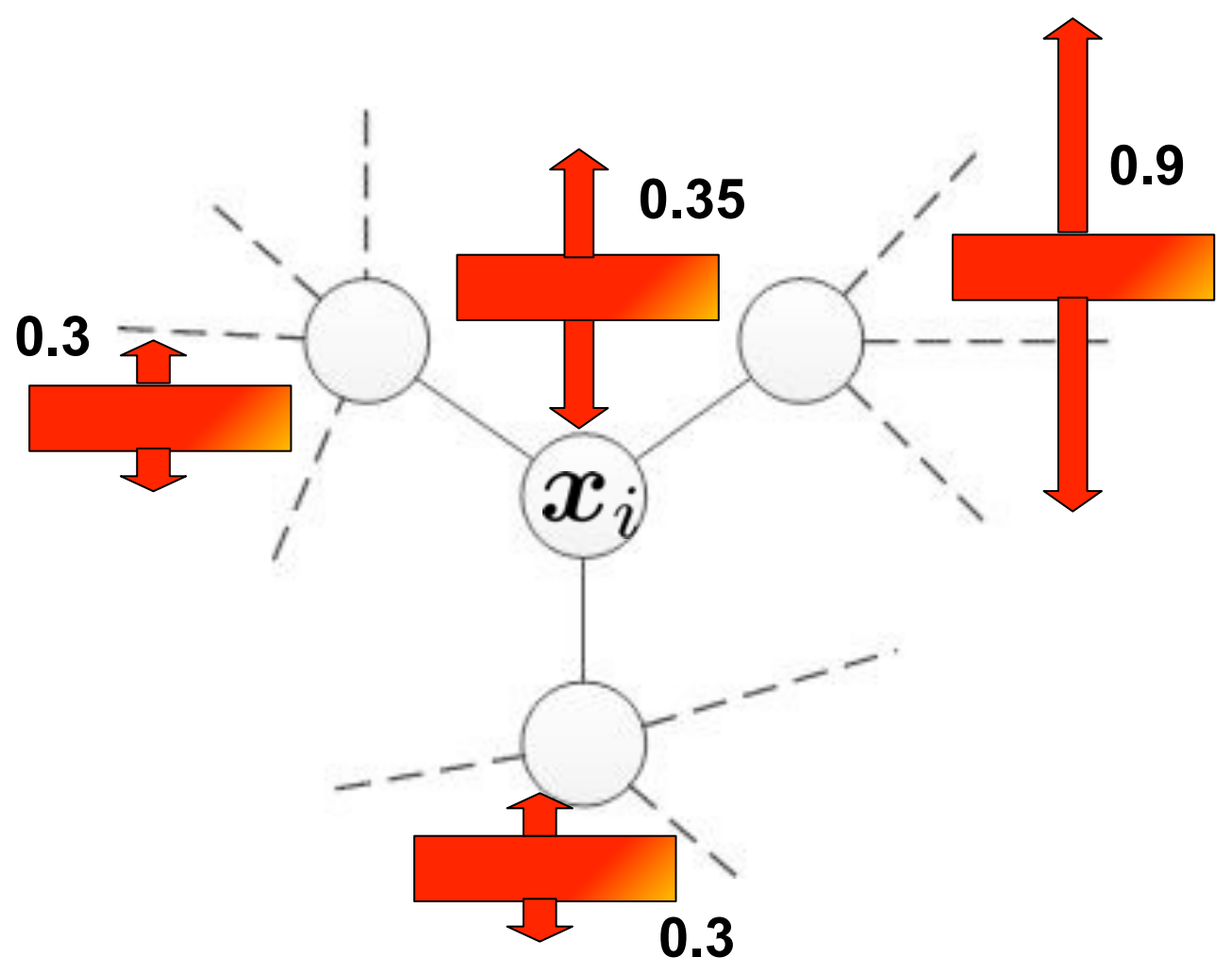


- Average Neighbors
- Consider Confidence

TACO: Confidence Update

$$\sigma_{i,r}^{(t)} \leftarrow \frac{\beta}{2\alpha} + \frac{1}{2\alpha}\sqrt{\beta^2 + 2\alpha\left[\sum_{j=1}^{n} w_{i,j}\left(\mu_{i,r}^{(t-1)} - \mu_{j,r}^{(t-1)}\right)^2 + \delta_l(i)\left(\mu_{i,r}^{(t-1)} - y_{i,r}\right)^2\right]}$$



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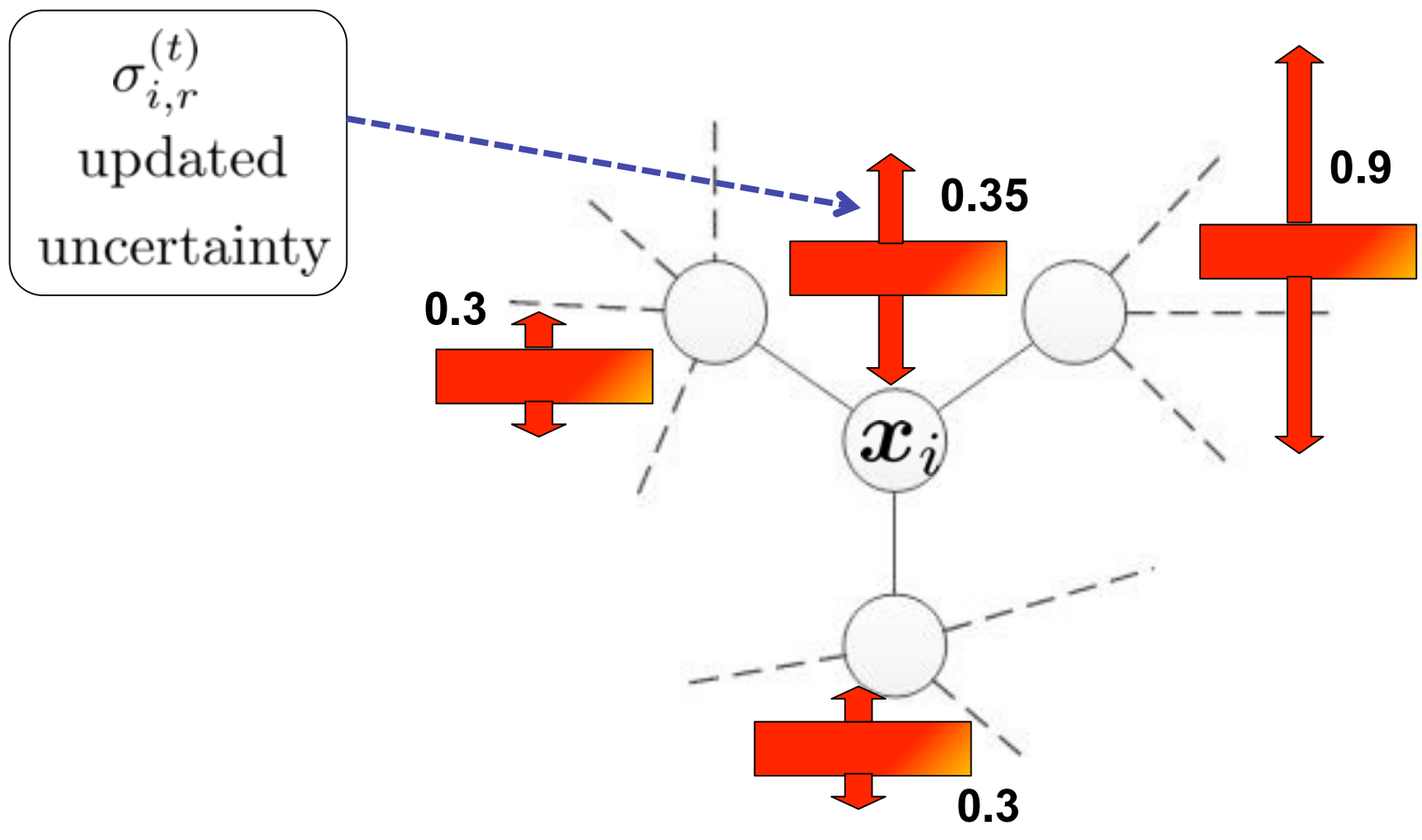

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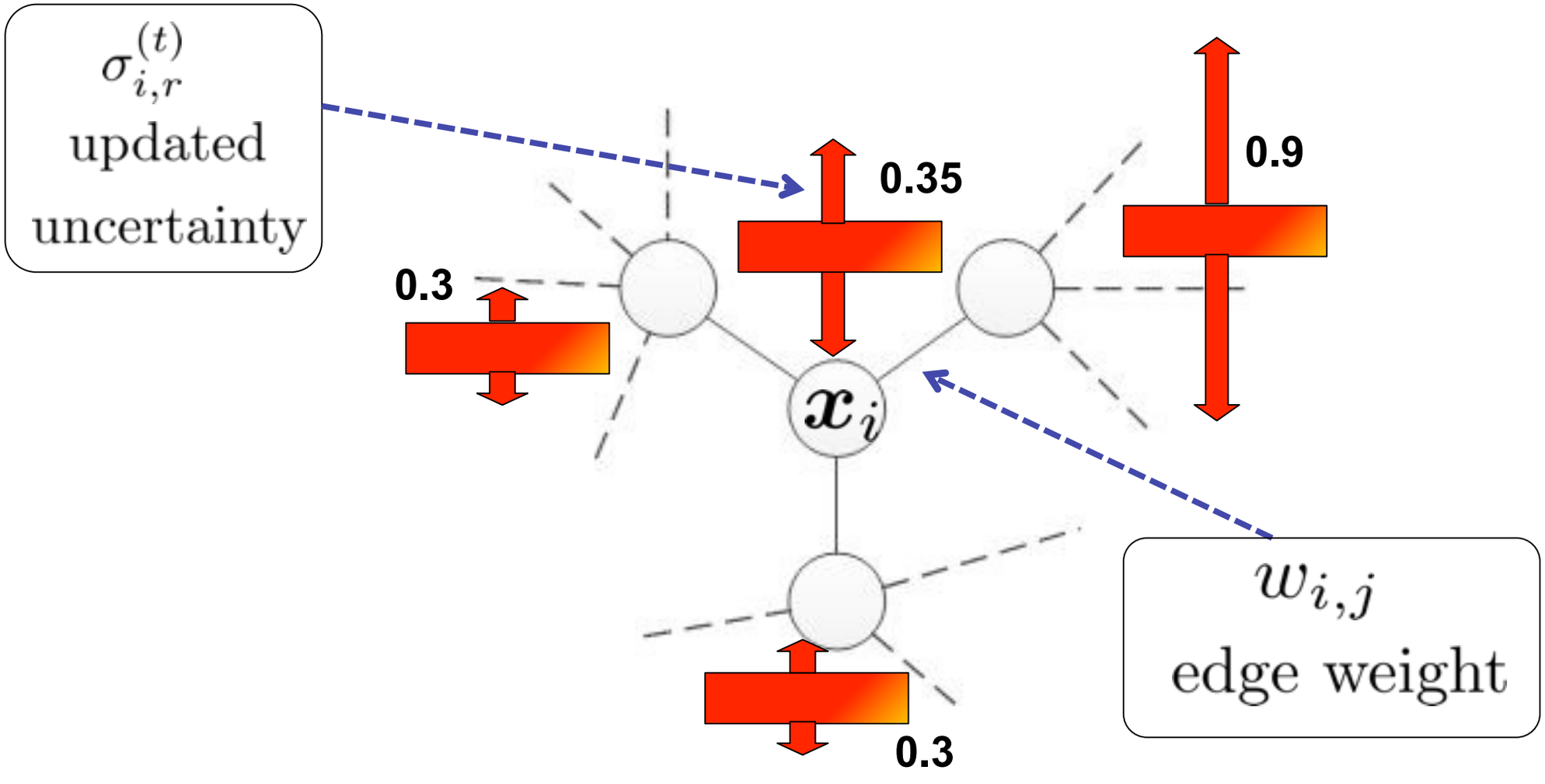

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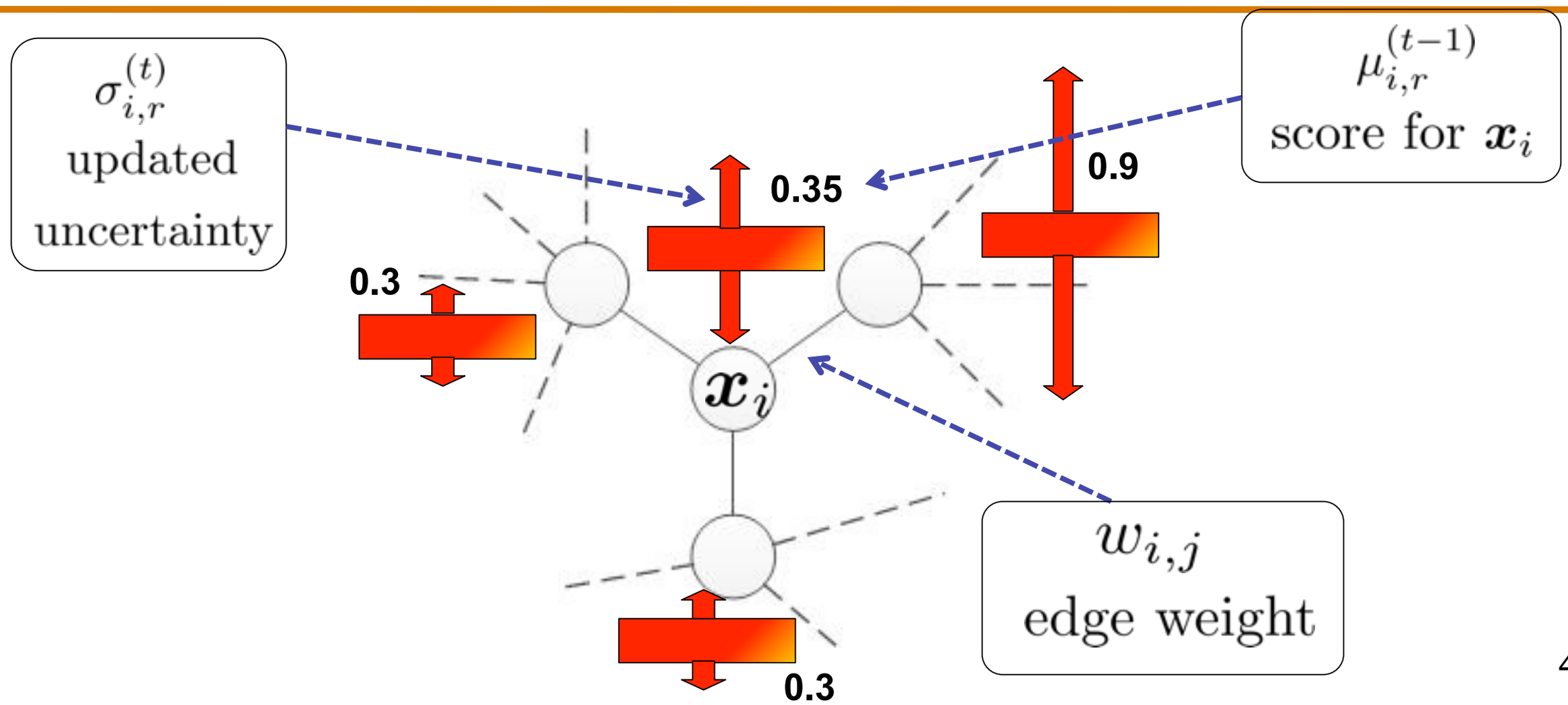

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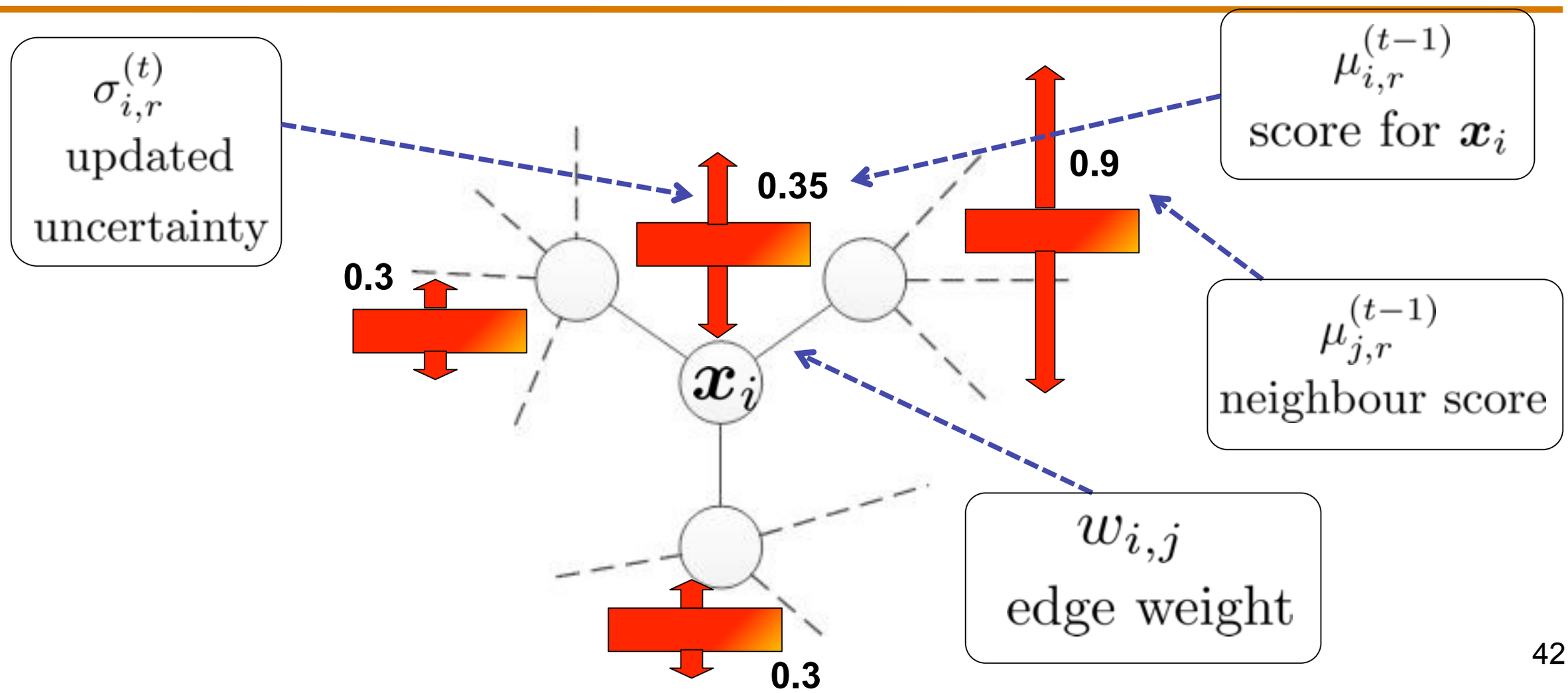

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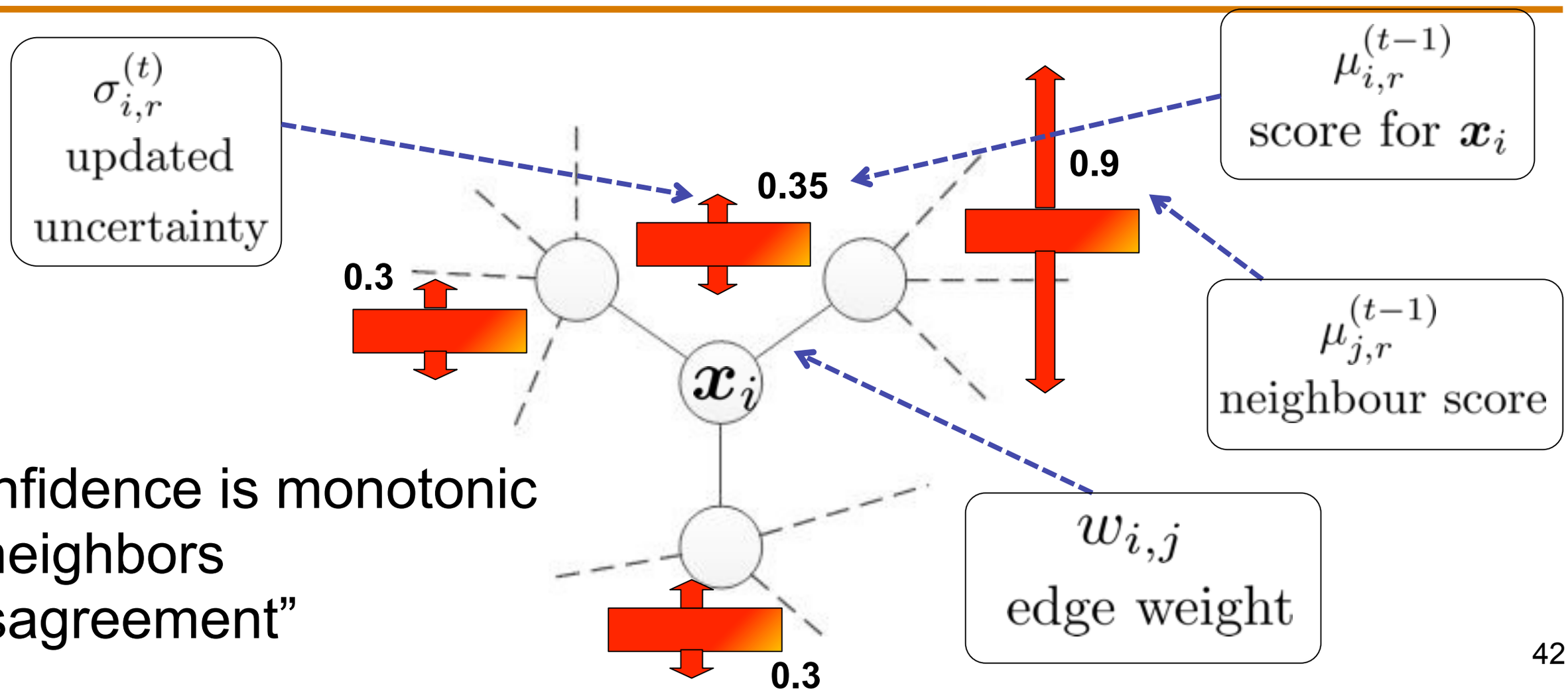

Confidence is monotonic in neighbors "disagreement"

Outline

- Motivation
- Graph Construction
- Inference Methods
  - Label Propagation
  - Modified Adsorption
  - Transduction with Confidence
  - Manifold Regularization
  - Measure Propagation
  - Sparse Label Propagation
- Scalability
- Applications
- Conclusion & Future Work

Manifold Regularization

[Belkin et al., JMLR 2006]

$$f^* = \arg\min_f \frac{1}{l} \sum_{i=1}^{l} V(y_i, f(x_i)) + \beta \; f^T L f + \gamma ||f||_K^2$$

Manifold Regularization

[Belkin et al., JMLR 2006]

Training Data Loss

$$f^* = \arg\min_f \frac{1}{l} \sum_{i=1}^{l} \boxed{V(y_i, f(x_i))} + \beta \; f^T L f + \gamma ||f||_K^2$$

Loss Function
(e.g., soft margin)

Manifold Regularization

[Belkin et al., JMLR 2006]

Training Data Loss

Smoothness Regularizer

$$f^* = \arg\min_f \frac{1}{l} \sum_{i=1}^{l} \boxed{V(y_i, f(x_i))} + \boxed{\beta \; f^T L f} + \gamma ||f||_K^2$$

Loss Function (e.g., soft margin)

Laplacian of graph over labeled and unlabeled data

Manifold Regularization

[Belkin et al., JMLR 2006]

Training Data Loss

Smoothness Regularizer

Regularizer (e.g., L2)

$$f^* = \arg\min_f \frac{1}{l} \sum_{i=1}^{l} \boxed{V(y_i, f(x_i))} + \boxed{\beta \; f^T L f} + \boxed{\gamma ||f||_K^2}$$

Loss Function (e.g., soft margin)

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Loss Function (e.g., soft margin)

Laplacian of graph over labeled and unlabeled data

Trains an <u>inductive</u> classifier which can generalize to unseen instances

Outline

- Motivation
- Graph Construction
- Inference Methods
  - Label Propagation
  - Modified Adsorption
  - Transduction with Confidence
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  - Measure Propagation
  - Sparse Label Propagation
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Measure Propagation (MP)

[Subramanya and Bilmes, EMNLP 2008, NIPS 2009, JMLR 2011]

C_{KL}

$$\arg\min_{\{p_i\}} \sum_{i=1}^{l} D_{KL}(r_i||p_i) + \mu \sum_{i,j} w_{ij} D_{KL}(p_i||p_j) - \nu \sum_{i=1}^{n} H(p_i)$$

$$\text{s.t. } \sum_y p_i(y) = 1, \; p_i(y) \geq 0, \; \forall y, i$$

Measure Propagation (MP)

[Subramanya and Bilmes, EMNLP 2008, NIPS 2009, JMLR 2011]

C_{KL}

Divergence on seed nodes

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Seed and estimated label distributions (normalized) on node *i*

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KL Divergence

$$D_{KL}(p_i||p_j) = \sum_y p_i(y) \log \frac{p_i(y)}{p_j(y)}$$

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Smoothness (divergence across edge)

Entropic Regularizer

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$$H(p_i) = -\sum_y p_i(y) \log p_i(y)$$

C_{KL} is convex (with non-negative edge weights and hyper-parameters)

MP is related to Information Regularization [Corduneanu and Jaakkola, 2003]

Solving MP Objective

- For ease of optimization, reformulate MP objective:

CMP

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New probability measure, one for each vertex, similar to p_i

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Encourages agreement between p_i and q_i

$$\operatorname*{argmin}_{p \in \Delta^n} C_{KL}(p) = \lim_{\alpha \to \infty} \operatorname*{argmin}_{p,q \in \Delta^n} C_{MP}(p, q)$$

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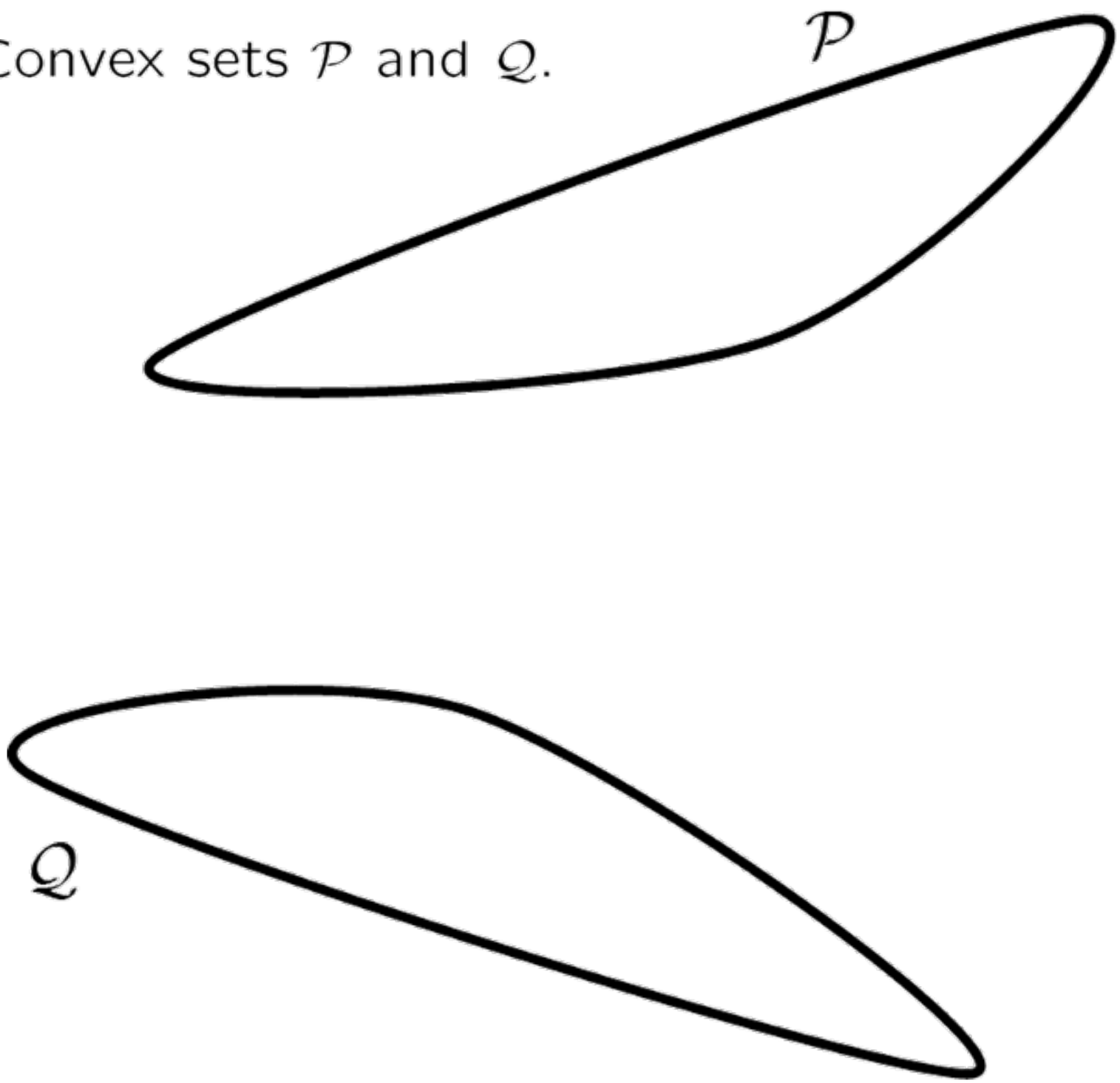
C_{MP} can be solved using Alternating Minimization (AM)

Alternating Minimization

Convex sets $\mathcal{P}$ and $\mathcal{Q}$.



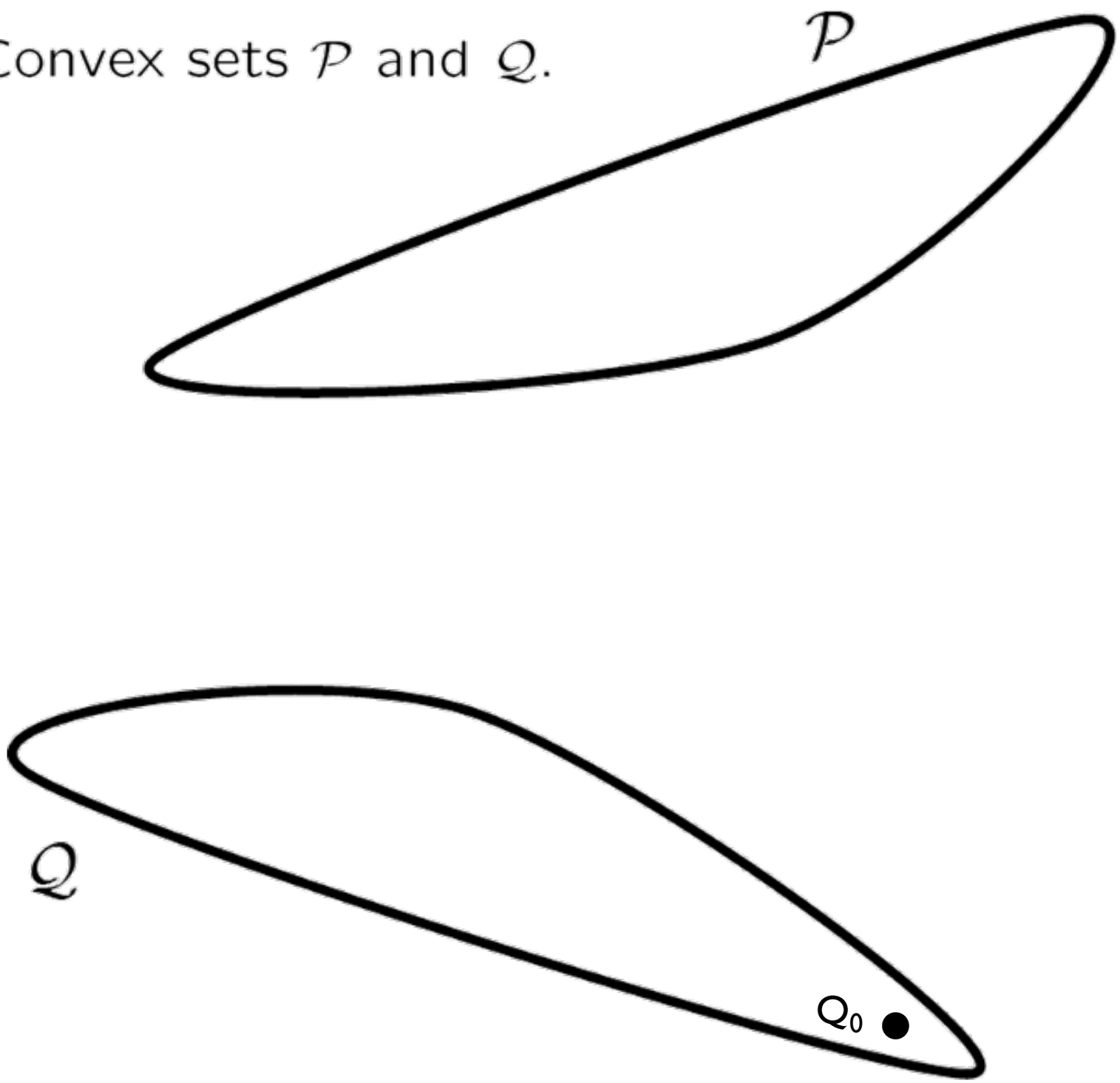
Given distance $d(P, Q)$ with $P \in \mathcal{P}$ and $Q \in \mathcal{Q}$.

Alternating Minimization

Convex sets $\mathcal{P}$ and $\mathcal{Q}$.

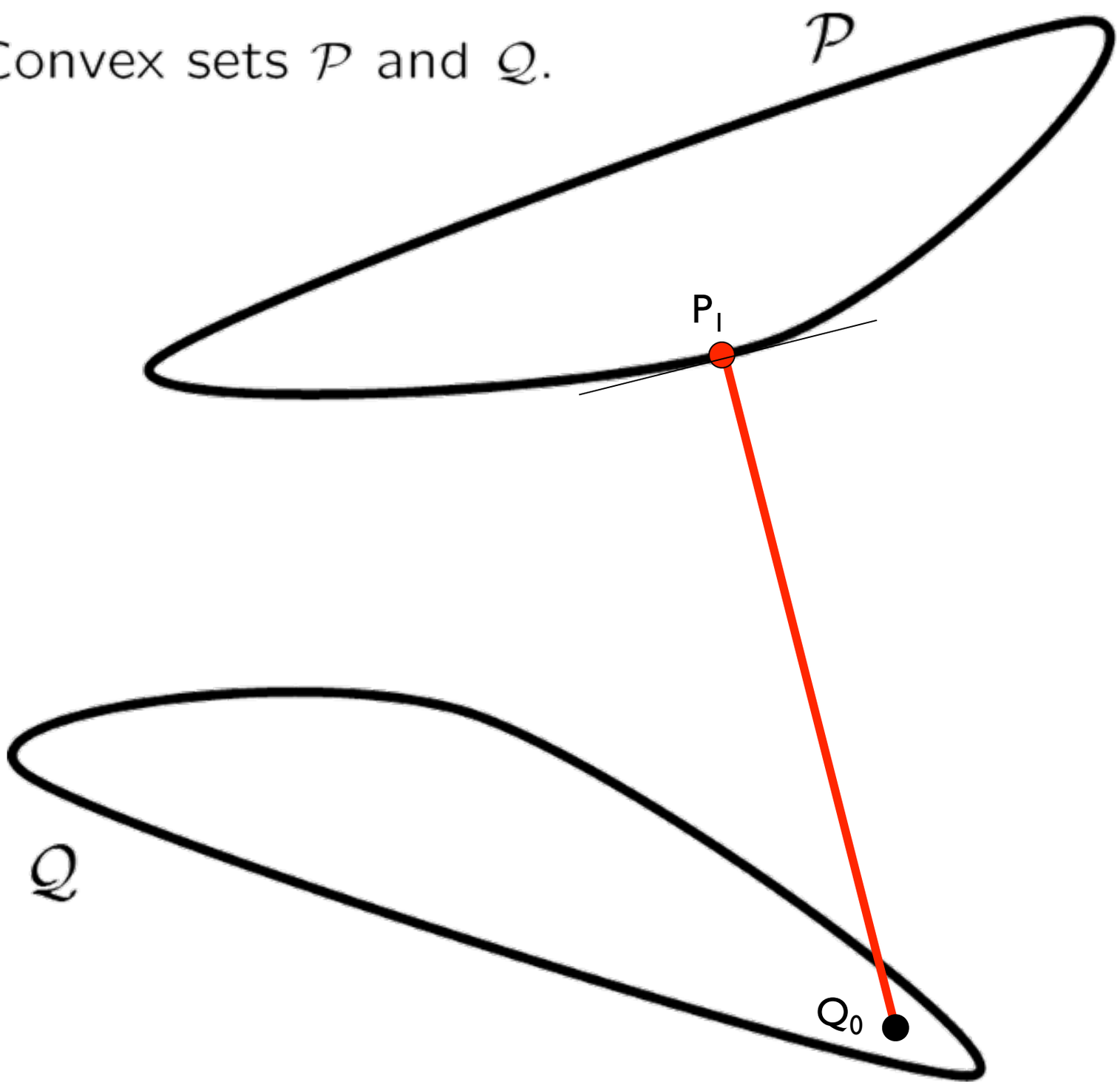
Given distance $d(P, Q)$ with $P \in \mathcal{P}$ and $Q \in \mathcal{Q}$.

Start with $Q_0 \in \mathcal{Q}$



Alternating Minimization

Convex sets $\mathcal{P}$ and $\mathcal{Q}$.



Given distance $d(P, Q)$ with $P \in \mathcal{P}$ and $Q \in \mathcal{Q}$.

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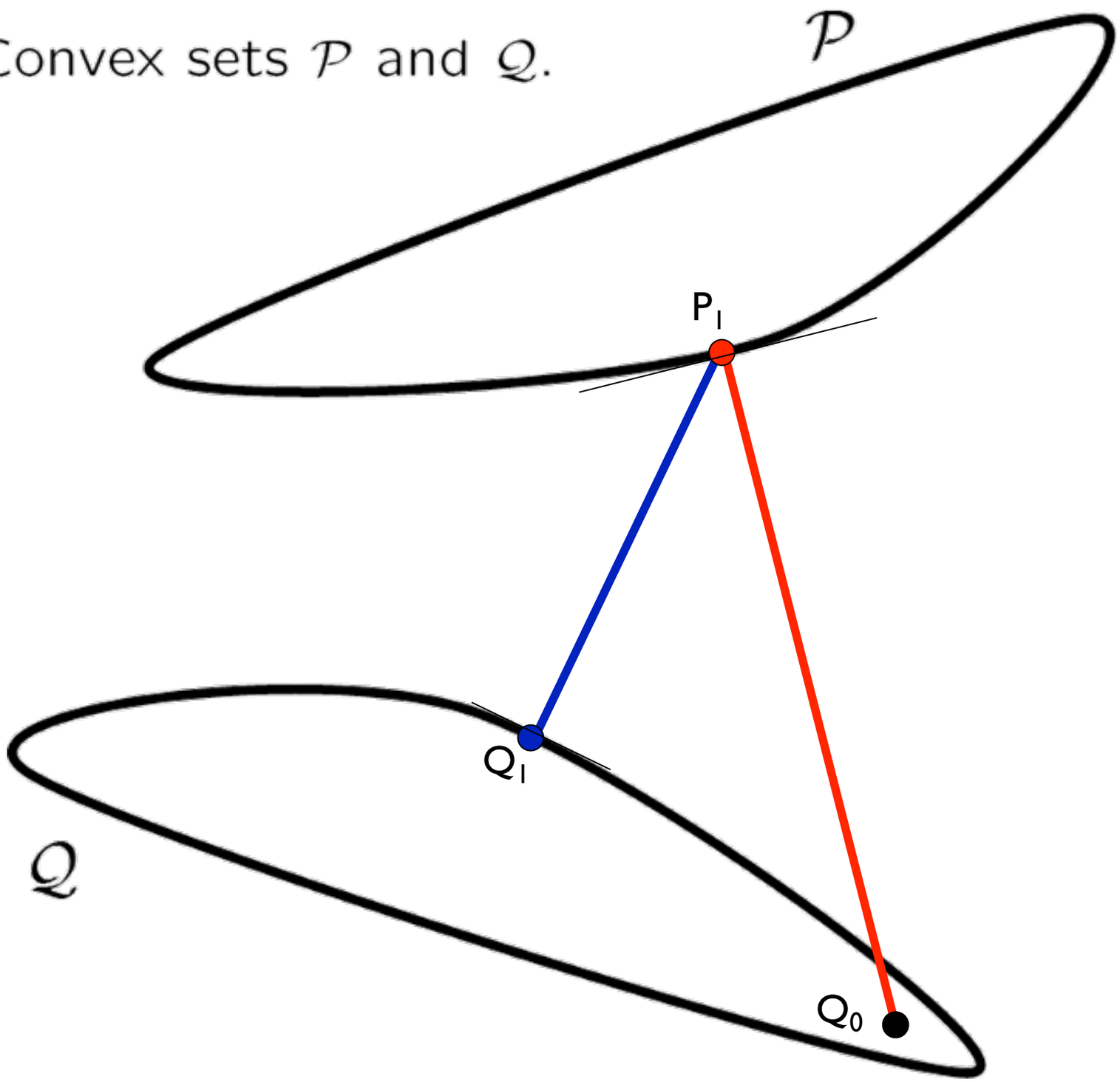
$$P_1 = \operatorname*{argmin}_P d(P, Q_0)$$

Alternating Minimization

Convex sets $\mathcal{P}$ and $\mathcal{Q}$.



Given distance $d(P, Q)$ with $P \in \mathcal{P}$ and $Q \in \mathcal{Q}$.

Start with $Q_0 \in \mathcal{Q}$

$$P_1 = \operatorname*{argmin}_P d(P, Q_0)$$

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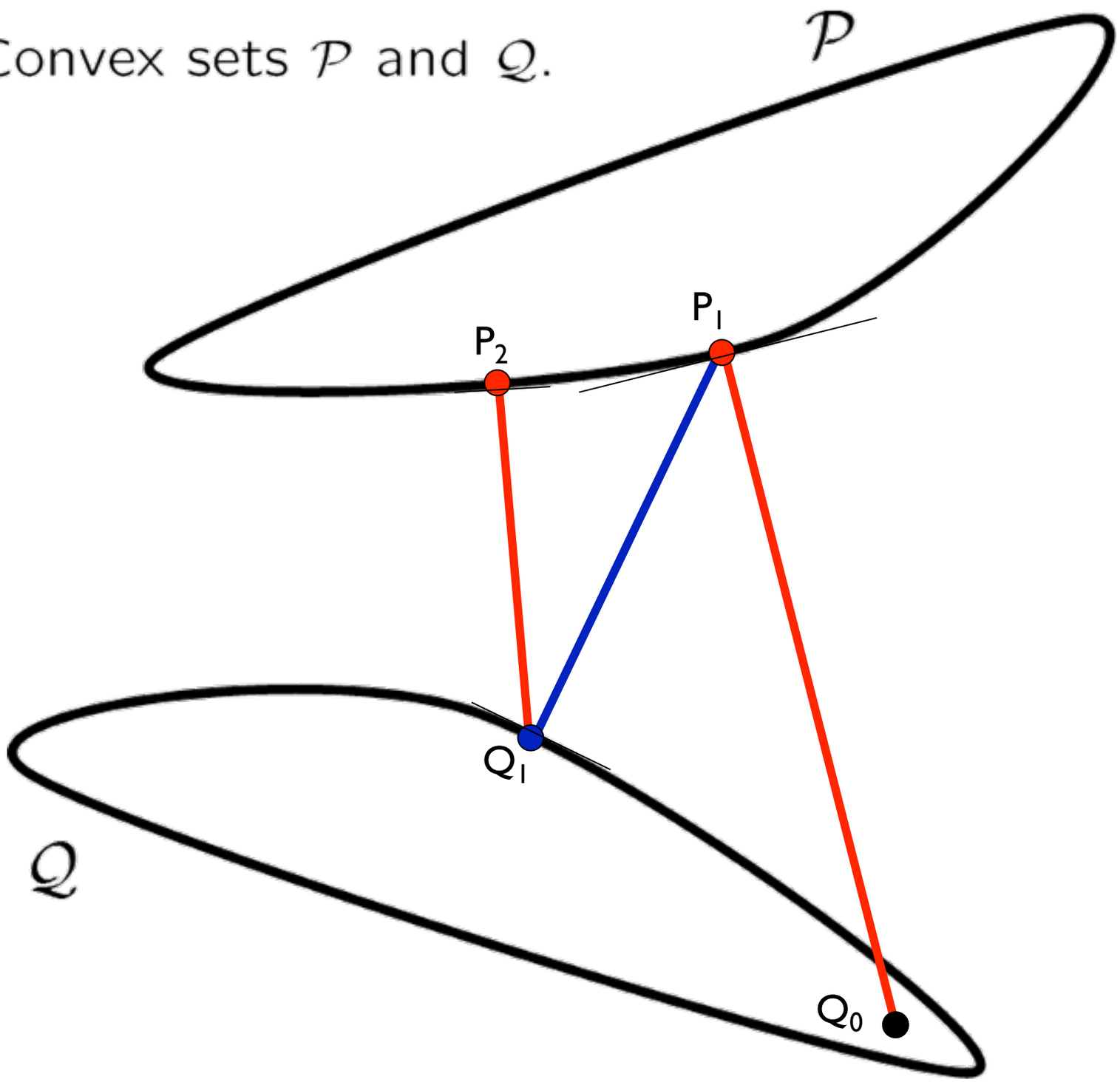
Alternating Minimization

Convex sets $\mathcal{P}$ and $\mathcal{Q}$.



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Start with $Q_0 \in \mathcal{Q}$

$$P_1 = \operatorname*{argmin}_P d(P, Q_0)$$

$$Q_1 = \operatorname*{argmin}_Q d(P_1, Q)$$

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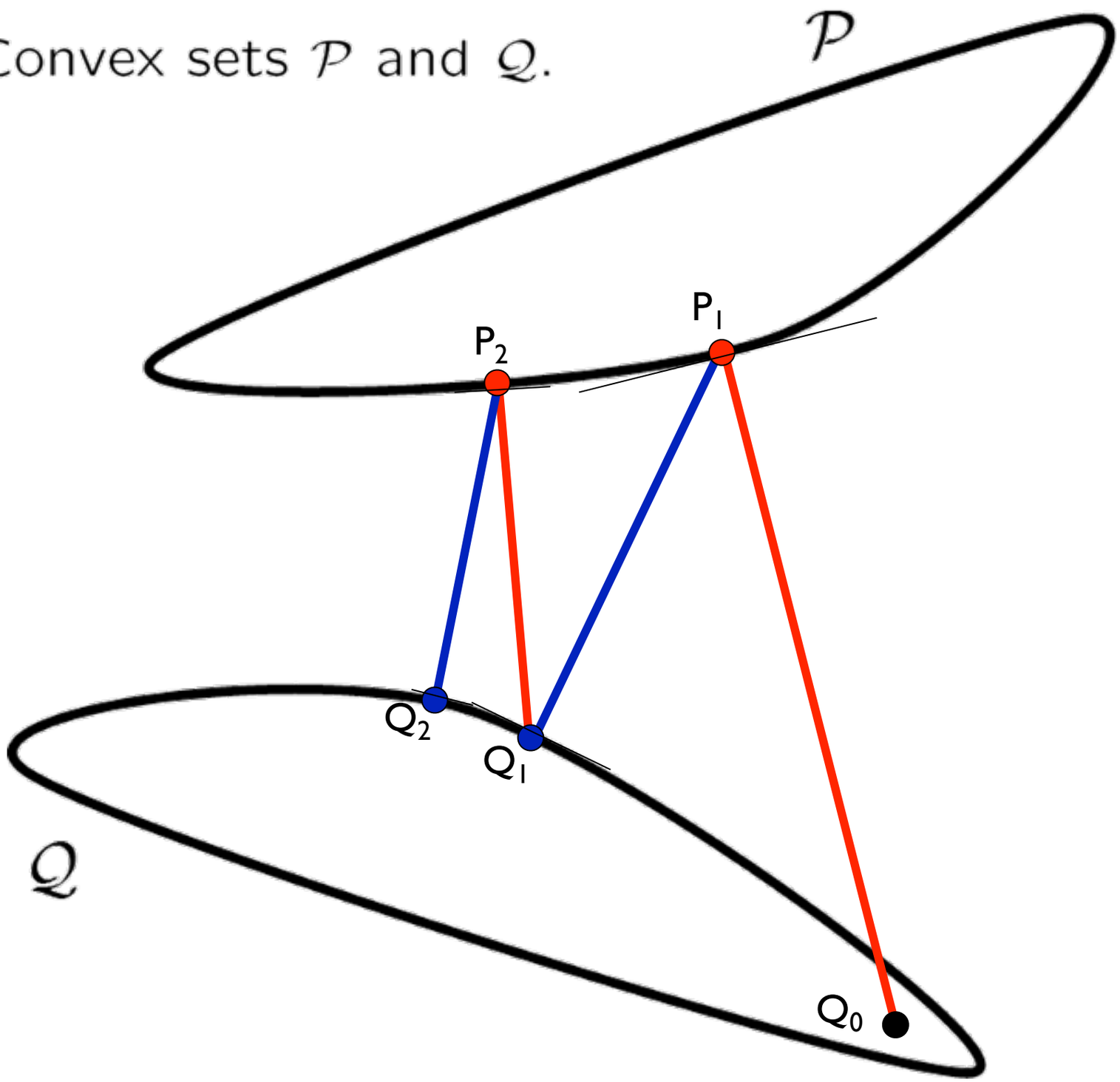
Alternating Minimization

Convex sets $\mathcal{P}$ and $\mathcal{Q}$.



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Start with $Q_0 \in \mathcal{Q}$

$$P_1 = \underset{P}{\operatorname{argmin}}\, d(P, Q_0)$$

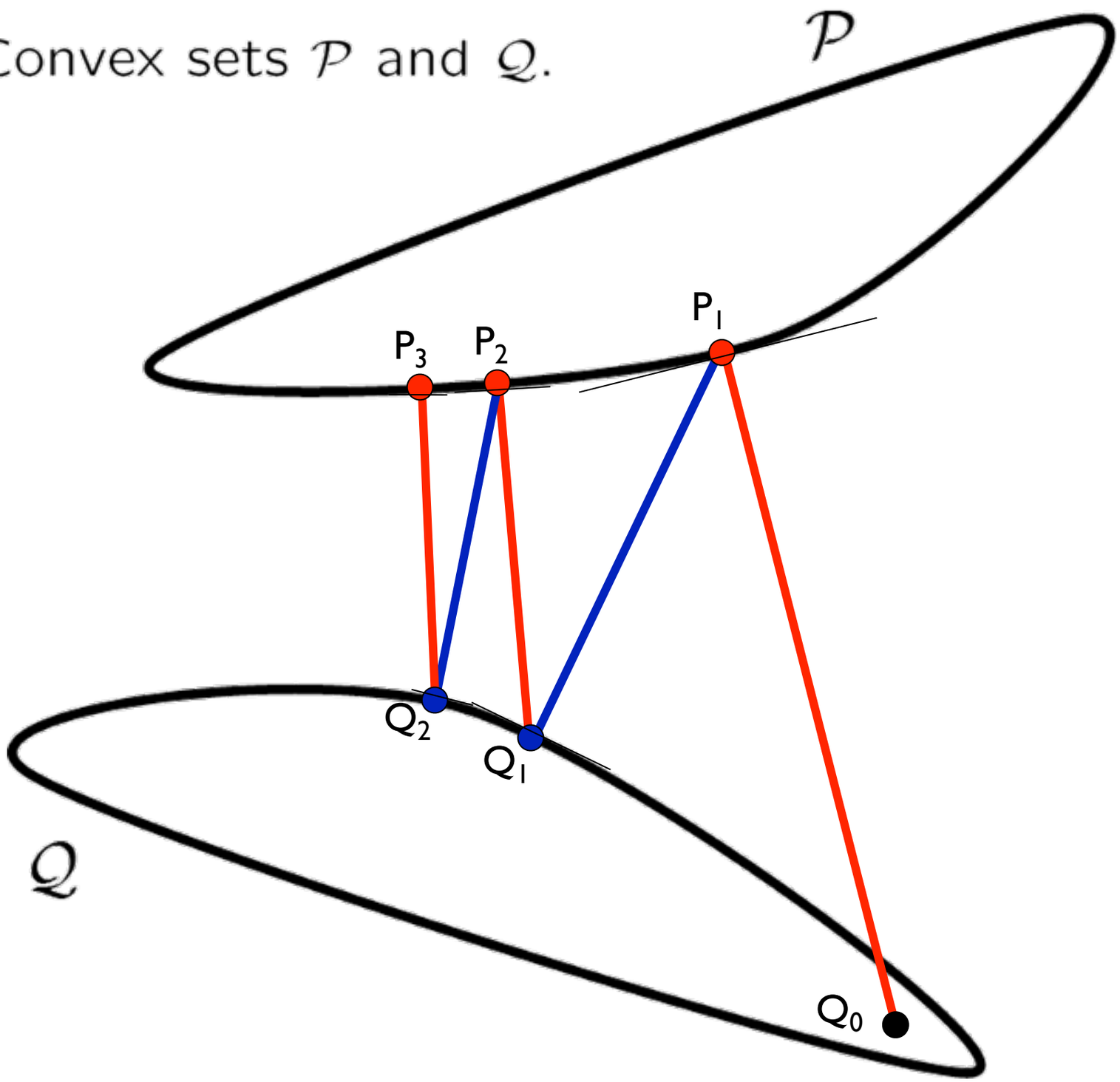
$$Q_1 = \underset{Q}{\operatorname{argmin}}\, d(P_1, Q)$$

$$P_2 = \underset{P}{\operatorname{argmin}}\, d(P, Q_1)$$

$$Q_2 = \underset{Q}{\operatorname{argmin}}\, d(P_2, Q)$$

Alternating Minimization

Convex sets $\mathcal{P}$ and $\mathcal{Q}$.



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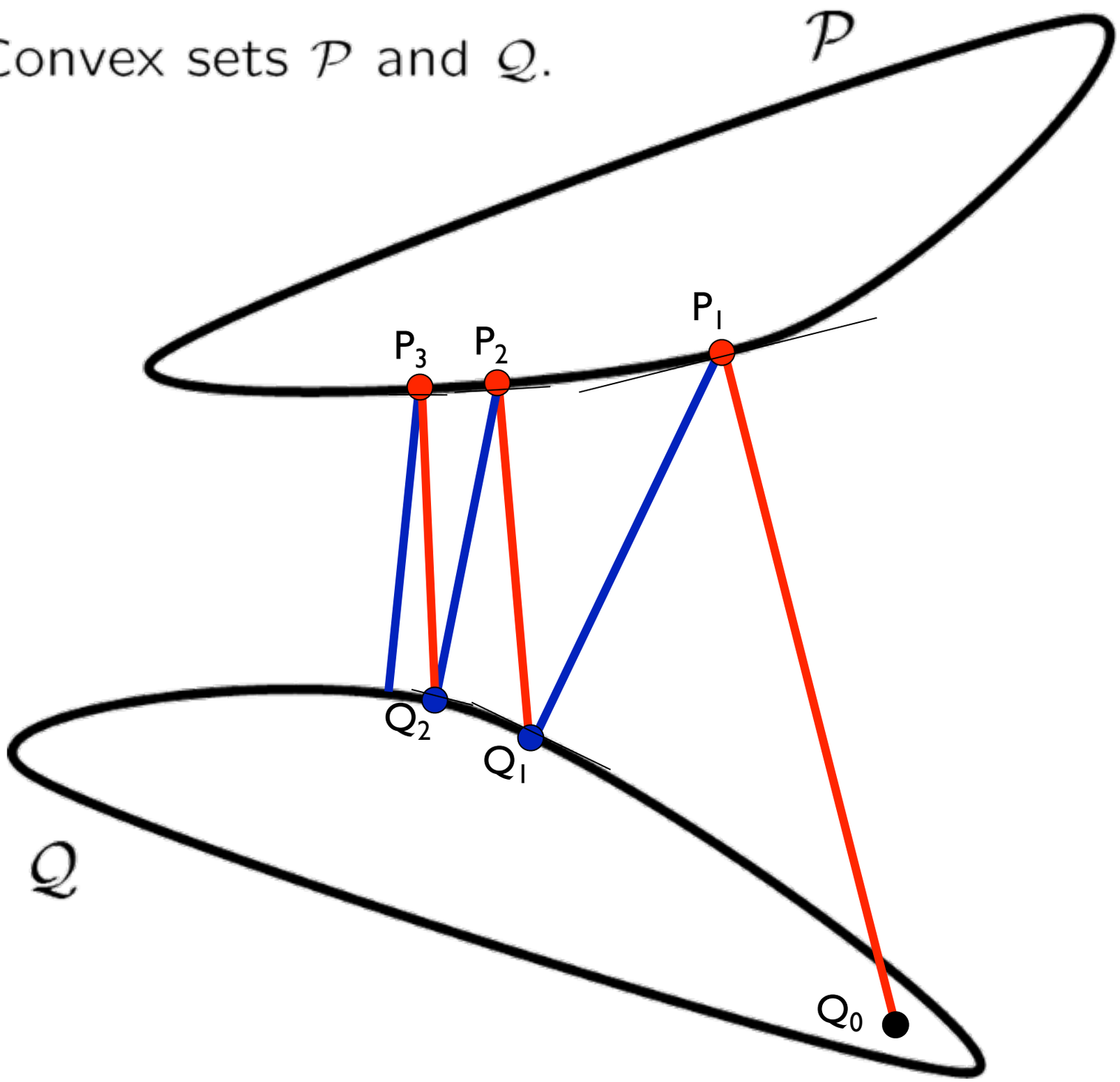

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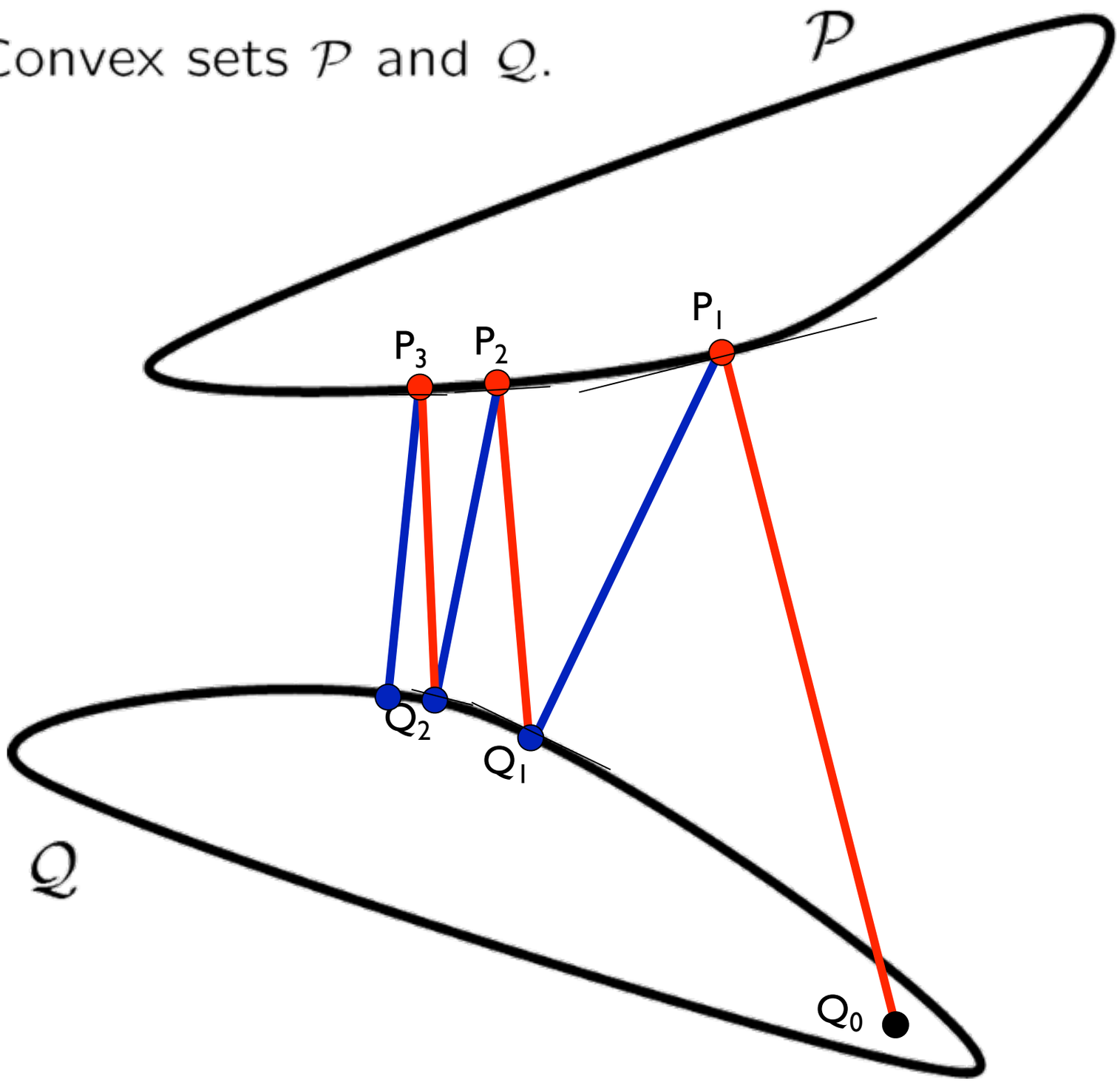

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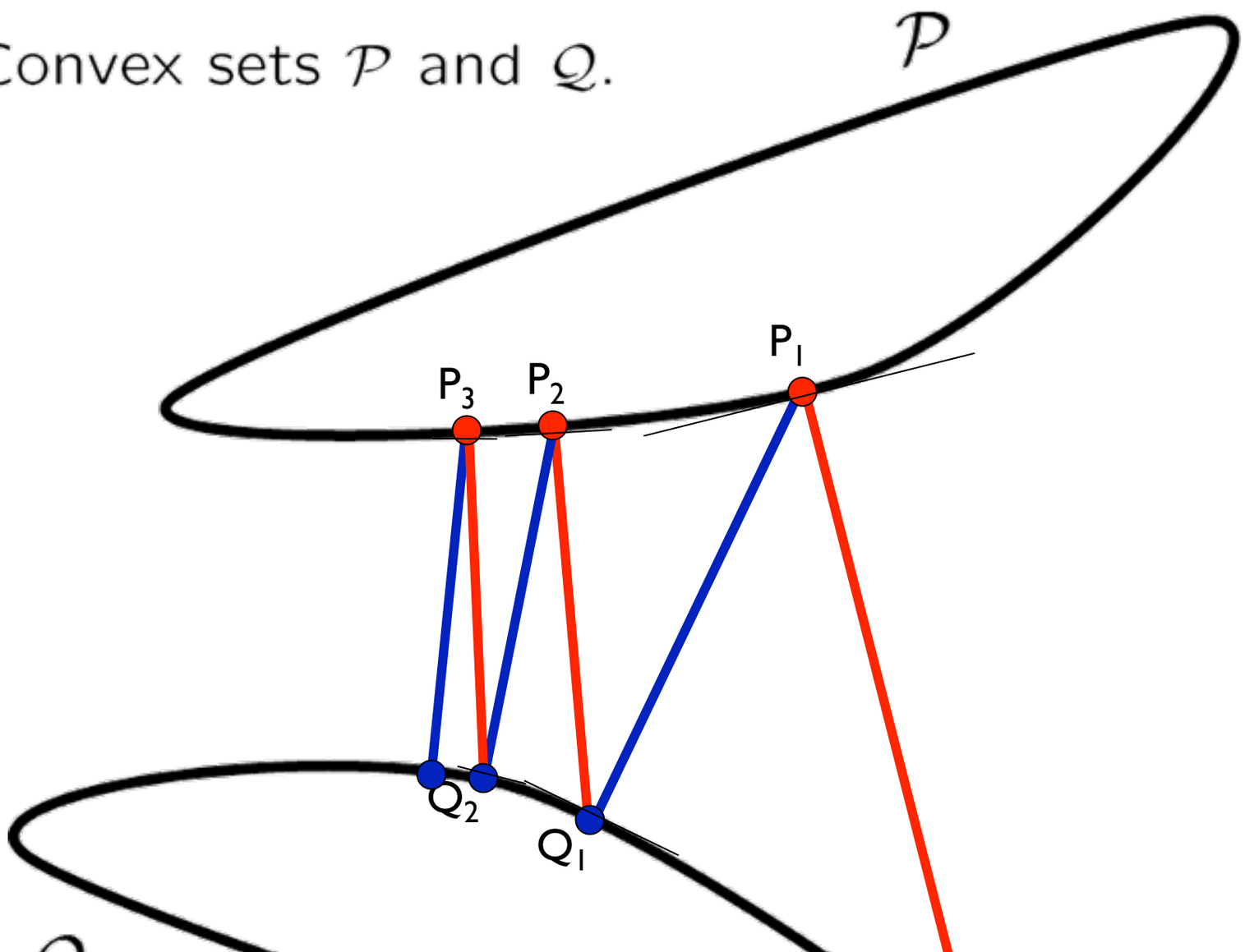

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$$Q_1 = \underset{Q}{\operatorname{argmin}}\, d(P_1, Q)$$

$$P_2 = \underset{P}{\operatorname{argmin}}\, d(P, Q_1)$$

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C_{MP} satisfies the necessary conditions for AM to converge [Subramanya and Bilmes, JMLR 2011]

Why AM?

Why AM?

Criteria	MOM	AM
Iterative	YES	YES
Learning Rate	Armijo Rule	None
Number of Hyper-parameters	7	1 (α)
Test for Convergence	Requires Tuning	Automatic
Update Equations	Not Intuitive	Intuitive and easily Parallelized

Table 1: There are two ways to solving the proposed objective, namely, the popular numerical optimization tool method of multipliers (MOM), and the proposed approach based on alternating minimization (AM). This table compares the two approaches on various fronts.

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$$p_i^{(n)}(y) = \frac{\exp\{\frac{\mu}{\gamma_i}\sum_j w'_{ij}\log q_j^{(n-1)}(y)\}}{\sum_y \exp\{\frac{\mu}{\gamma_i}\sum_j w'_{ij}\log q_j^{(n-1)}(y)\}}$$

$$q_i^{(n)}(y) = \frac{r_i(y)\delta(i \leq l) + \mu\sum_j w'_{ji} p_j^{(n)}(y)}{\delta(i \leq l) + \mu\sum_j w'_{ji}}$$

$$\text{where } \gamma_i = \nu + \mu\sum_j w'_{ij}$$

Performance of SSL Algorithms

	COIL						OPT					
l	10	20	50	80	100	150	10	20	50	80	100	150
k-NN	34.5	53.9	66.9	77.9	79.2	83.5	79.6	83.9	85.5	90.5	92.0	93.8
SGT	40.1	61.2	78.0	88.5	89.0	89.9	90.4	90.6	91.4	94.7	**97.4**	**97.4**
LapRLS	**49.2**	61.4	78.4	80.1	84.5	87.8	89.7	**91.2**	92.3	96.1	**97.6**	**97.3**
SQ-Loss-I	**48.9**	63.0	**81.0**	87.5	89.0	90.9	**92.2**	90.2	**95.9**	**97.2**	**97.3**	**97.7**
MP	47.7	**65.7**	78.5	**89.6**	**90.2**	**91.1**	90.6	**90.8**	94.7	**96.6**	**97.0**	**97.1**

Comparison of accuracies for different number of labeled samples across COIL (6 classes) and OPT (10 classes) datasets

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Comparison of accuracies for different number of labeled samples across COIL (6 classes) and OPT (10 classes) datasets

Graph SSL can be effective when the data satisfies manifold assumption. More results and discussion in Chapter 21 of the SSL Book (Chapelle et al.)

Outline

- Motivation
- Graph Construction
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  - Manifold Regularization
  - Spectral Graph Transduction
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  - Sparse Label Propagation
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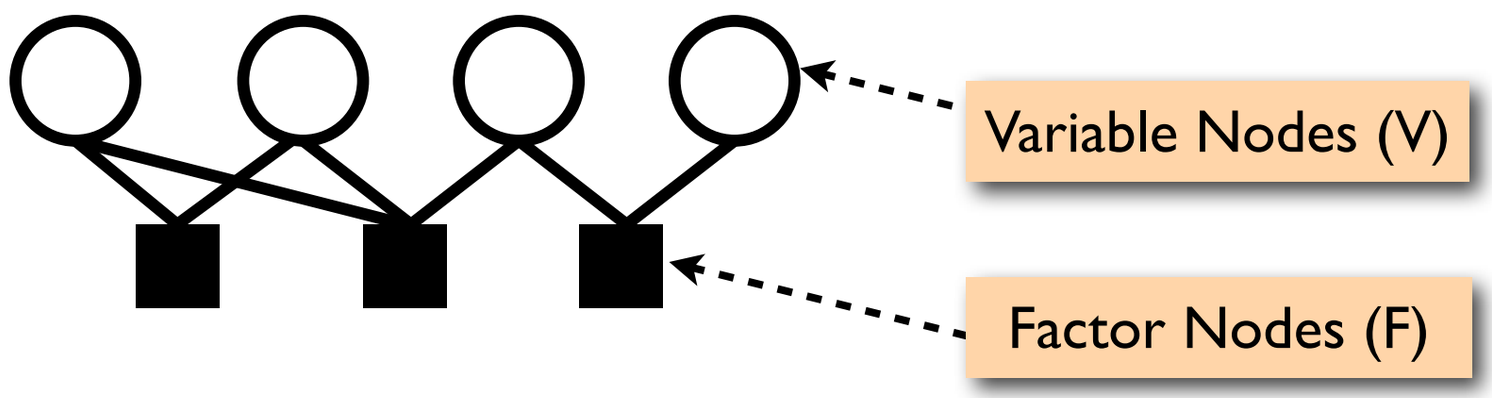
Background: Factor Graphs
[Kschischang et al., 2001]

Factor Graph

- bipartite graph
- variable nodes (e.g., label distribution on a node)
- factor nodes: fitness function over variable assignment



Distribution over all variables' values

$$\log P\left(\{v\}_{v\in V}\right) \quad = \quad -\log Z + \sum_{f\in F} \log \alpha_f \left(\{v\}_{(v,f)\in E}\right)$$

variables connected to factor f

Factor Graph Interpretation of Graph SSL [Zhu et al., ICML 2003] [Das and Smith, NAACL 2012]

3-term Graph SSL Objective (common to many algorithms)

min Seed Matching Loss (if any) + Edge Smoothness Loss + Regularization Loss

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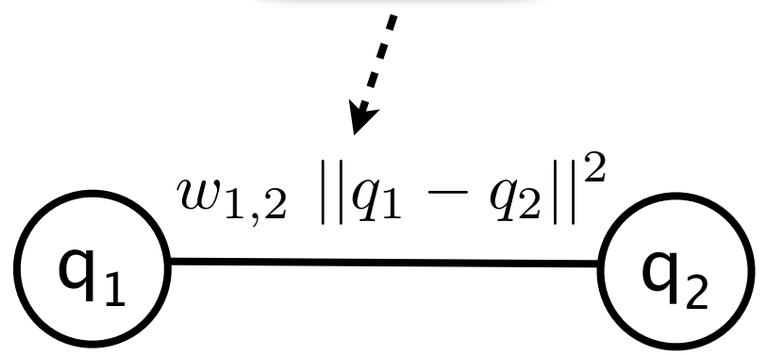

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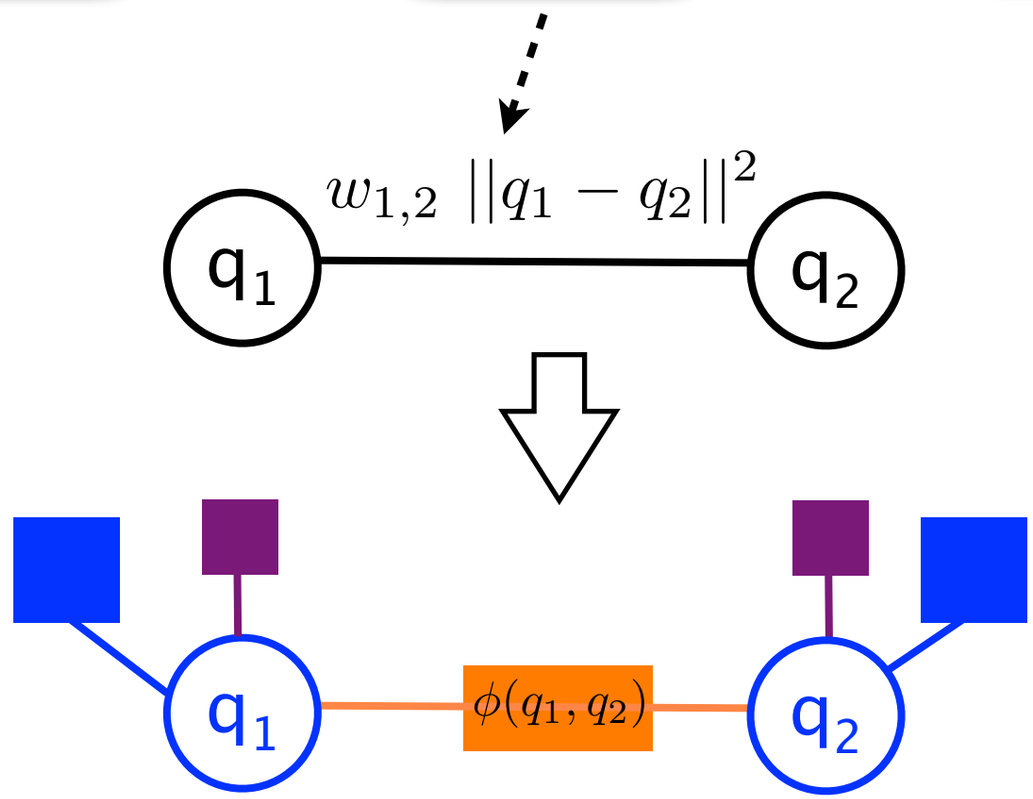

Factor Graph Interpretation of Graph SSL [Zhu et al., ICML 2003] [Das and Smith, NAACL 2012]

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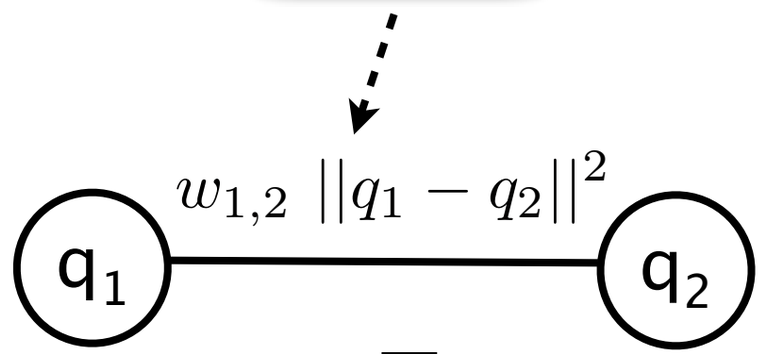

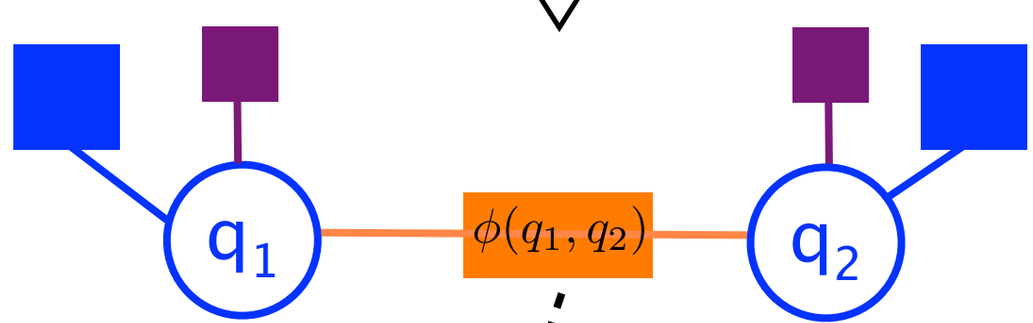

Smoothness Factor

$$\phi(q_1, q_2) \propto w_{1,2}\ ||q_1 - q_2||^2$$

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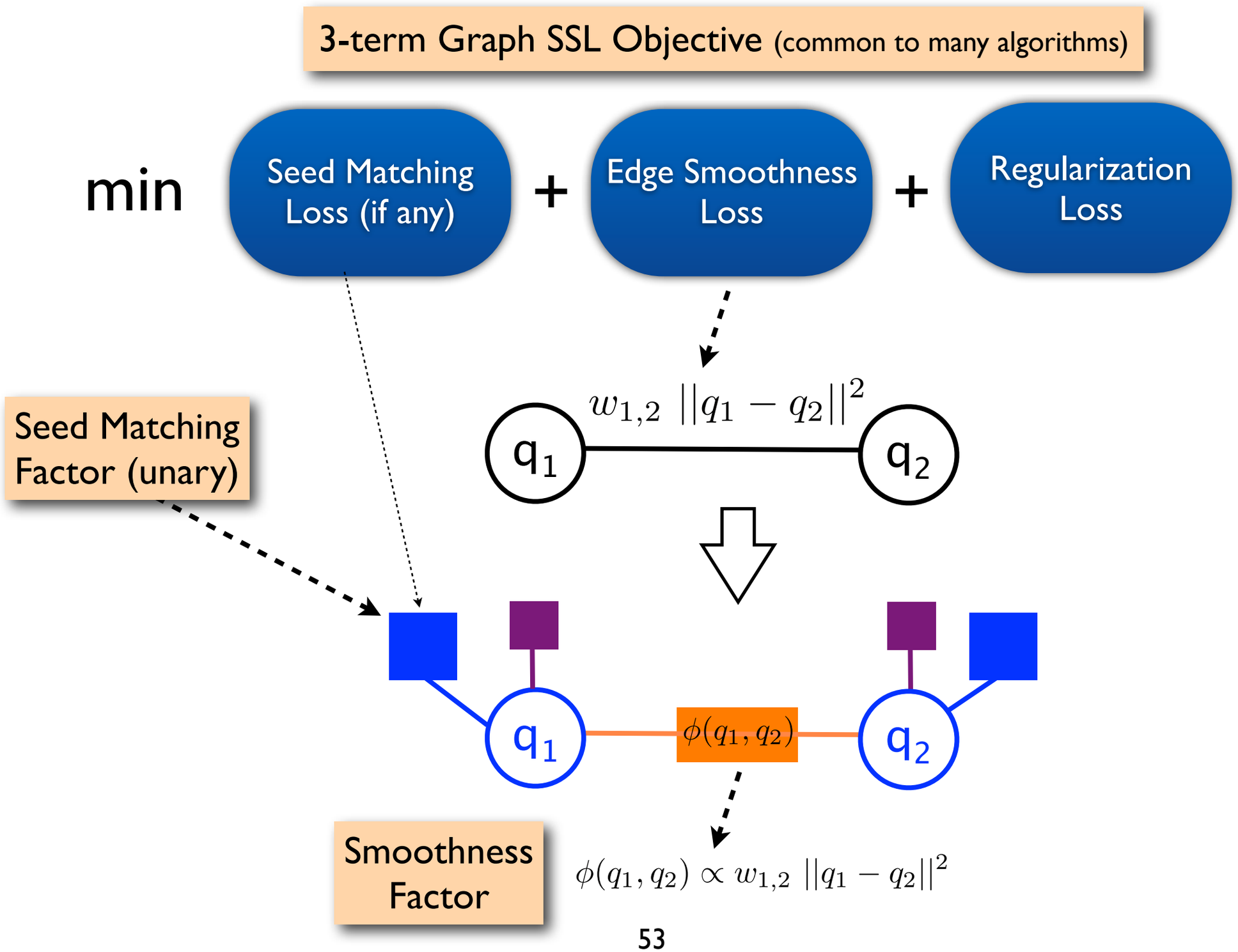

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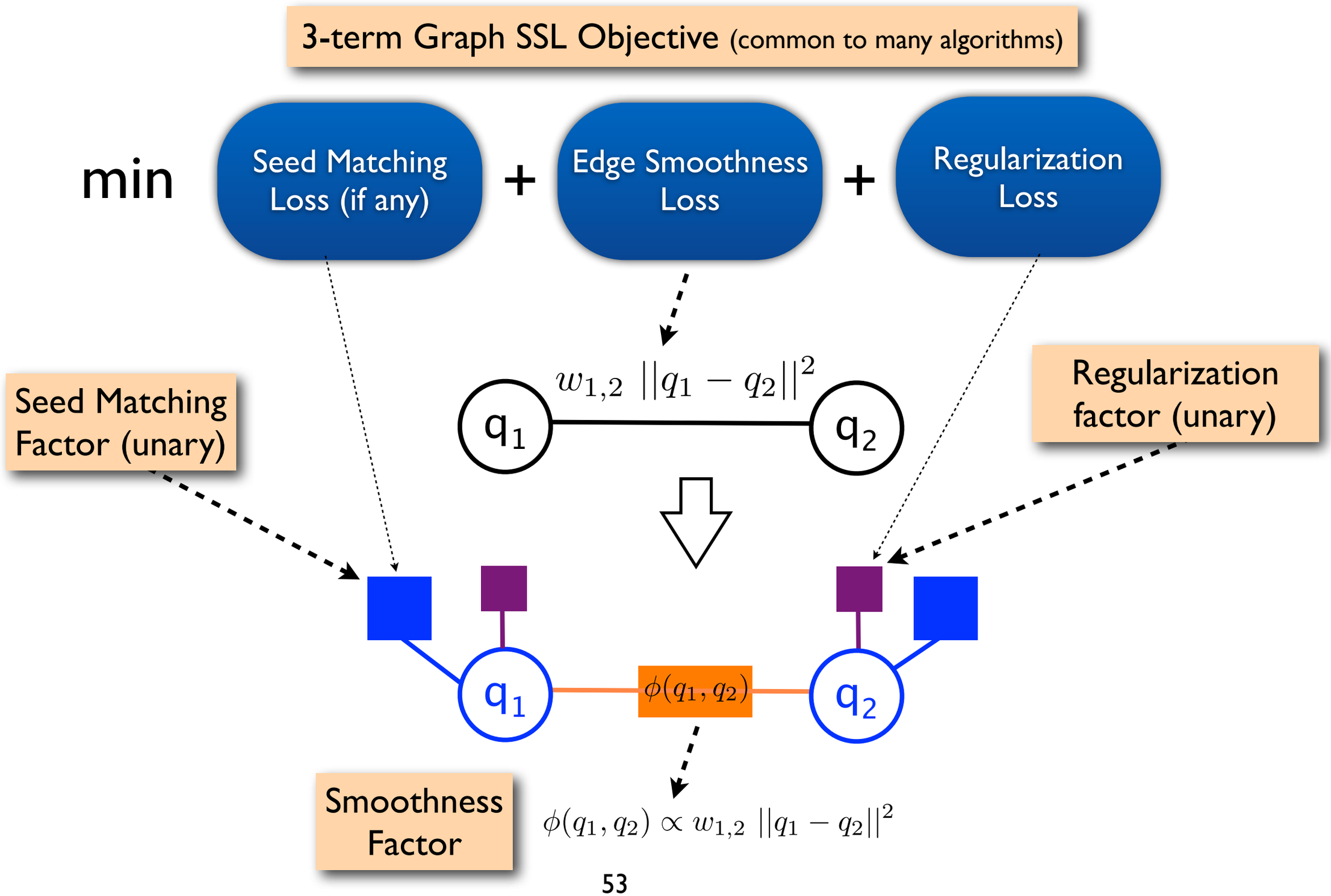

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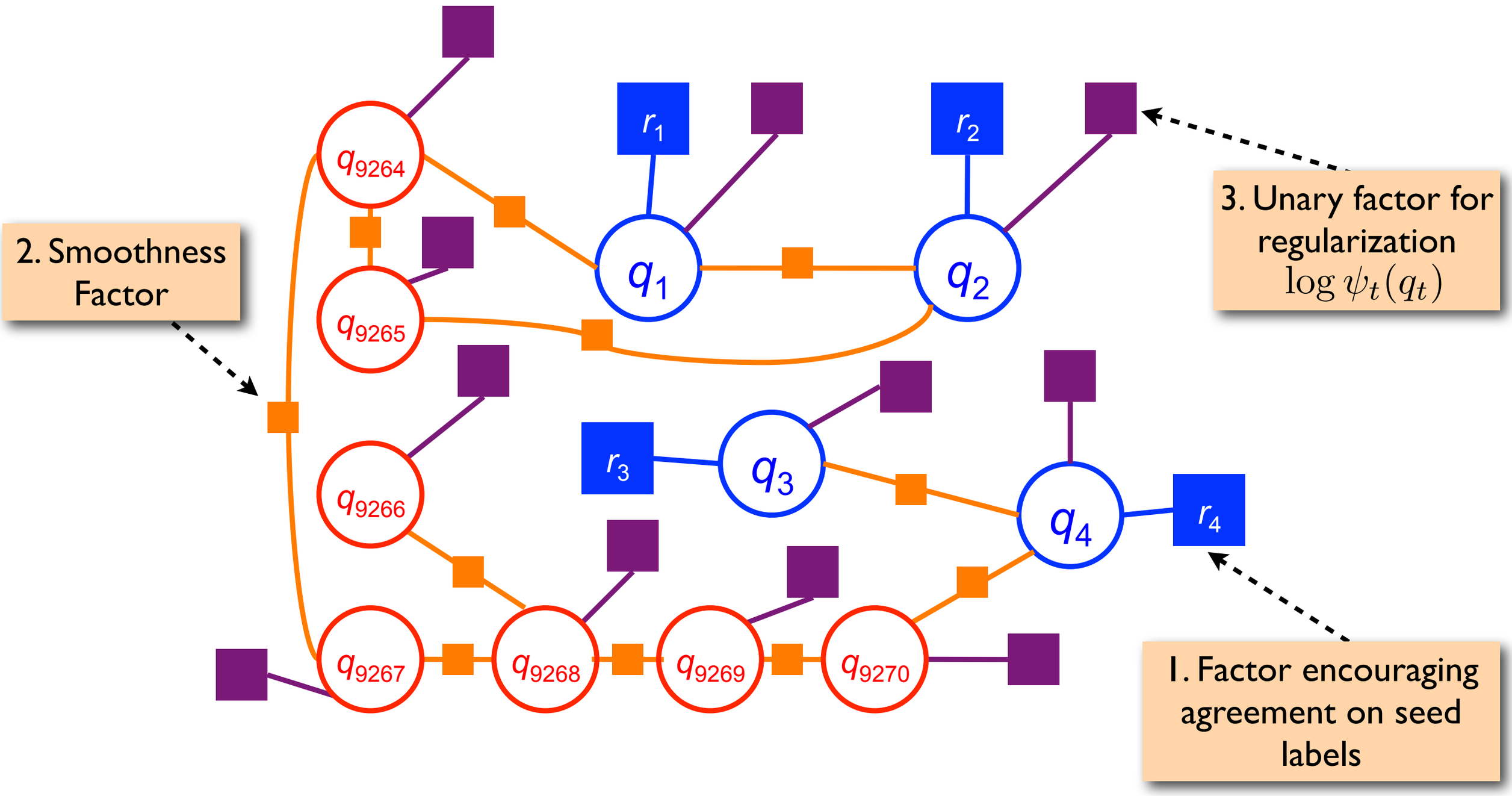

Label Propagation with Sparsity

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Enforce through sparsity inducing unary factor

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Lasso (Tibshirani, 1996) $\log \psi_t(q_t) = -\lambda \left\| q_t \right\|_1$

Elitist Lasso (Kowalski and Torrésani, 2009)

$$\log \psi_t(q_t) = -\lambda \left(\left\| q_t \right\|_1 \right)^2$$

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For more details, see [Das and Smith, NAACL 2012]

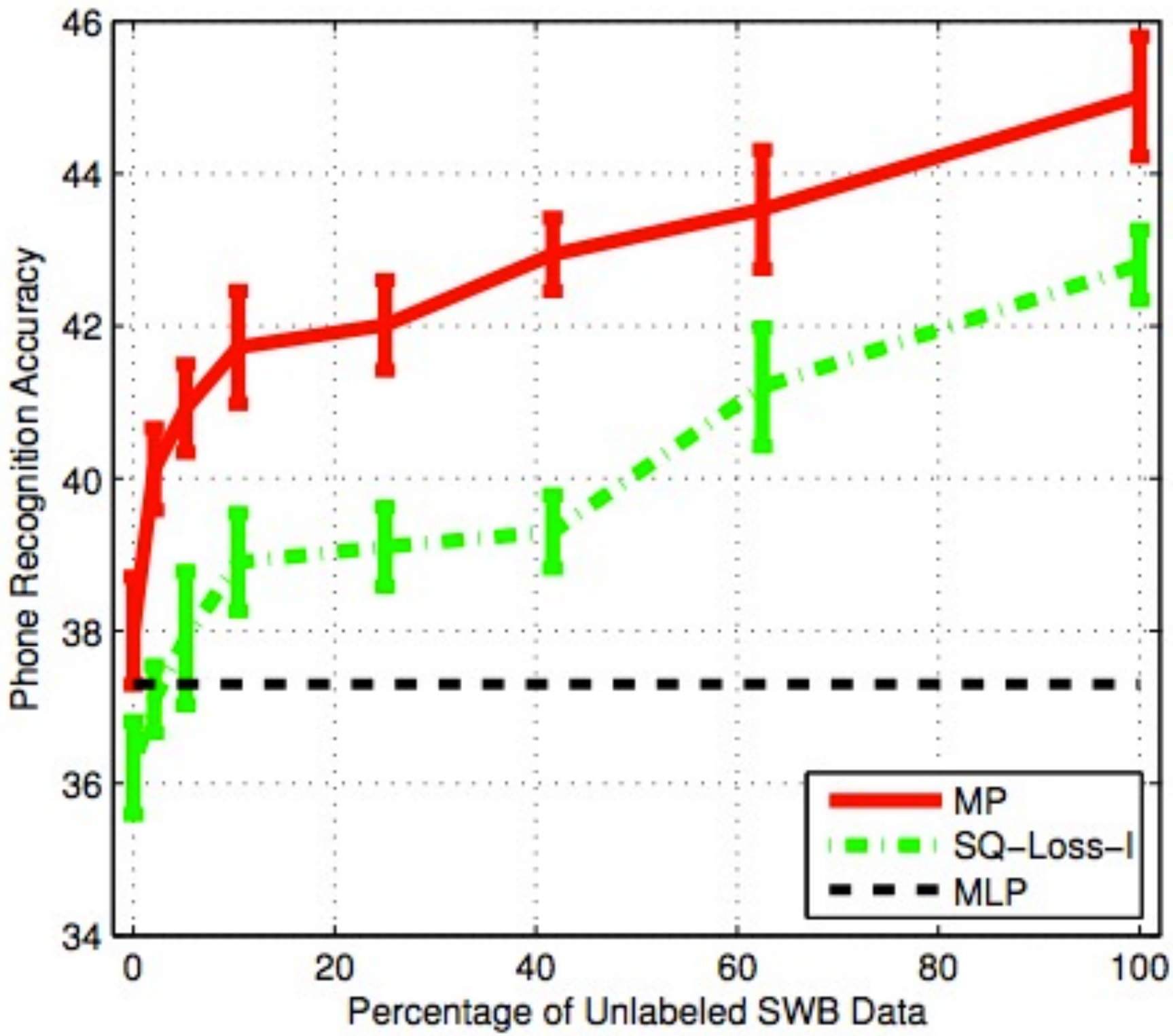
Other Graph-SSL Methods

- Spectral Graph Transduction [Joachims, ICML 2003]
- SSL on Directed Graphs
  - [Zhou et al, NIPS 2005], [Zhou et al., ICML 2005]
- Learning with dissimilarity edges
  - [Goldberg et al., AISTATS 2007]
- Learning to order: GraphOrder [Talukdar et al., CIKM 2012]
- Graph Transduction using Alternating Minimization
  - [Wang et al., ICML 2008]
- Graph as regularizer for Multi-Layered Perceptron
  - [Karlen et al., ICML 2008], [Malkin et al., Interspeech 2009]

Outline

- Motivation
- Graph Construction
- Inference Methods
- Scalability
  - Scalability Issues
  - Node reordering
  - MapReduce Parallelization
- Applications
- Conclusion & Future Work

More (Unlabeled) Data is Better Data



[Subramanya & Bilmes, JMLR 2011]

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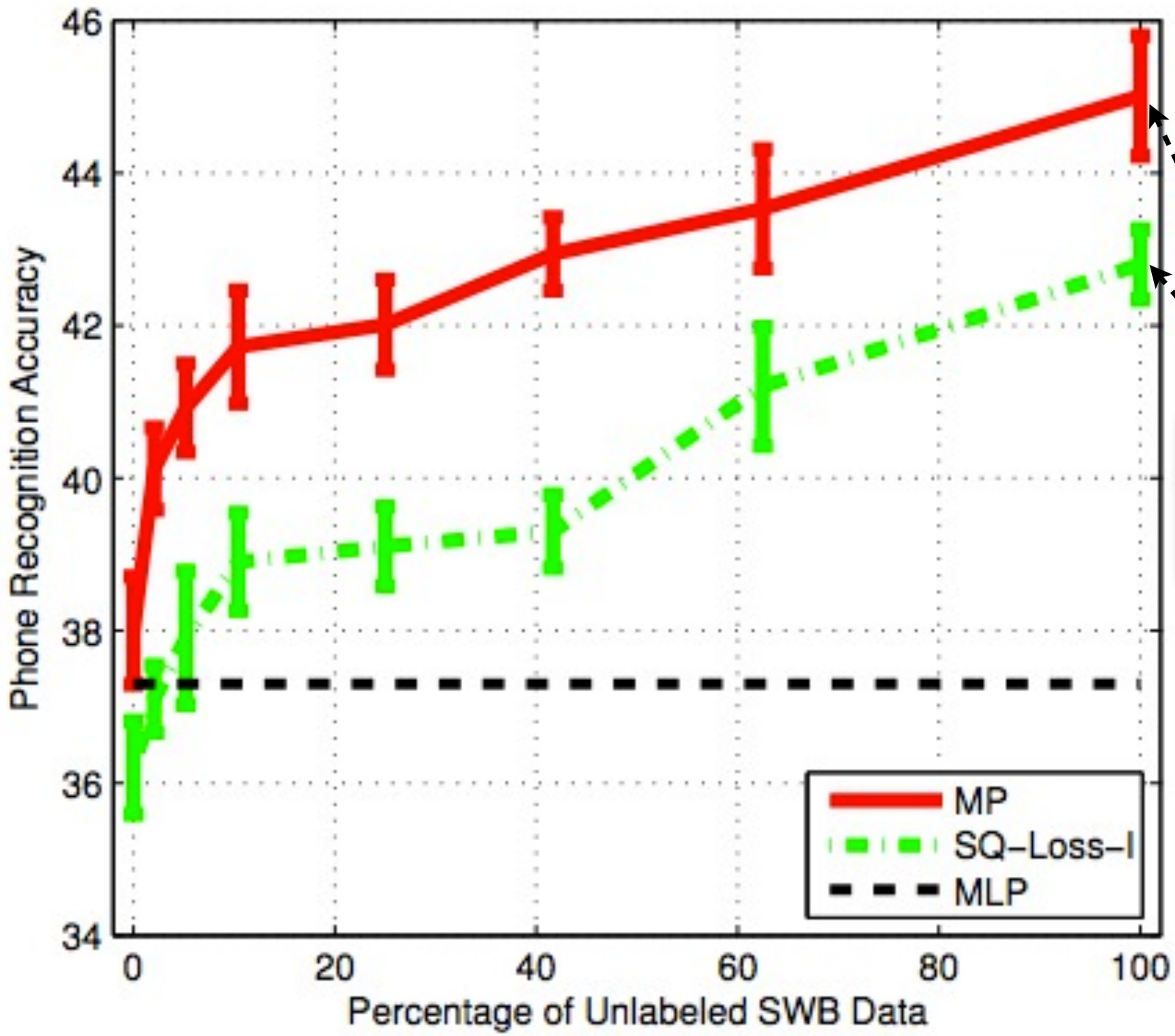

Graph with **120m** vertices

[Subramanya & Bilmes, JMLR 2011]

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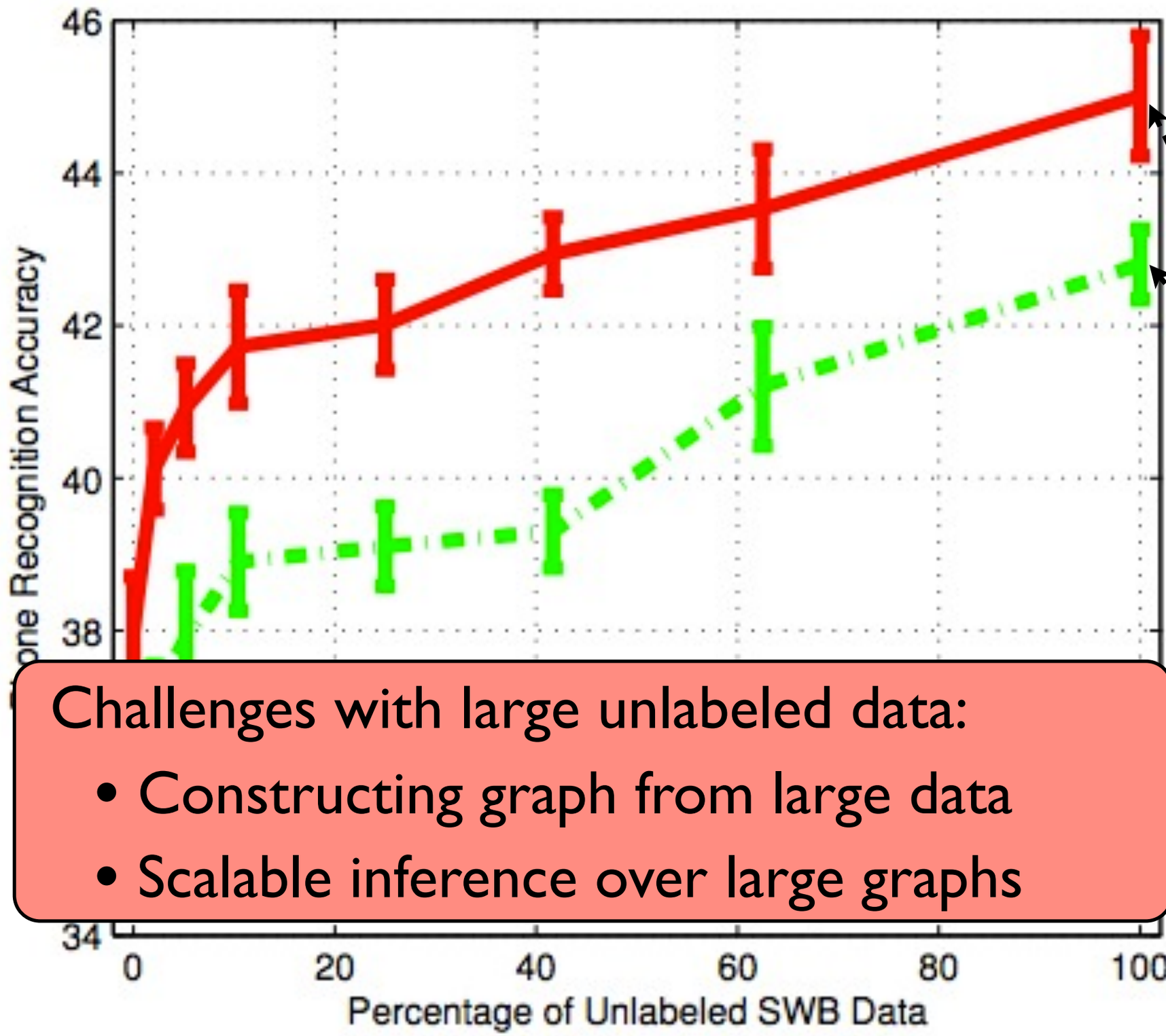

Graph with **120m** vertices

Challenges with large unlabeled data:

- Constructing graph from large data
- Scalable inference over large graphs

[Subramanya & Bilmes, JMLR 2011]

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Scalability Issues (I)

Graph Construction

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- Brute force (exact) k-NNG too expensive (quadratic)

Scalability Issues (I)

Graph Construction

- Brute force (exact) k-NNG too expensive (quadratic)
  - Approximate nearest neighbor using kd-tree [Friedman et al., 1977, also see http://www.cs.umd.edu/~mount/]

Scalability Issues (II)

Label Inference

- Sub-sample the data
  - Construct graph over a subset of a unlabeled data [Delalleau et al., AISTATS 2005]
  - Sparse Grids [Garcke & Griebel, KDD 2001]

Scalability Issues (II)

Label Inference

- Sub-sample the data
  - Construct graph over a subset of a unlabeled data [Delalleau et al., AISTATS 2005]
  - Sparse Grids [Garcke & Griebel, KDD 2001]
- How about using <u>more computation</u>? (next section)
  - Symmetric multi-processor (SMP)
  - Distributed Computer

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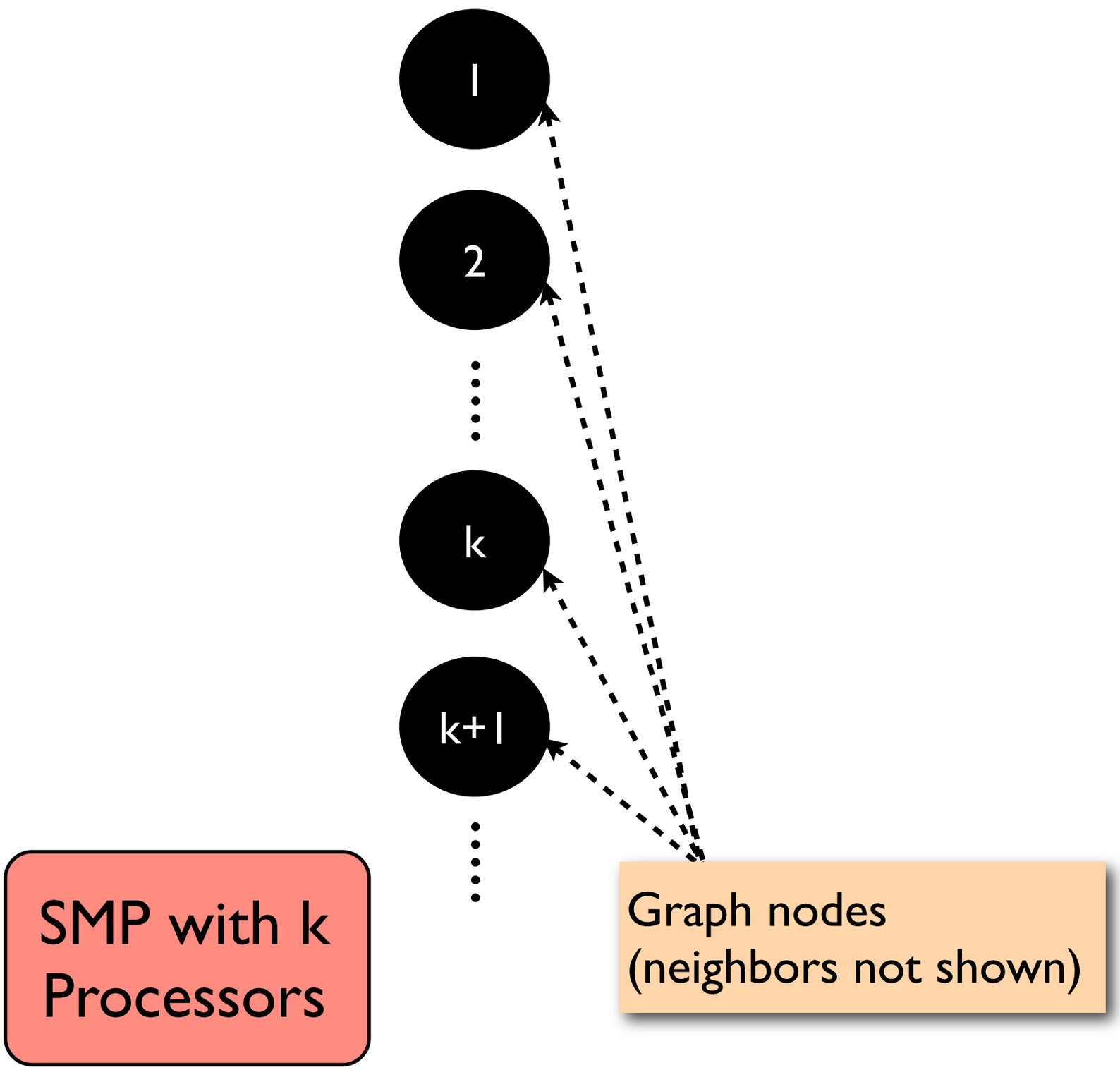
Scalability Issues

Node reordering
[Subramanya & Bilmes, JMLR 2011; Bilmes & Subramanya, 2011]

MapReduce Parallelization

Parallel computation on a SMP



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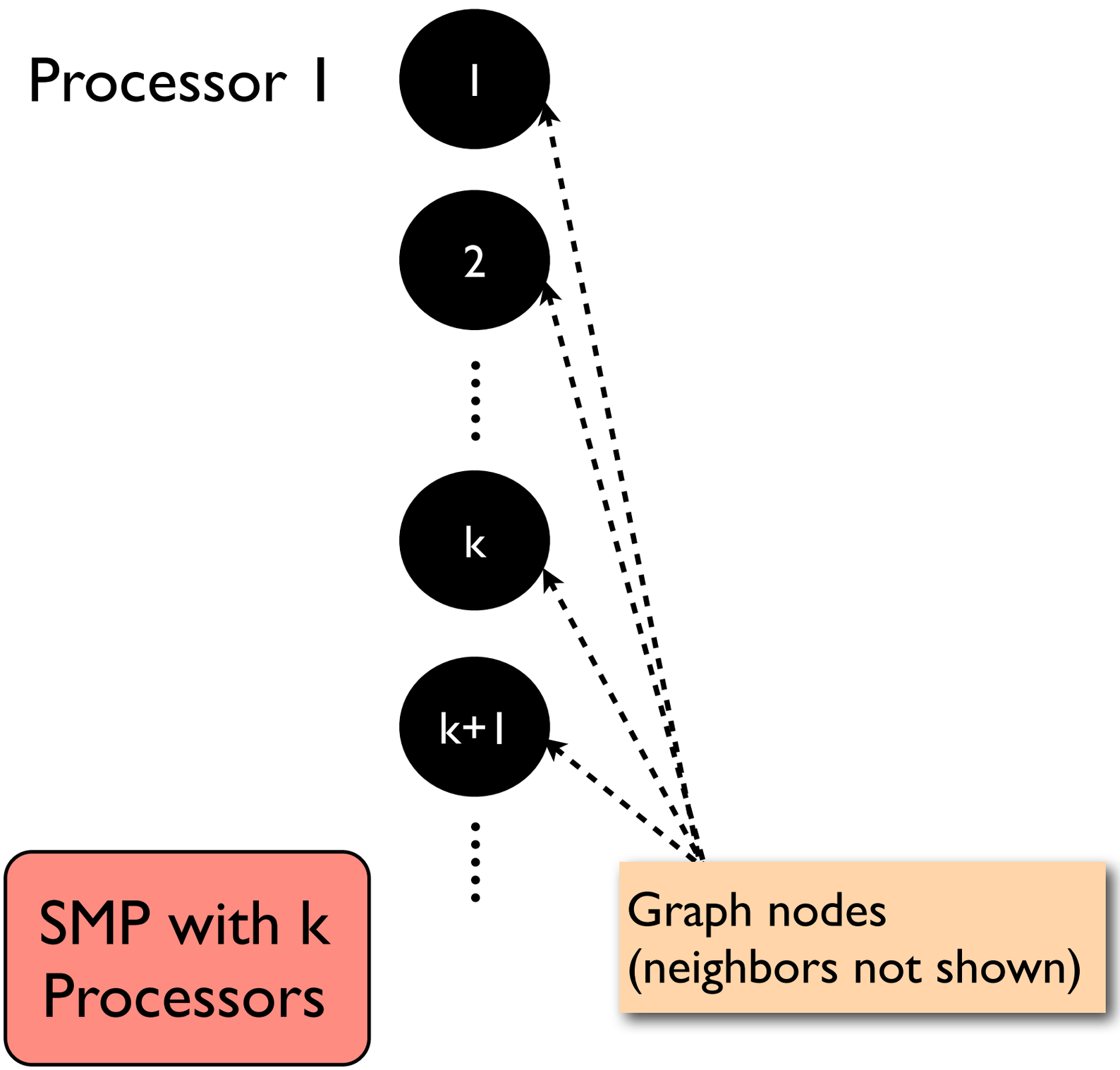

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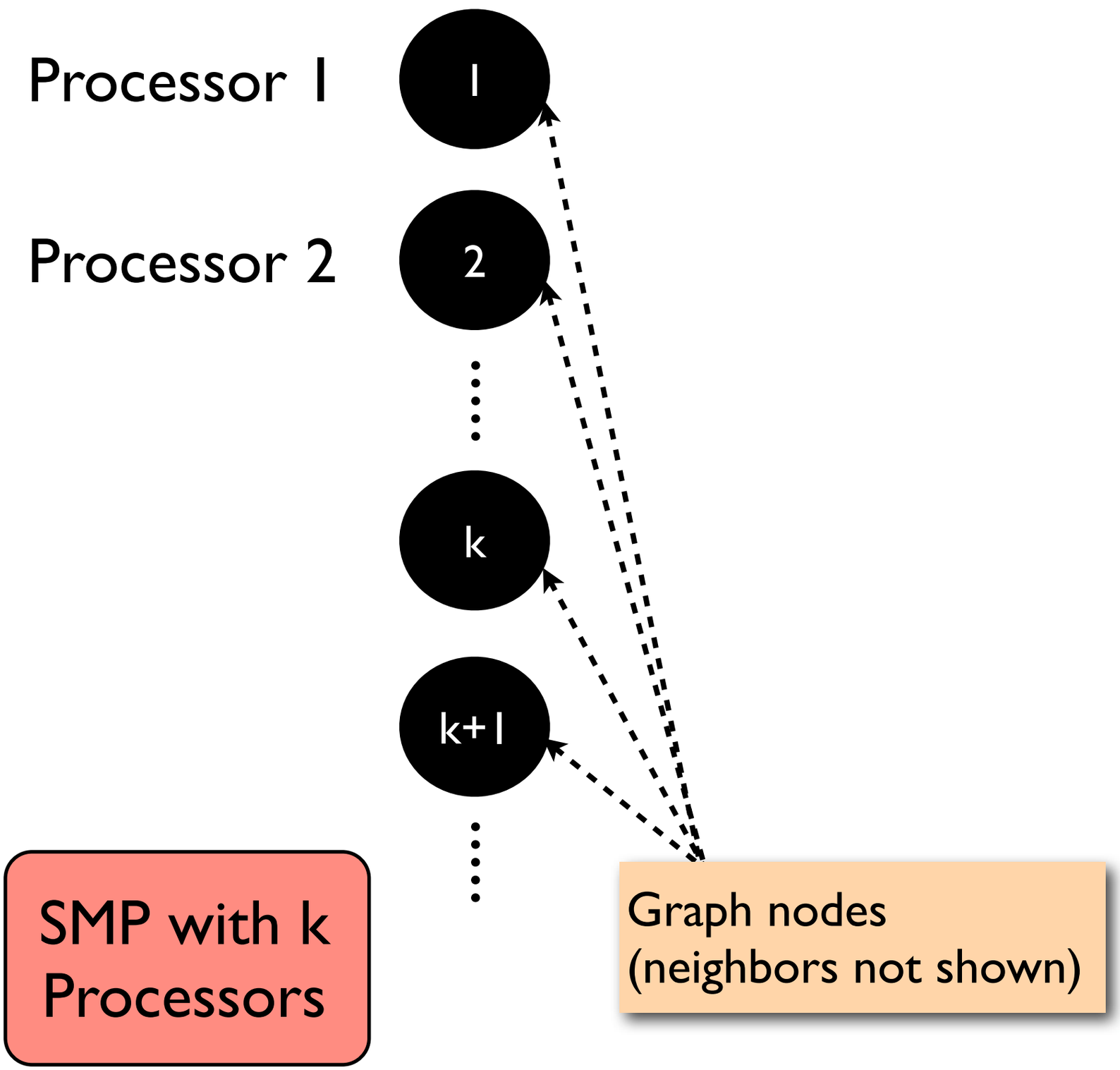

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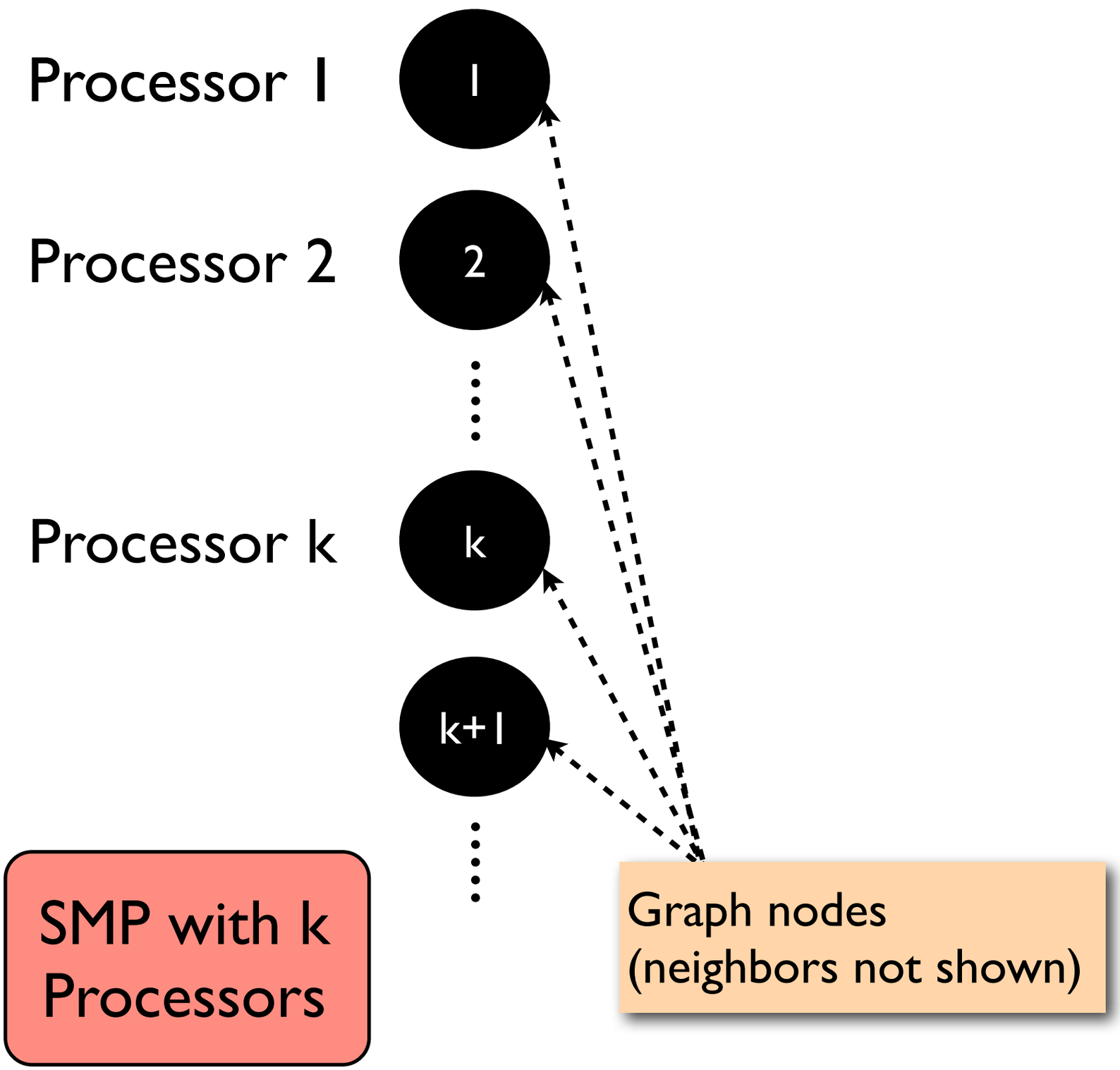

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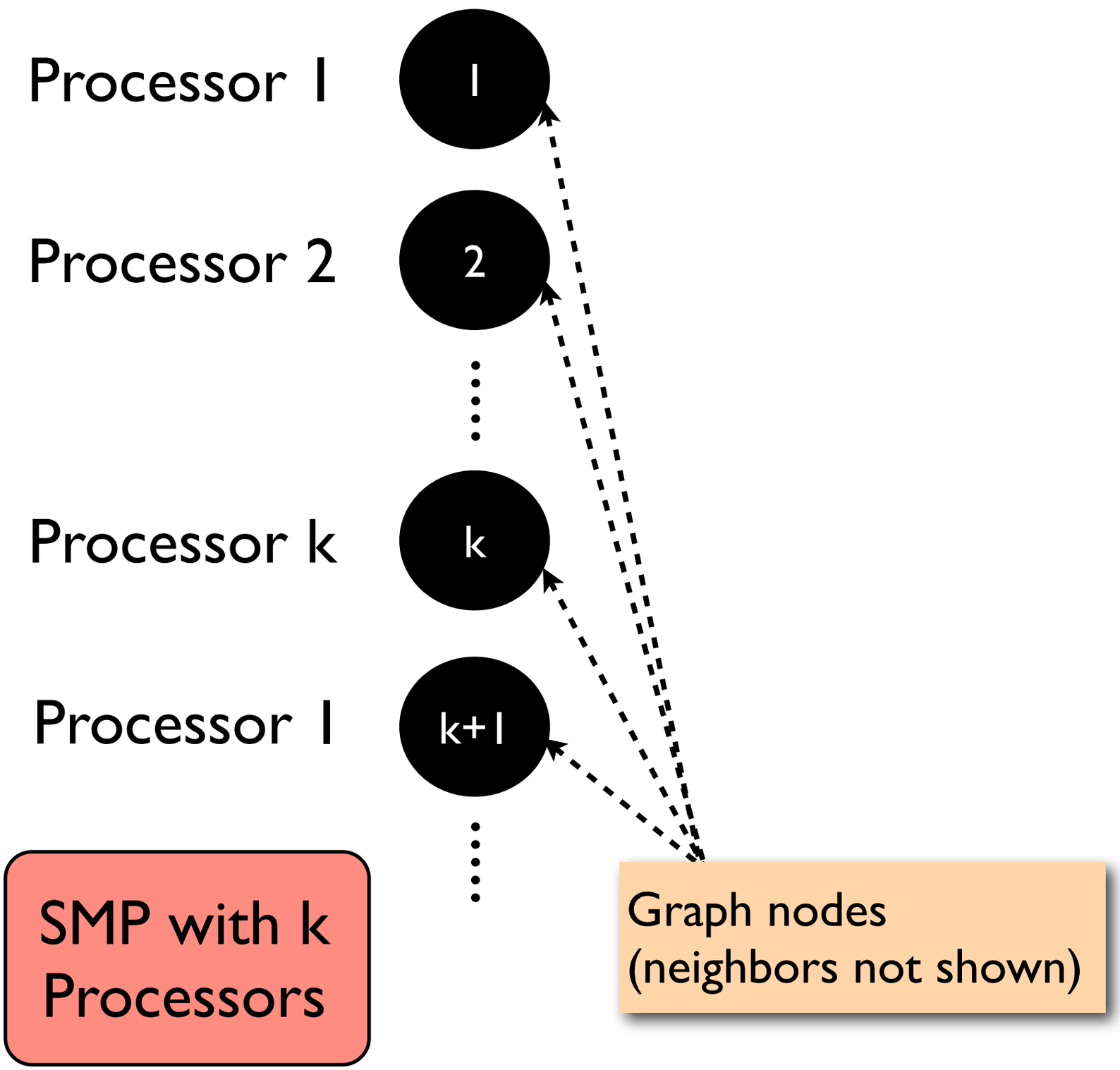
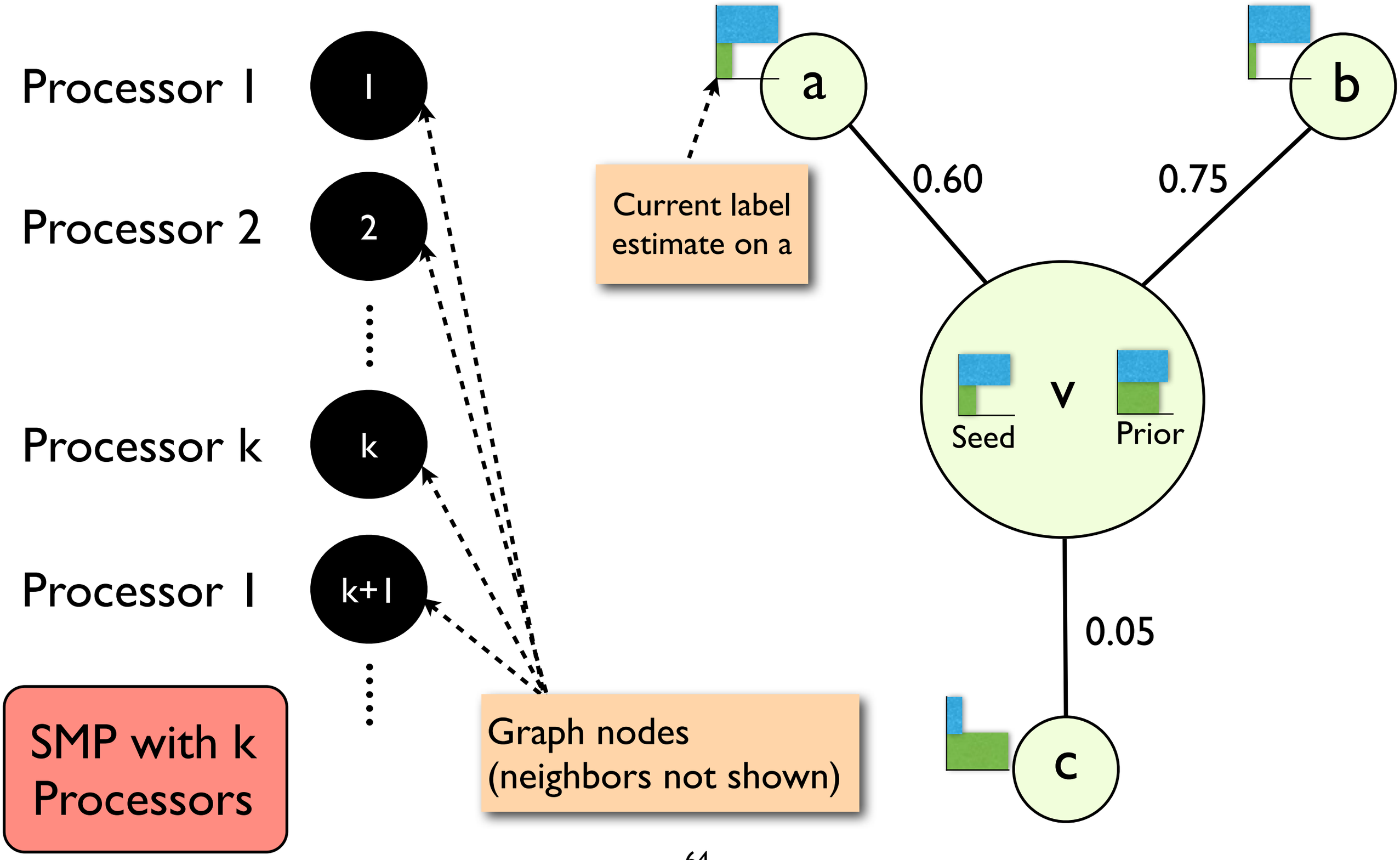


Label Update using Message Passing



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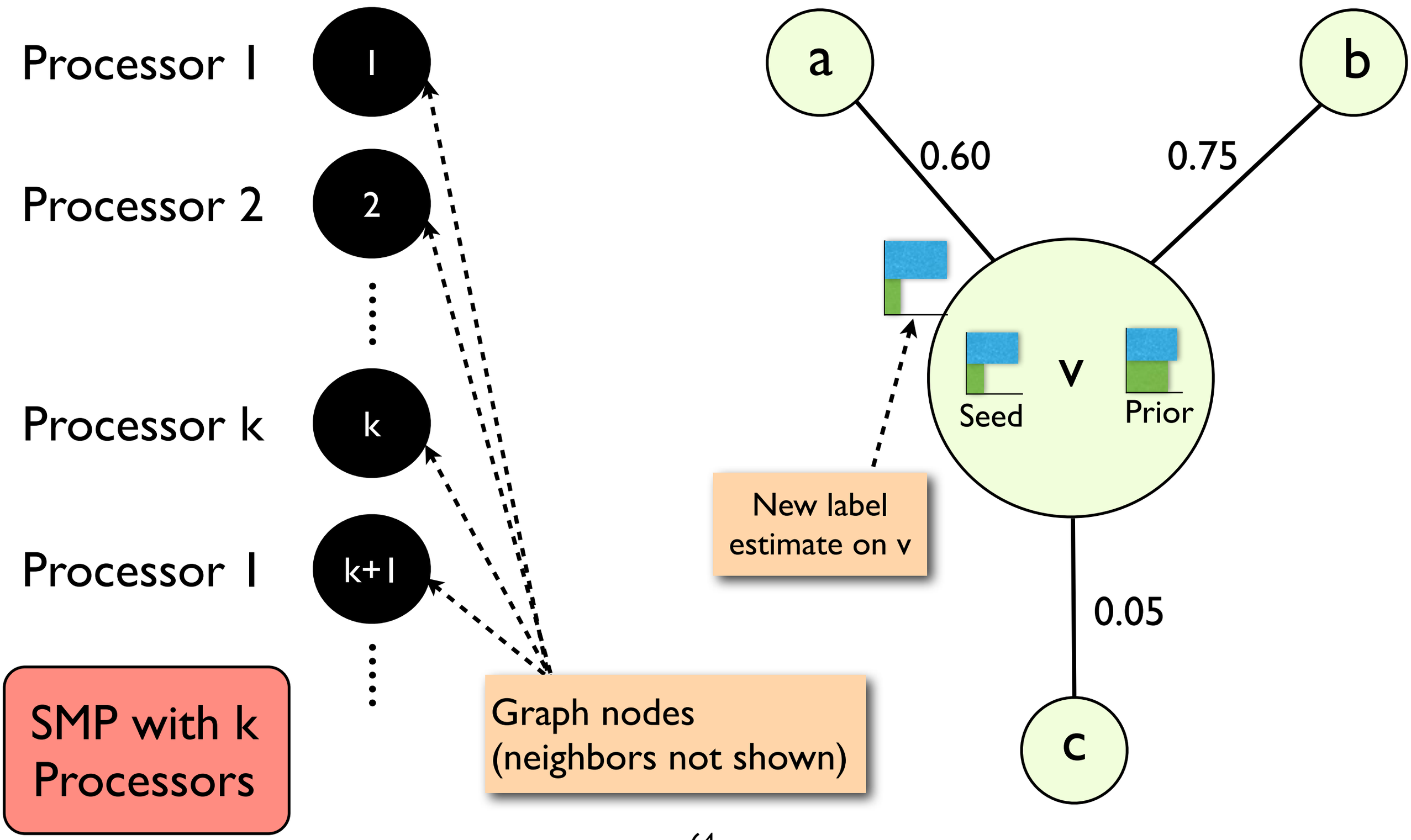
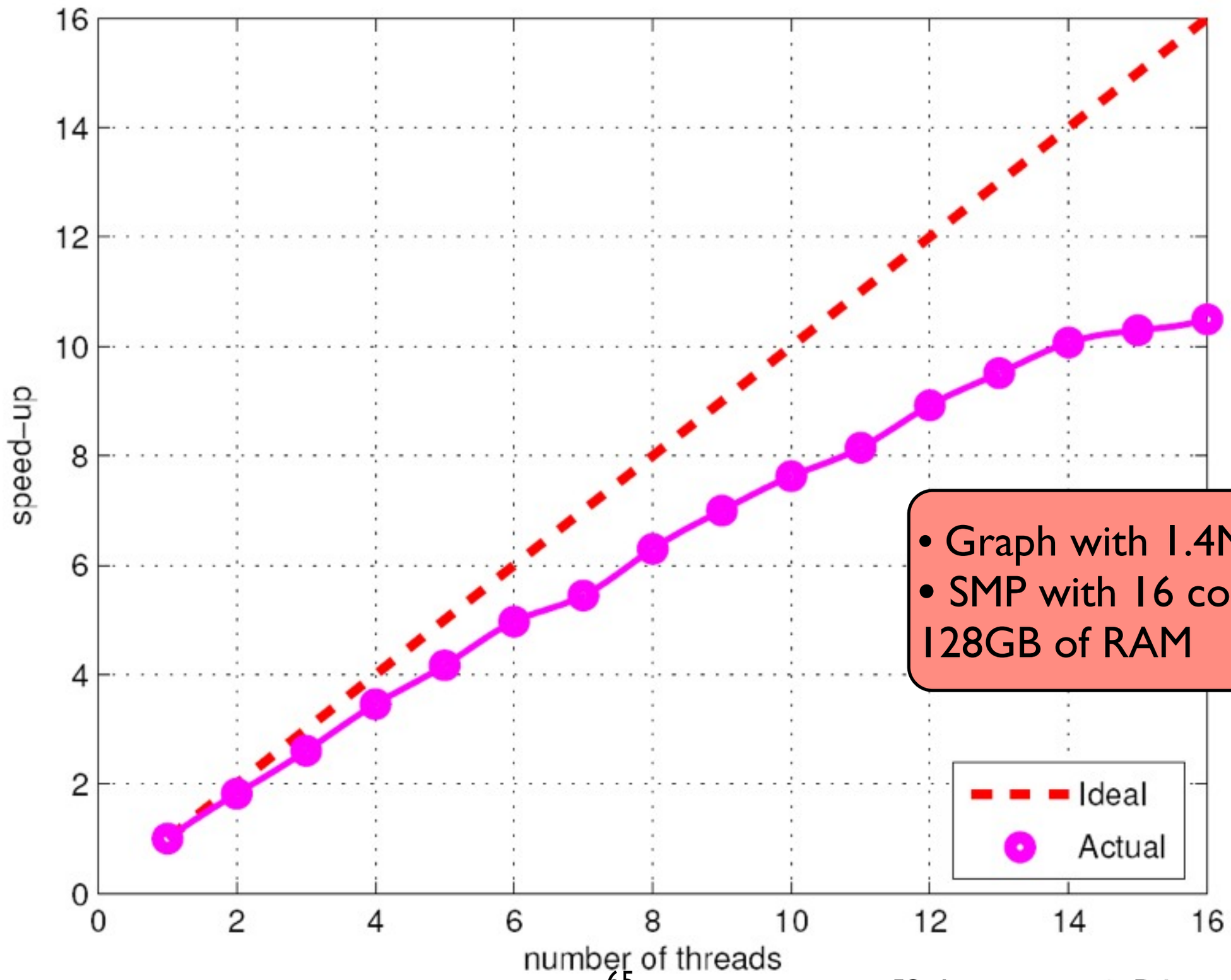


Speed-up on SMP



- Graph with 1.4M nodes
- SMP with 16 cores and 128GB of RAM

[Subramanya & Bilmes, JMLR, 2011]

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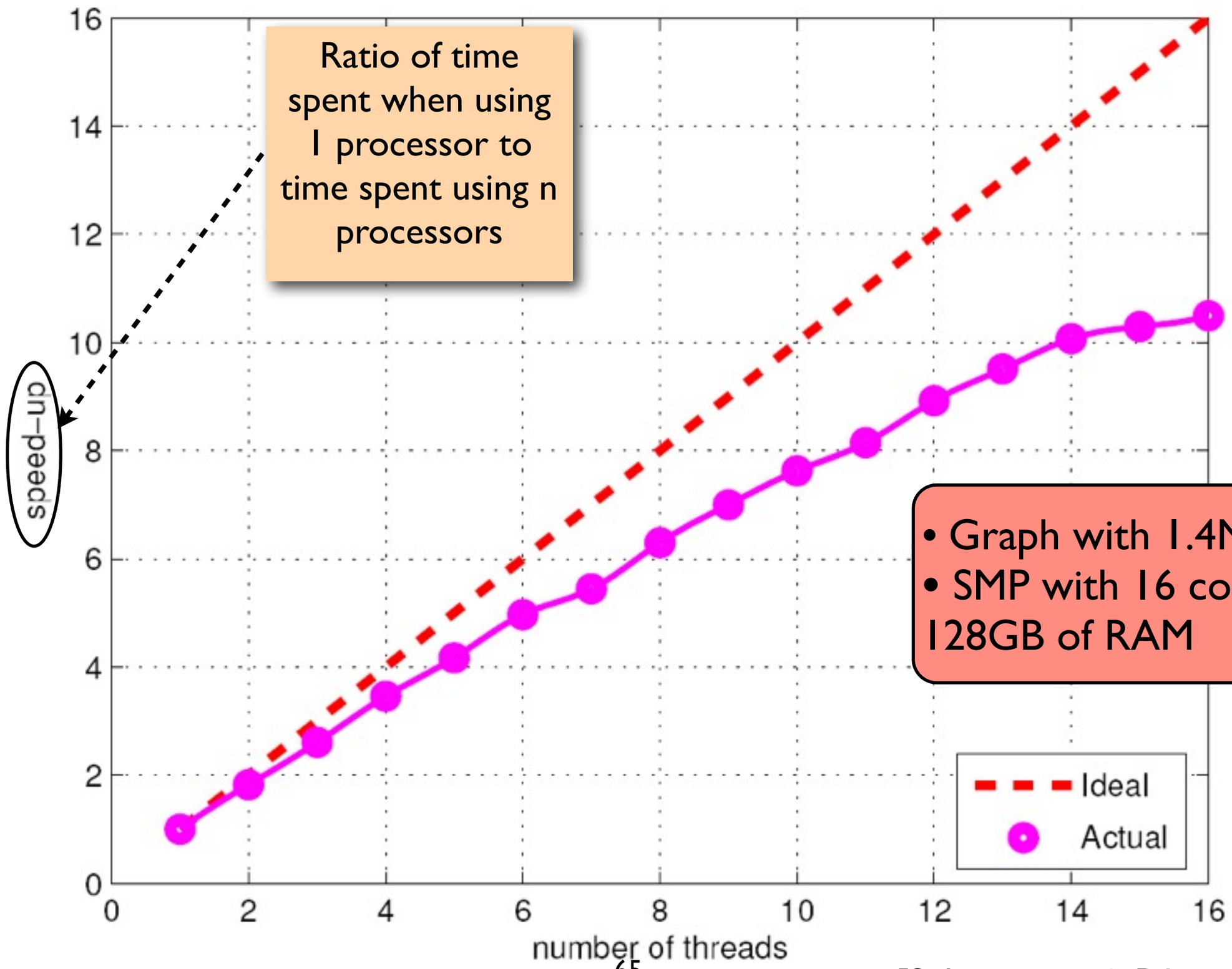

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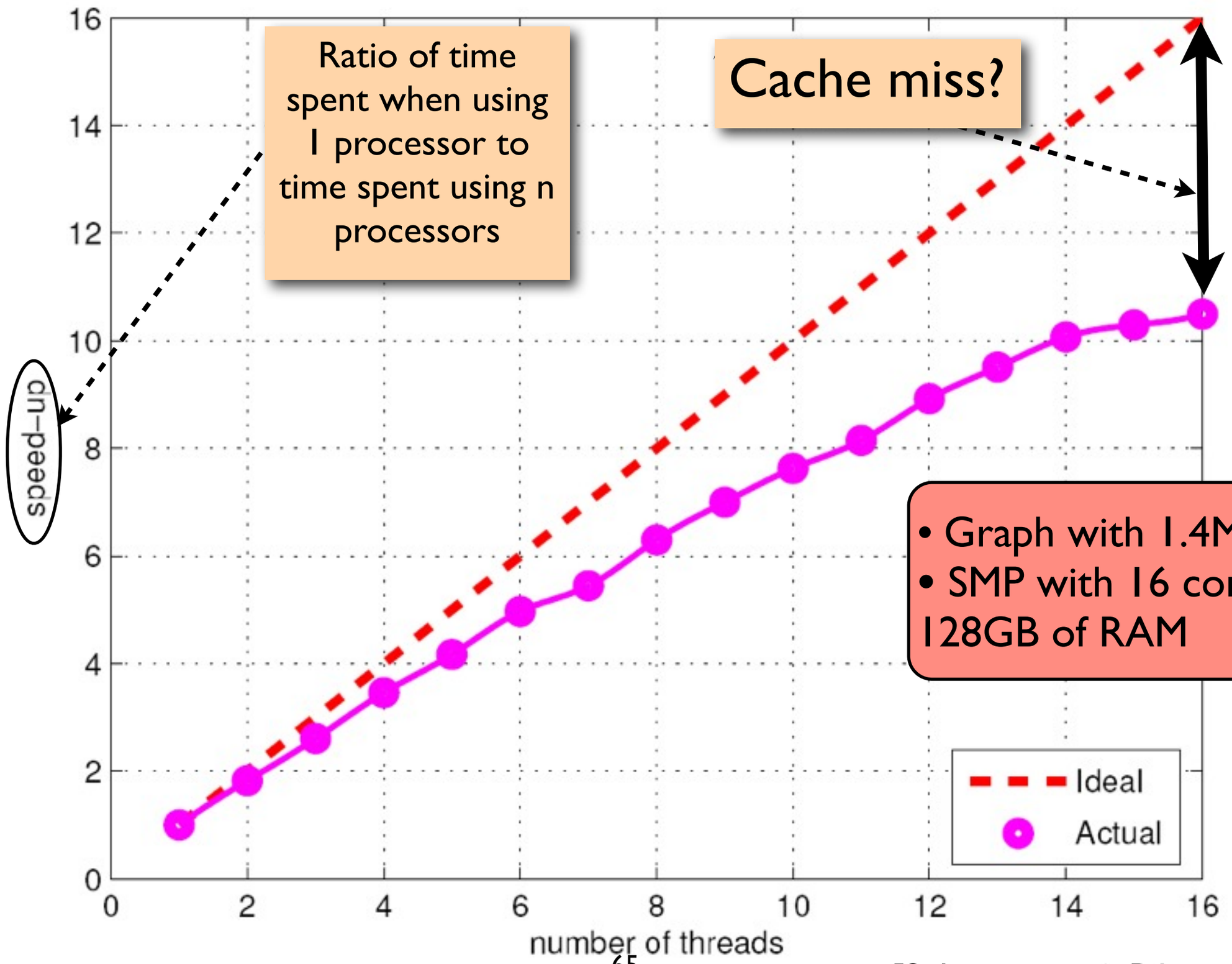

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[Subramanya & Bilmes, JMLR, 2011]

Node Reordering Algorithm

Input: Graph G = (V, E)

Result: Node ordered graph

1. Select an arbitrary node v
2. while unselected nodes remain do
   - 2.1. select an unselected node v` from among the neighbors' neighbors of v that has maximum overlap with v` neighbors
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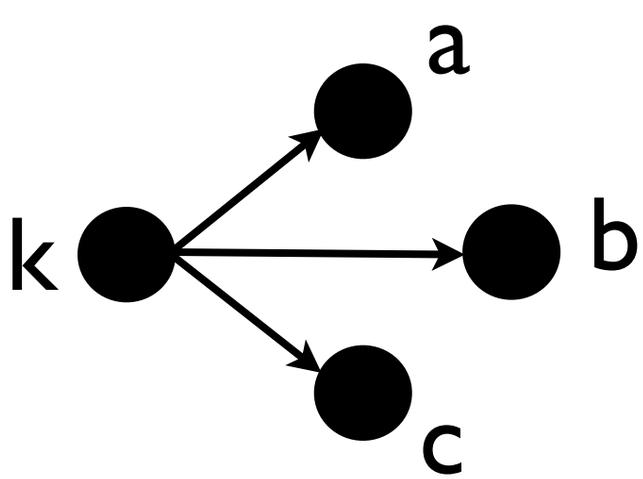
Exhaustive for sparse (e.g., k-NN) graphs

[Subramanya & Bilmes, JMLR, 2011]

Node Reordering Algorithm : Intuition



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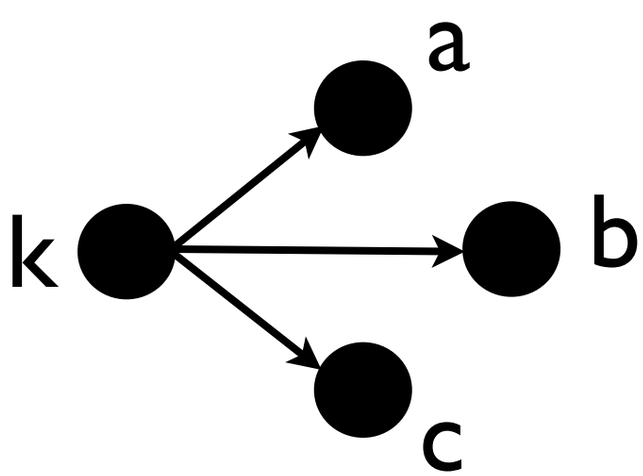

Which node should be placed after k to optimize cache performance?

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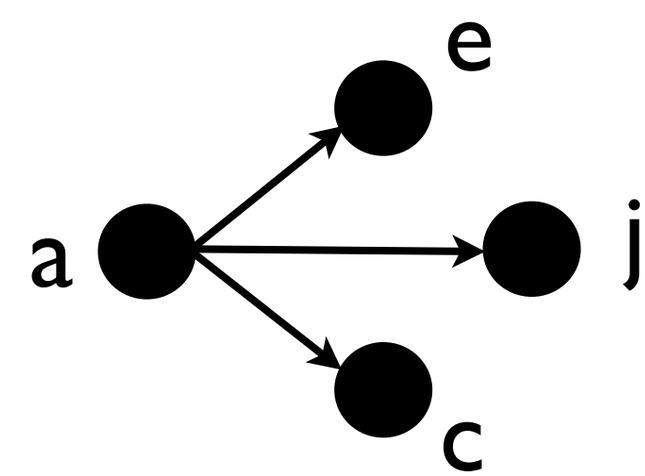

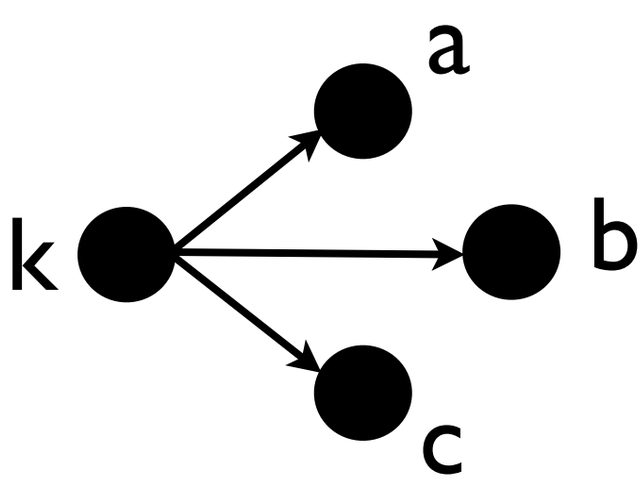

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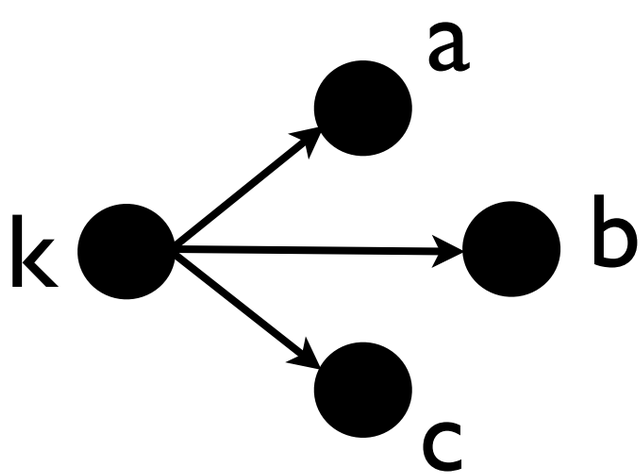

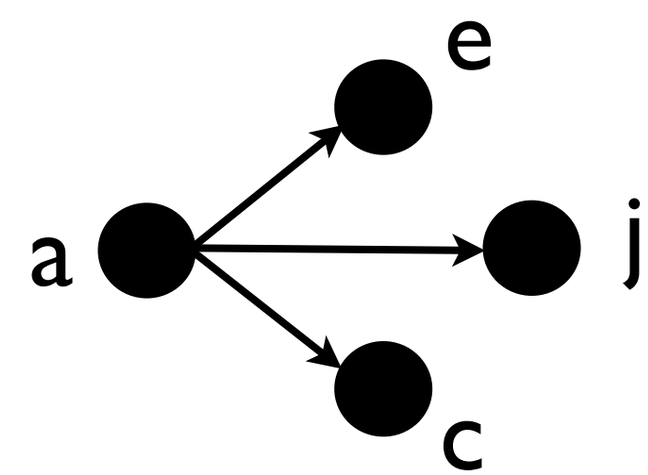

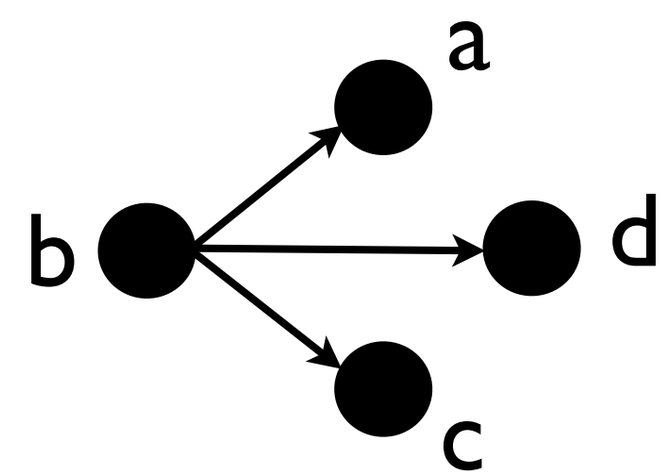

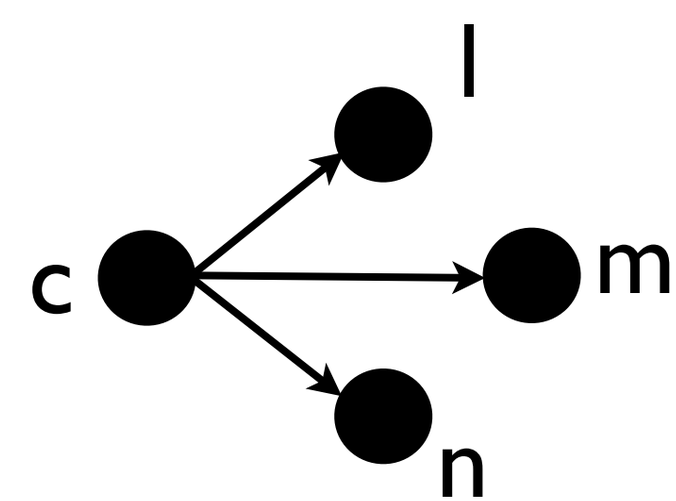

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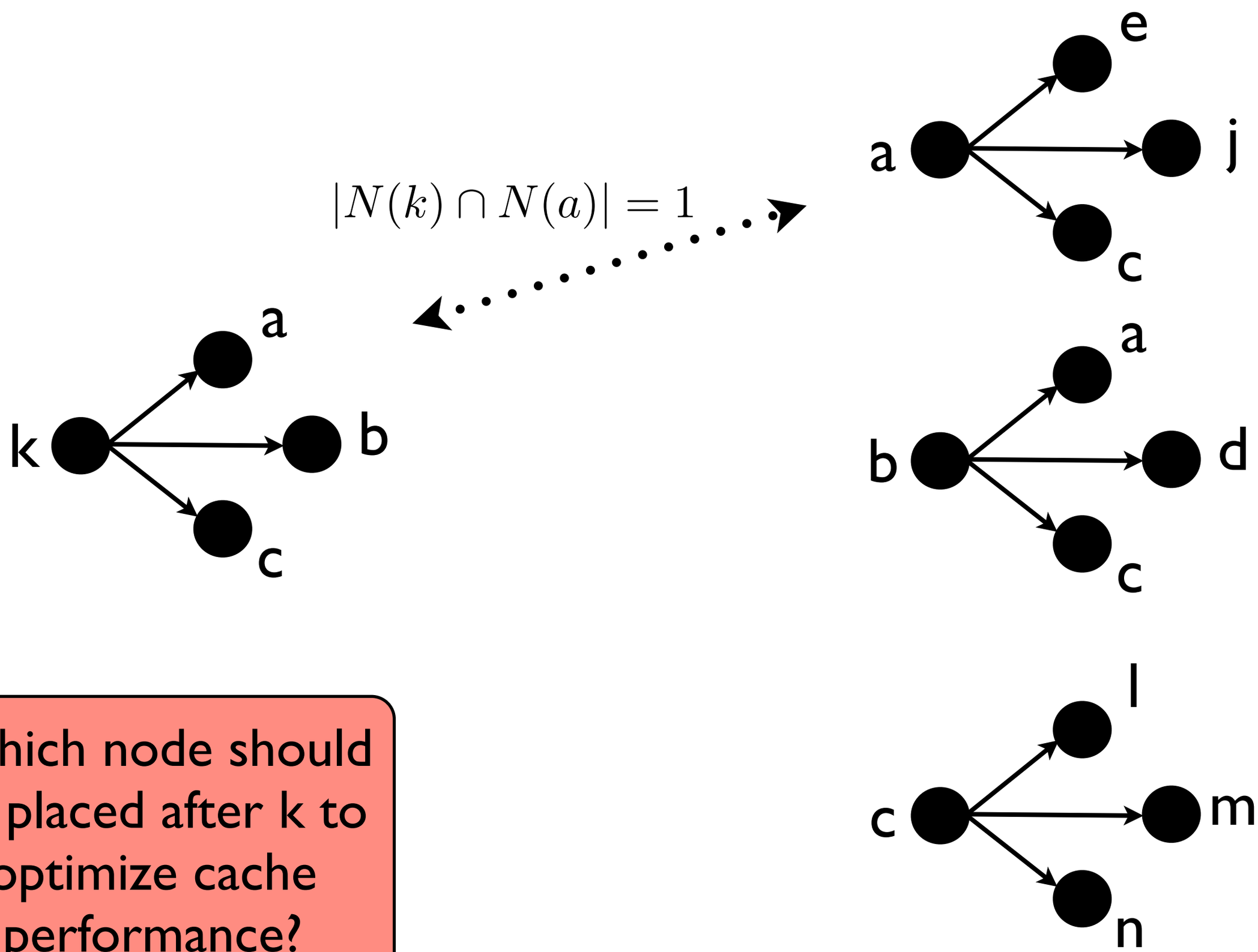

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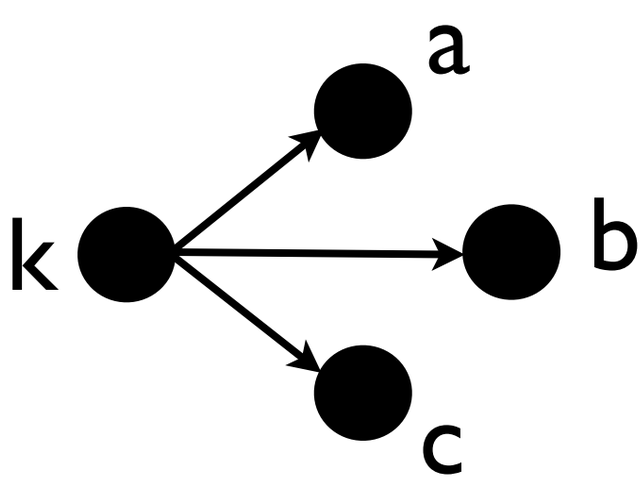
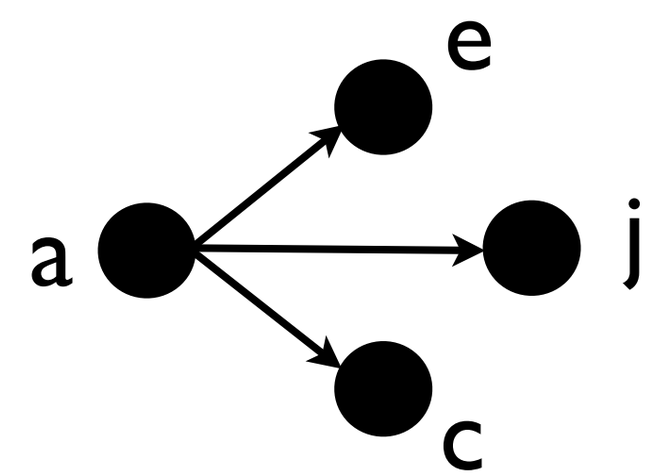
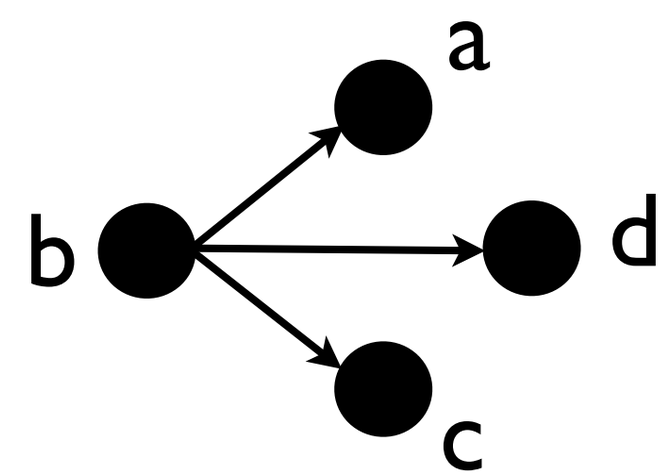
Node Reordering Algorithm : Intuition

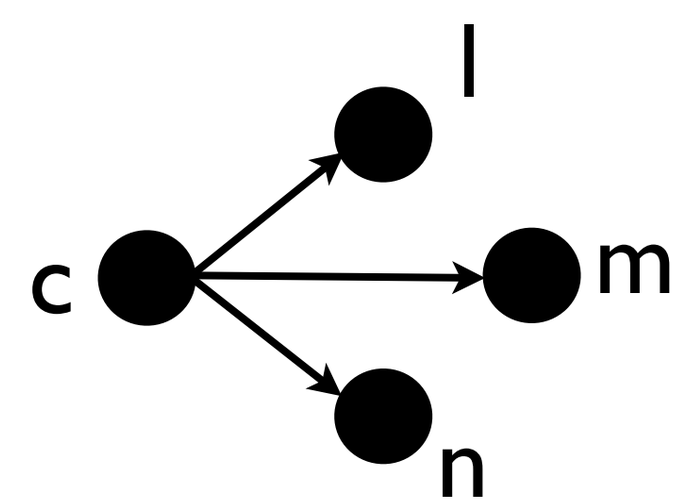
Cardinality of Intersection

$$|N(k) \cap N(a)| = 1$$









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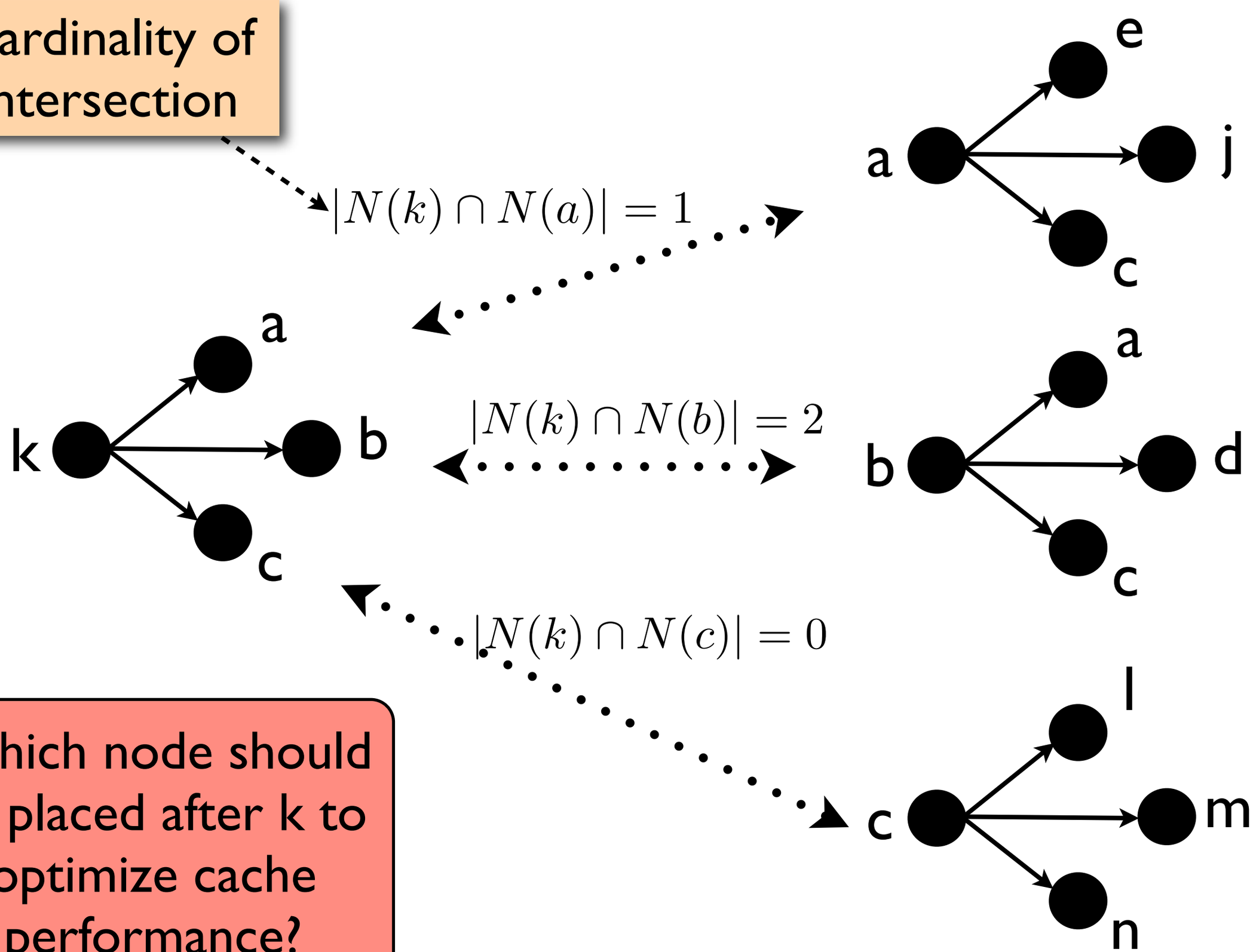

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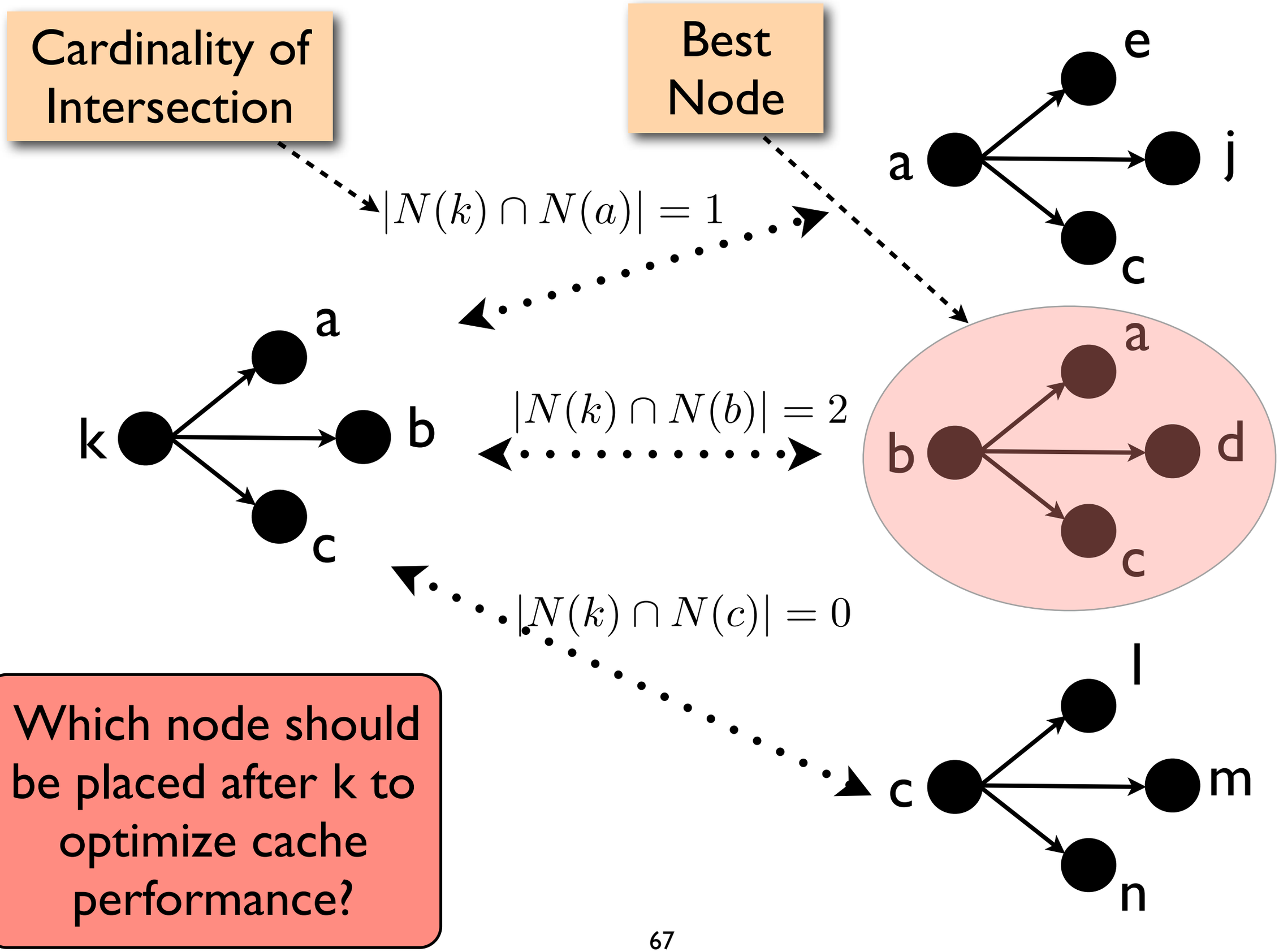
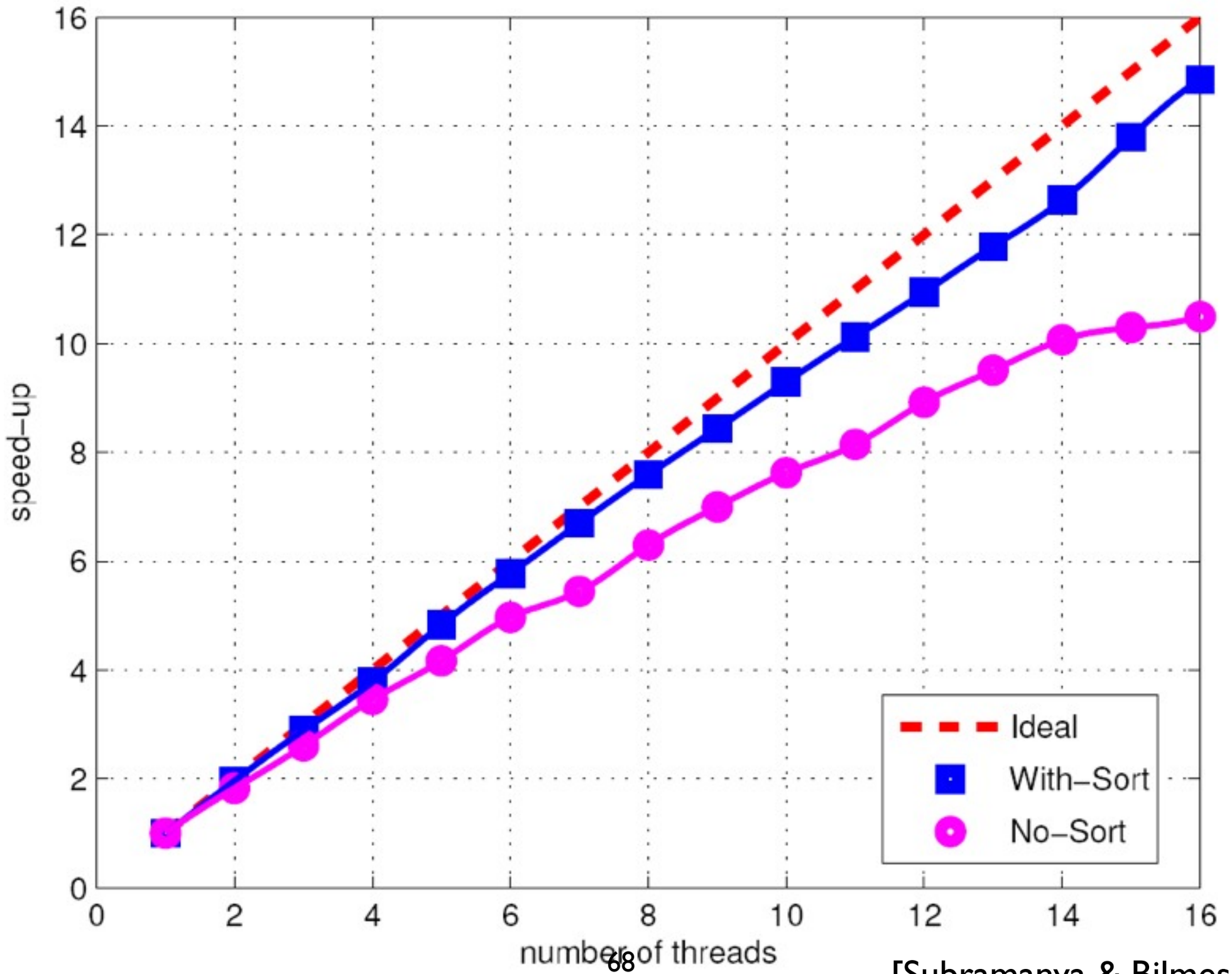


Which node should be placed after k to optimize cache performance?

Speed-up on SMP after Node Ordering

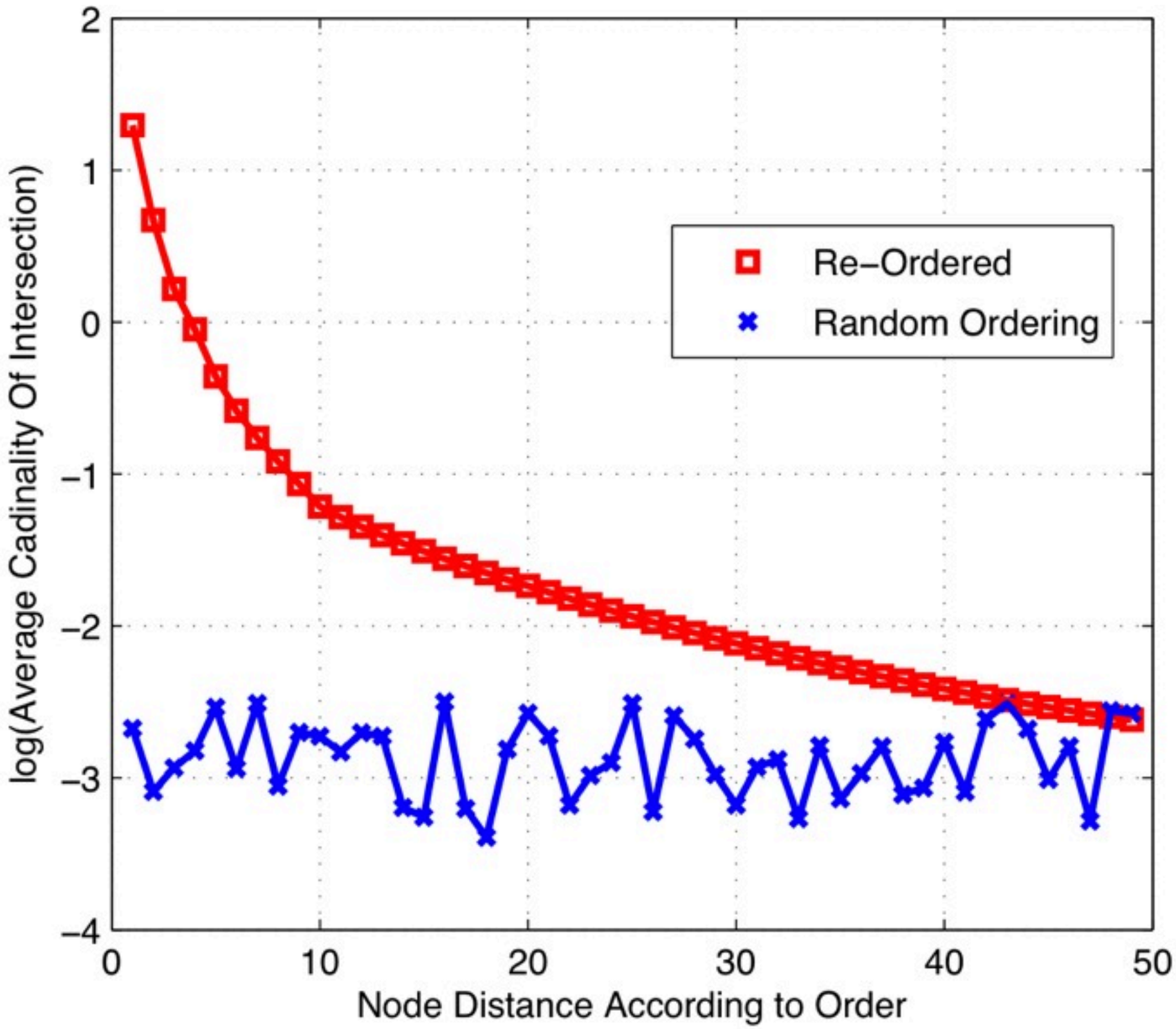


[Subramanya & Bilmes, JMLR, 2011]

Distributed Processing

- **Maximize** overlap between consecutive nodes within the same machine
- **Minimize** overlap across machines (reduce inter machine communication)

Distributed Processing



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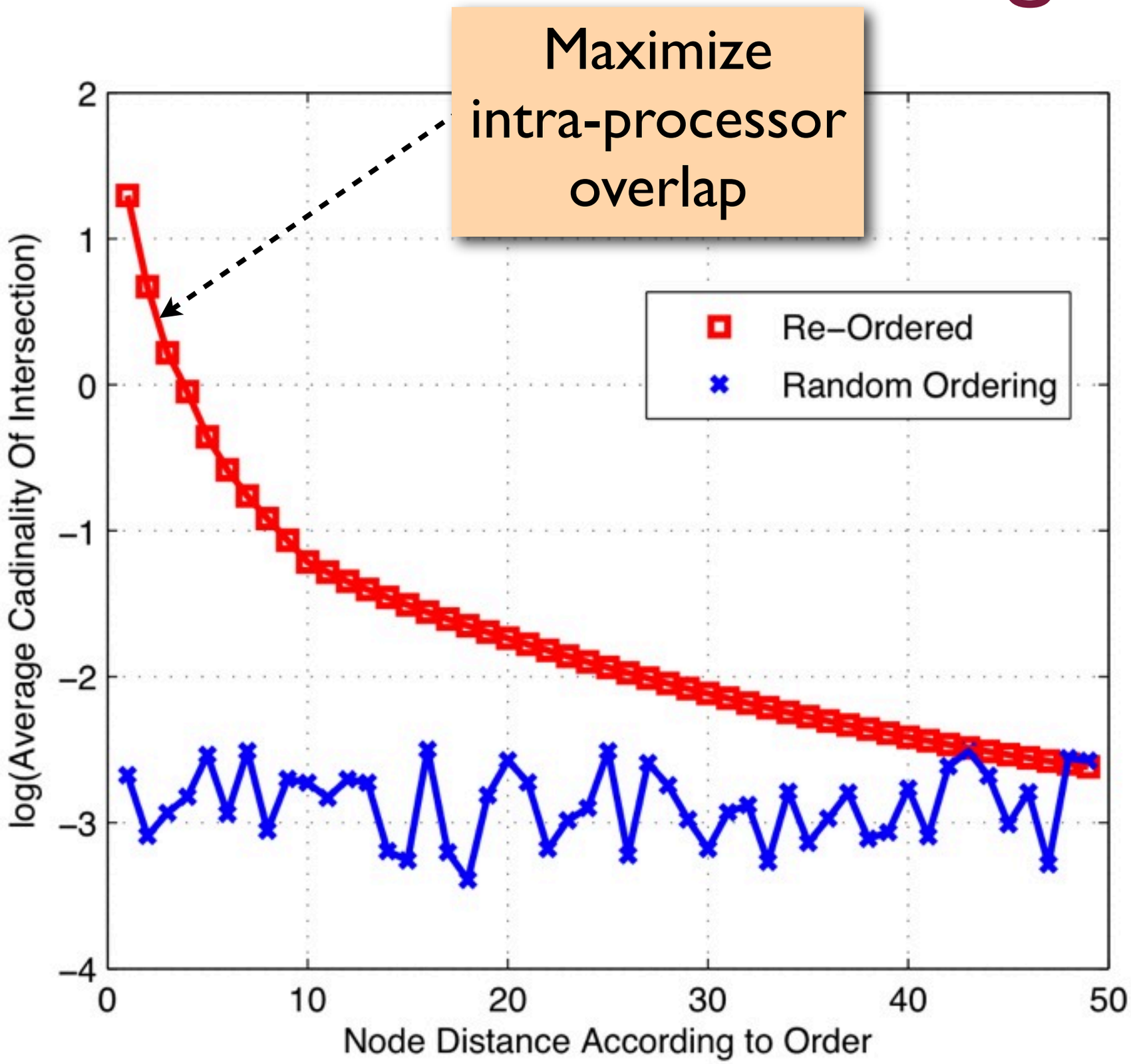

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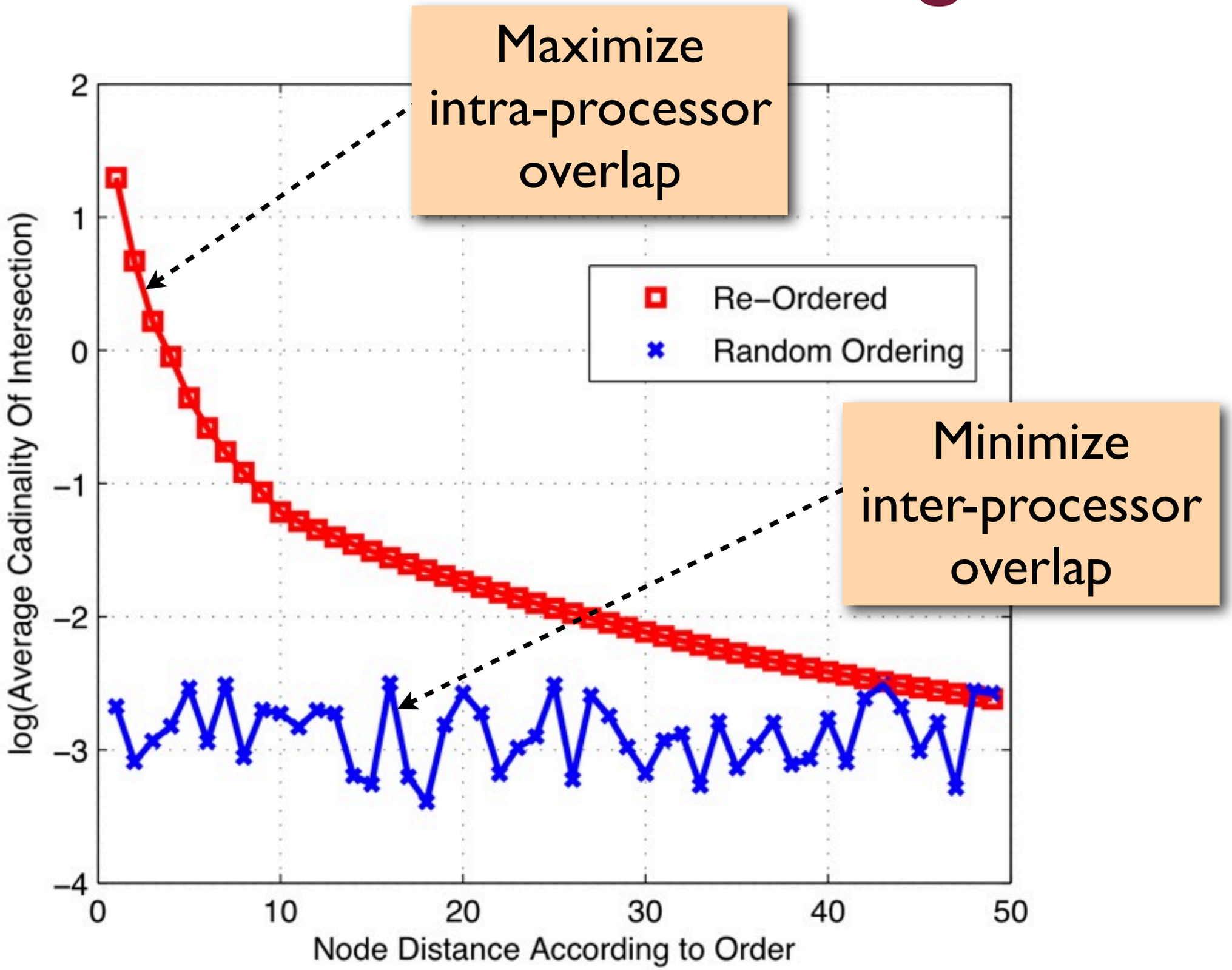

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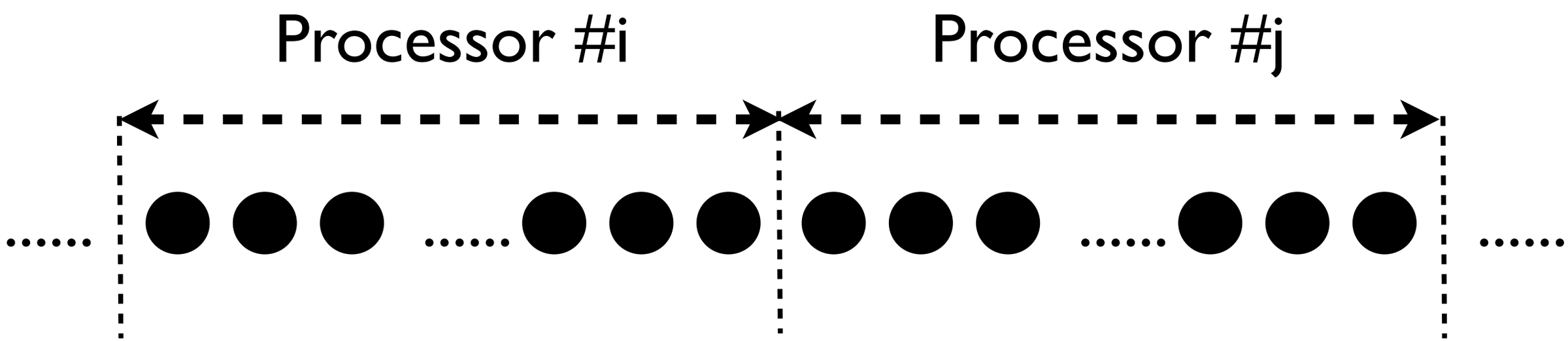
Node reordering for Distributed Computer



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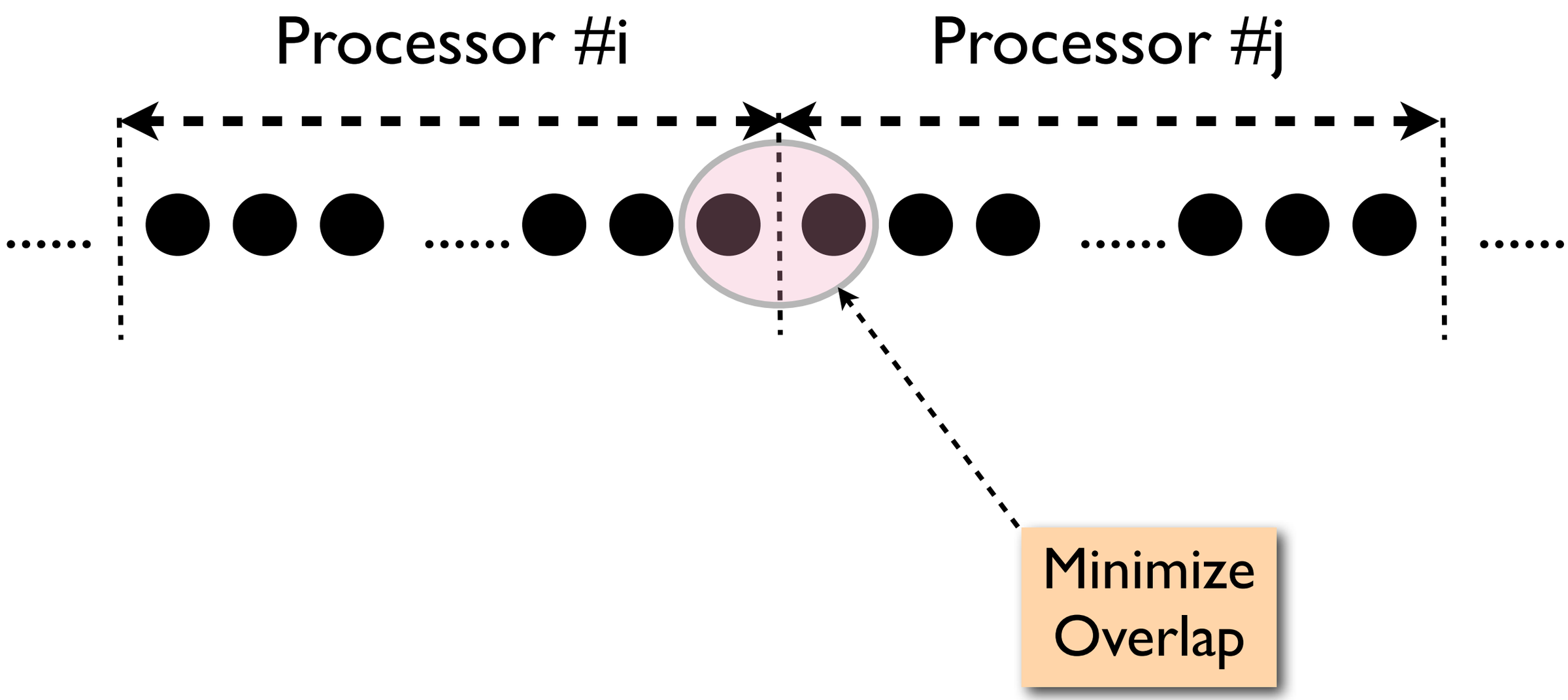

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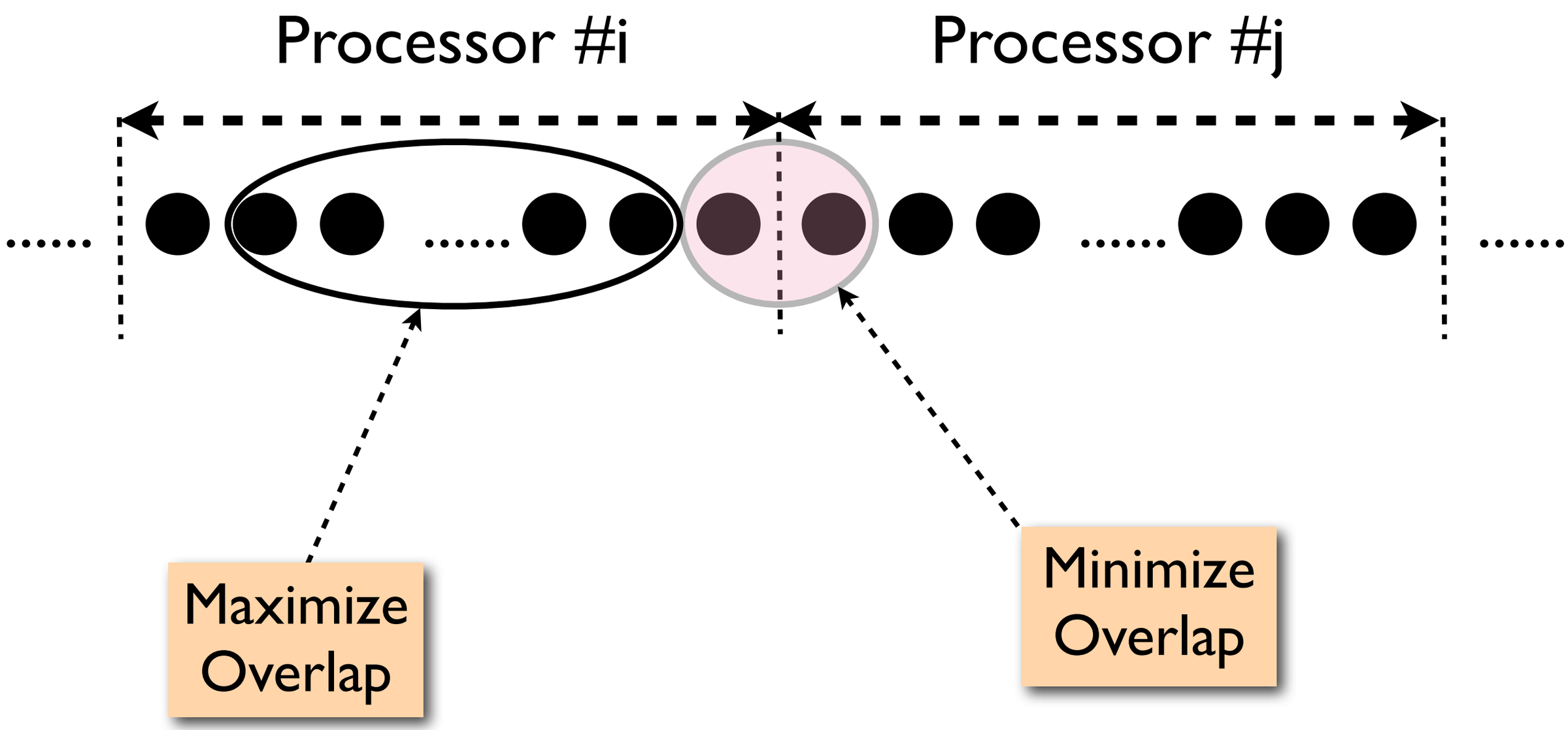
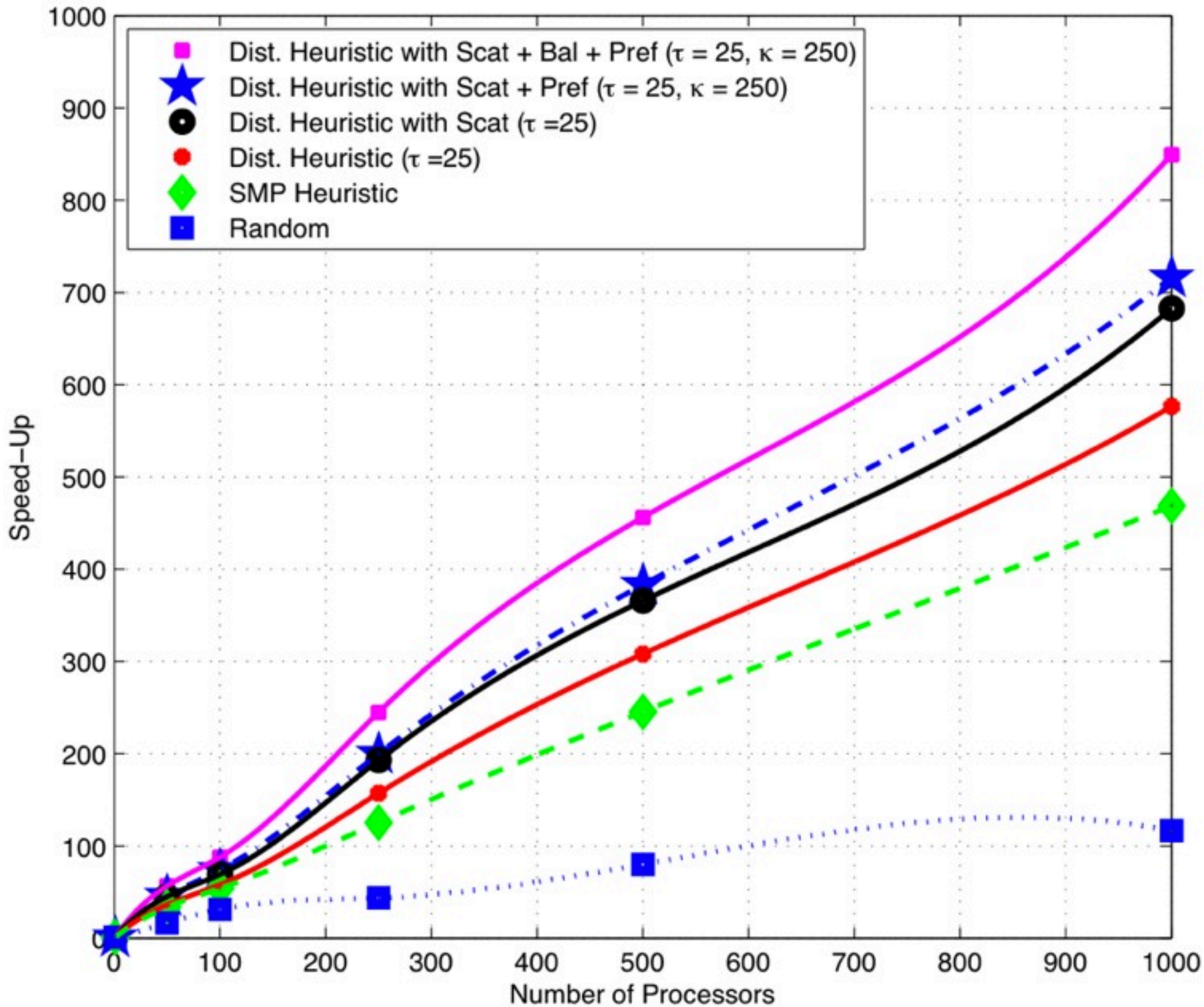


Distributed Processing Results



[Bilmes & Subramanya, 2011]

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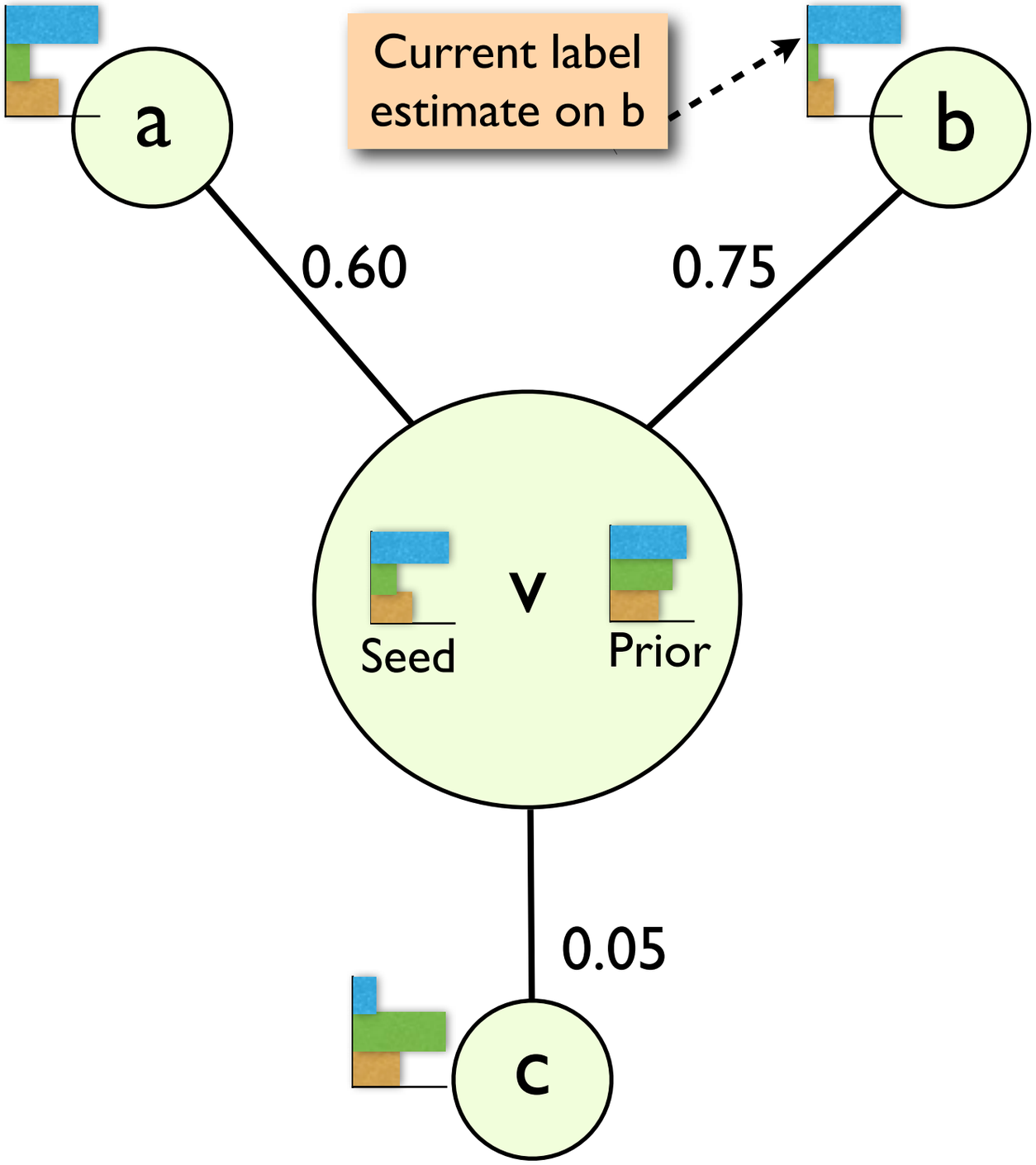
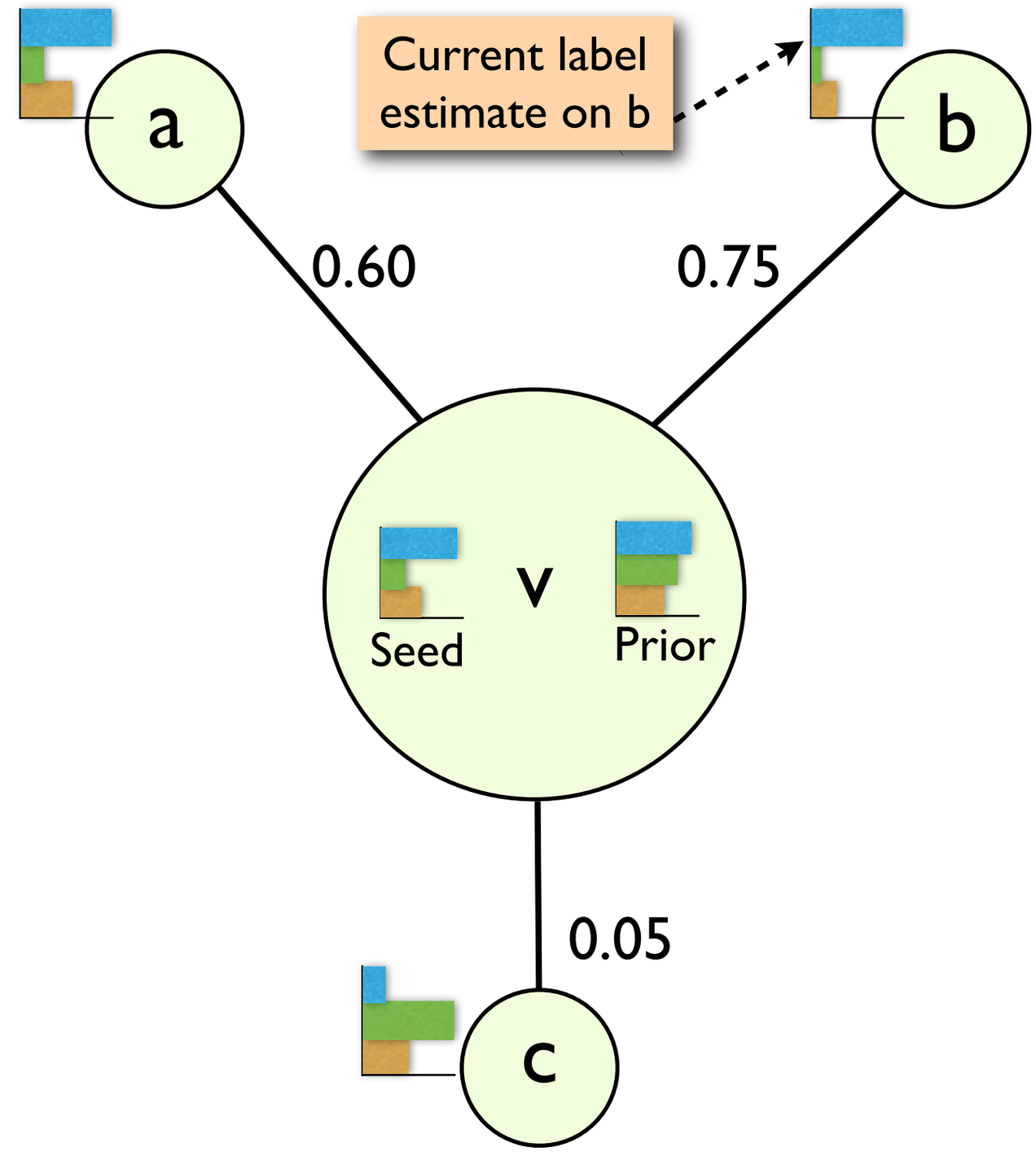
MapReduce Implementation of MAD



MapReduce Implementation of MAD

- Map
  - Each node send its current label assignments to its neighbors



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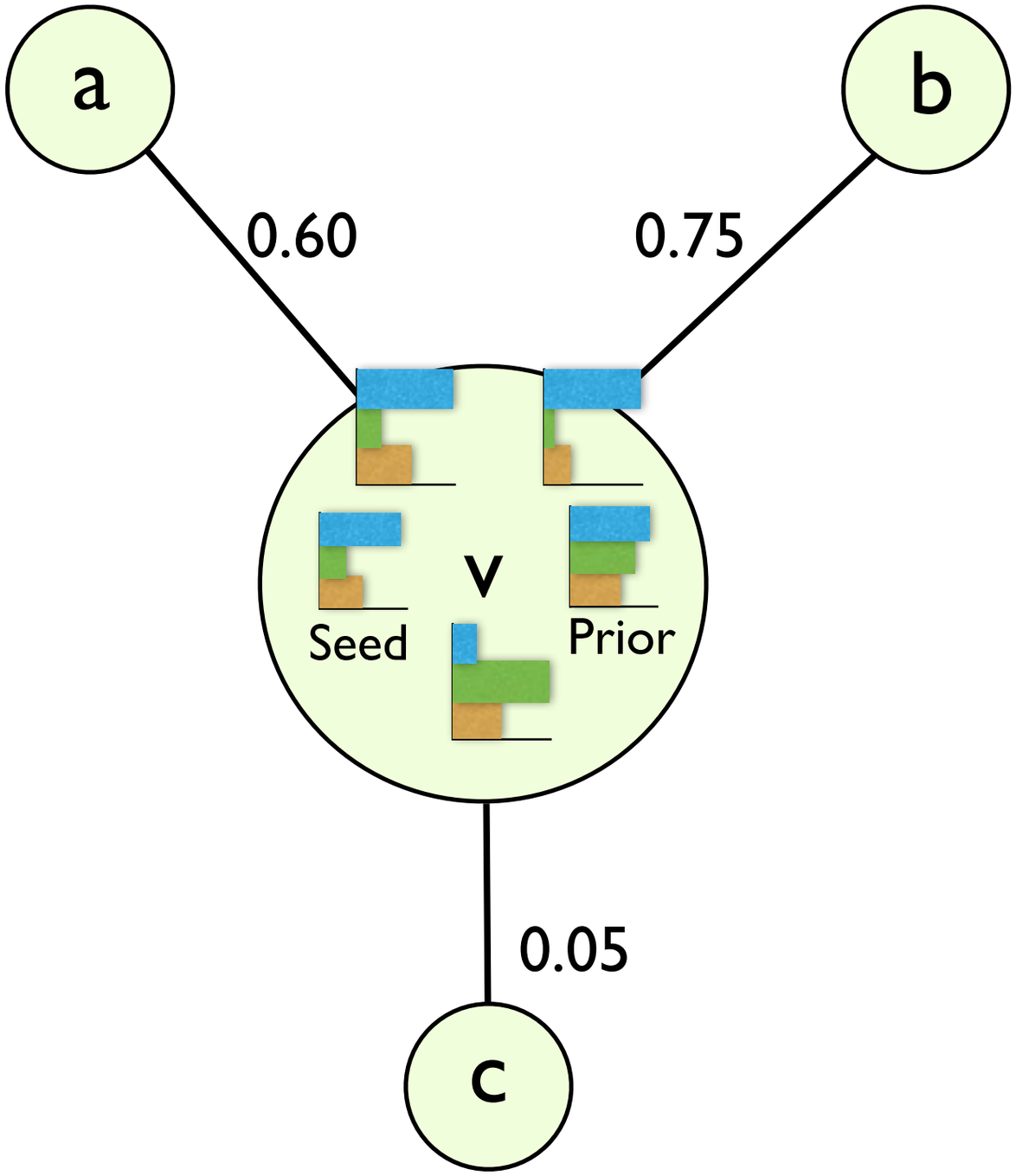
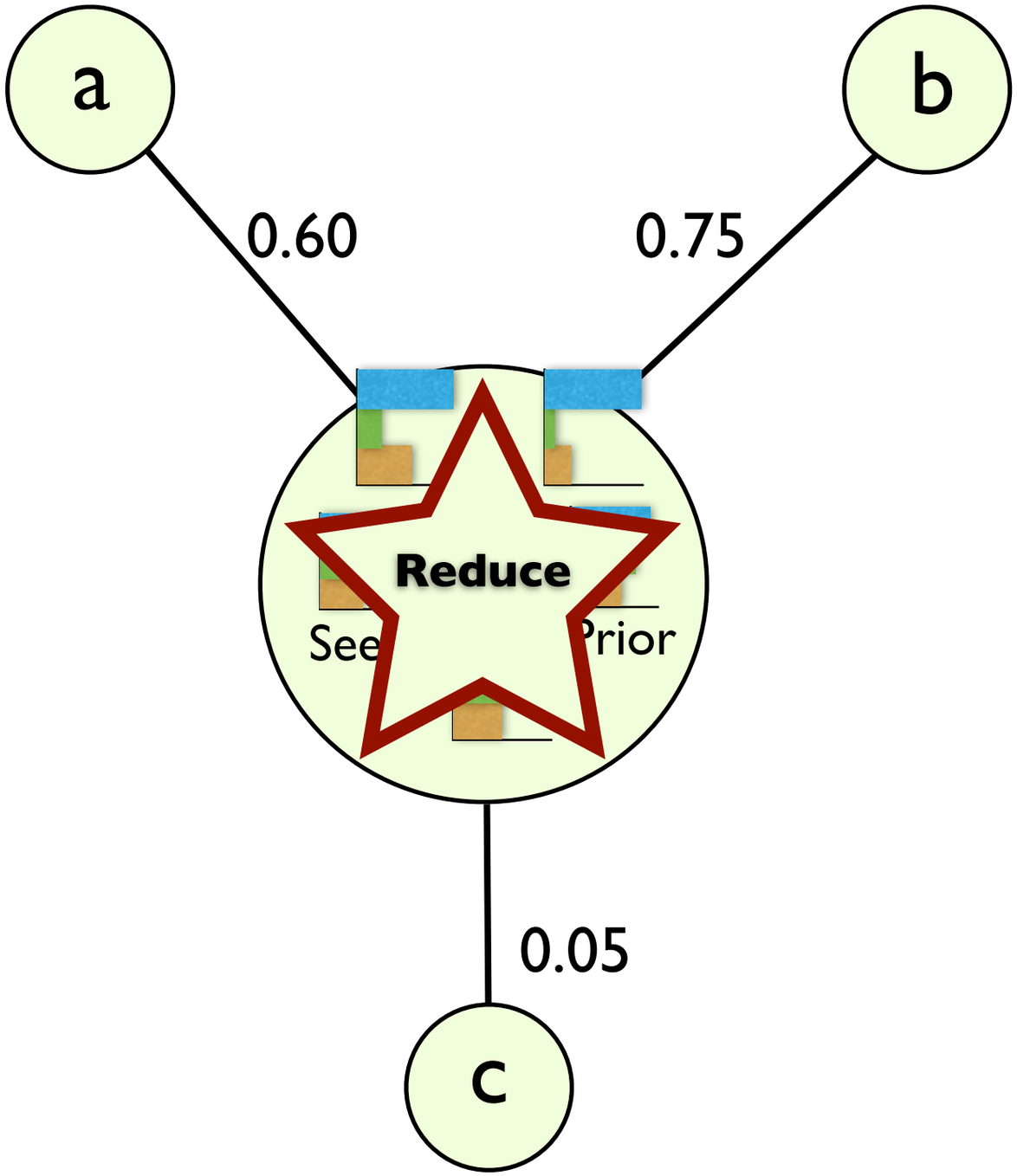


MapReduce Implementation of MAD

- Map
  - Each node send its current label assignments to its neighbors
- Reduce
  - Each node updates its own label assignment using messages received from neighbors, and its own information (e.g., seed labels, reg. penalties etc.)
- Repeat until convergence



MapReduce Implementation of MAD

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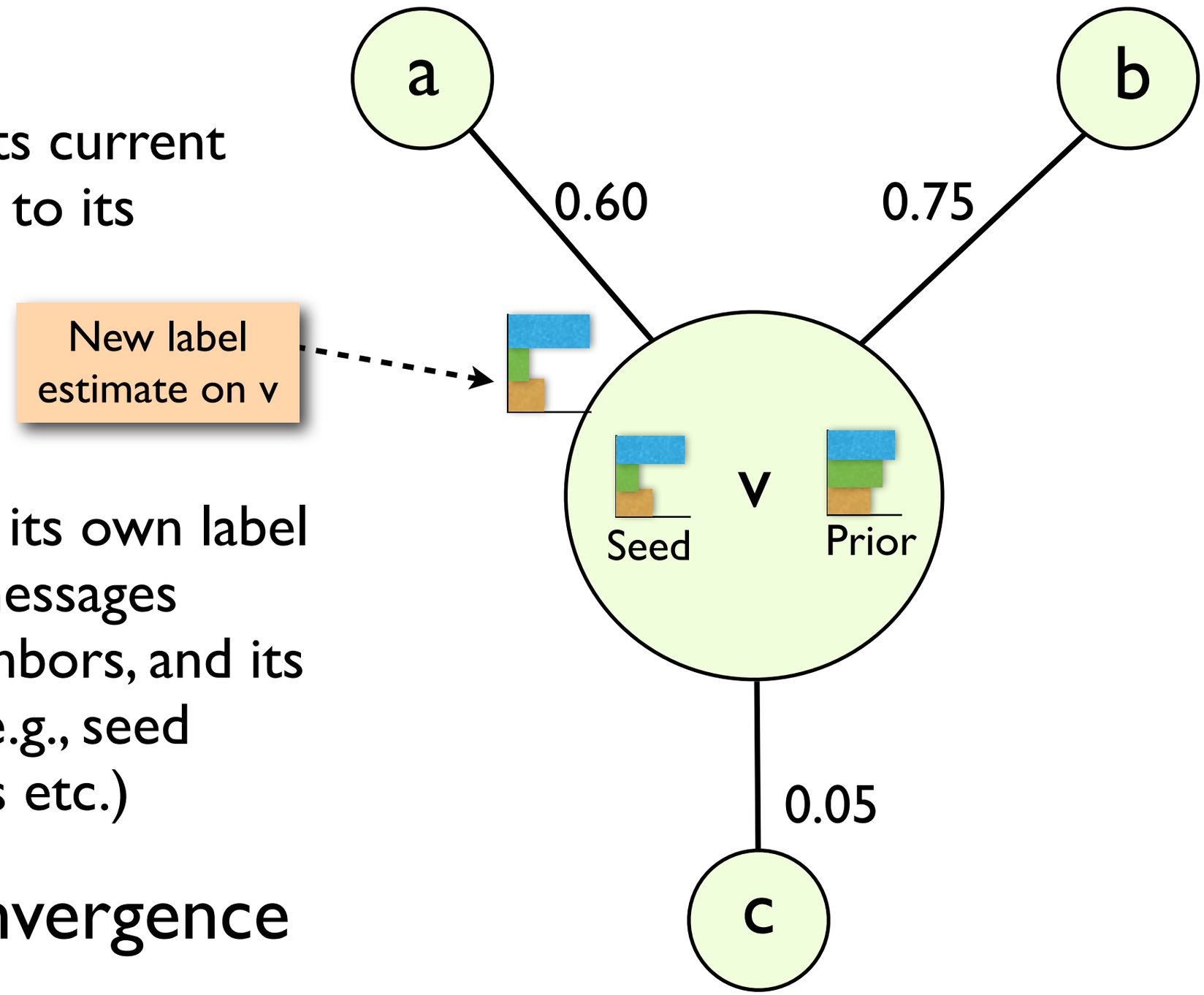
  - Each node send its current label assignments to its neighbors
- Reduce
  - Each node updates its own label assignment using messages received from neighbors, and its own information (e.g., seed labels, reg. penalties etc.)
- Repeat until convergence



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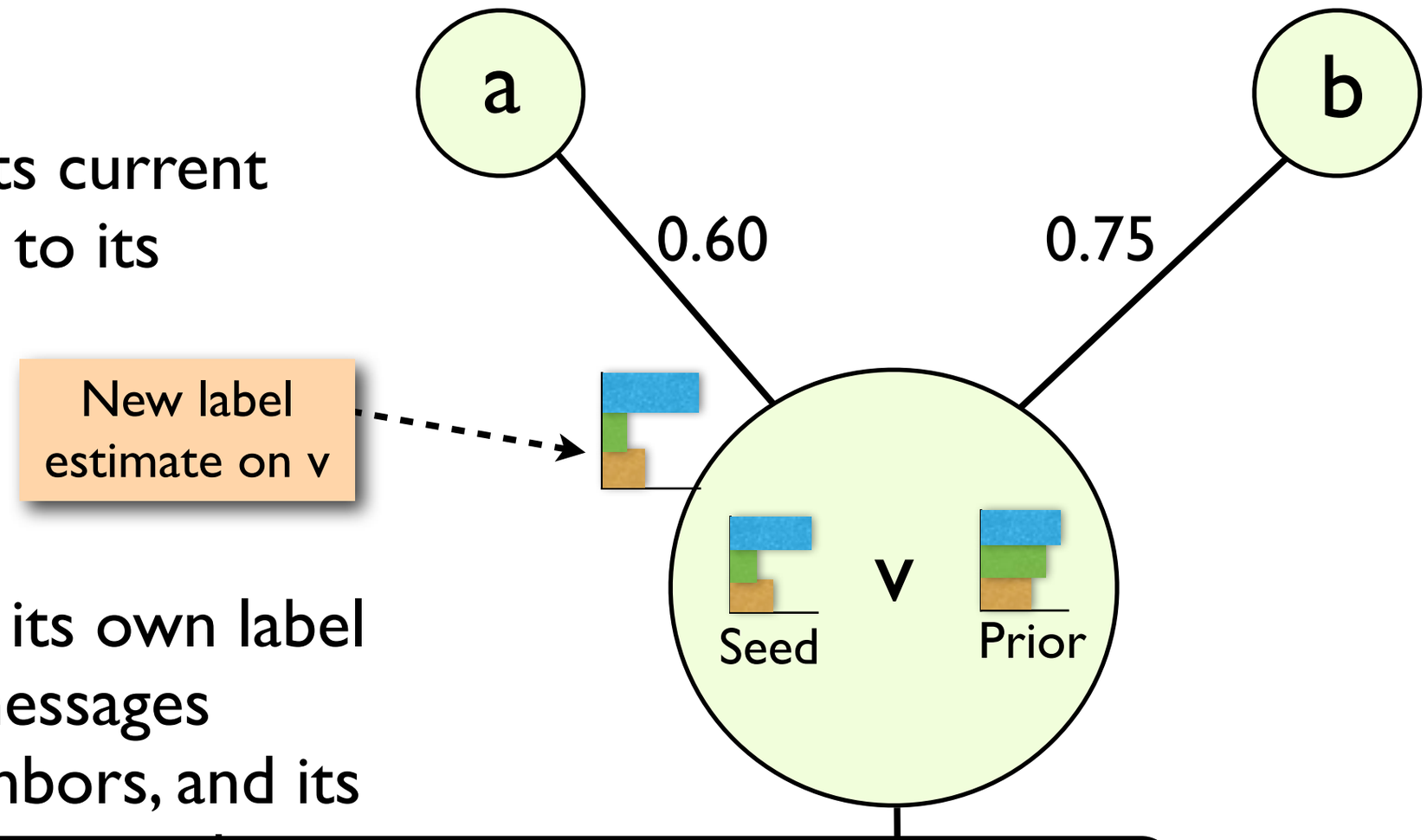

Code in Junto Label Propagation Toolkit

(includes Hadoop-based implementation)

http://code.google.com/p/junto/

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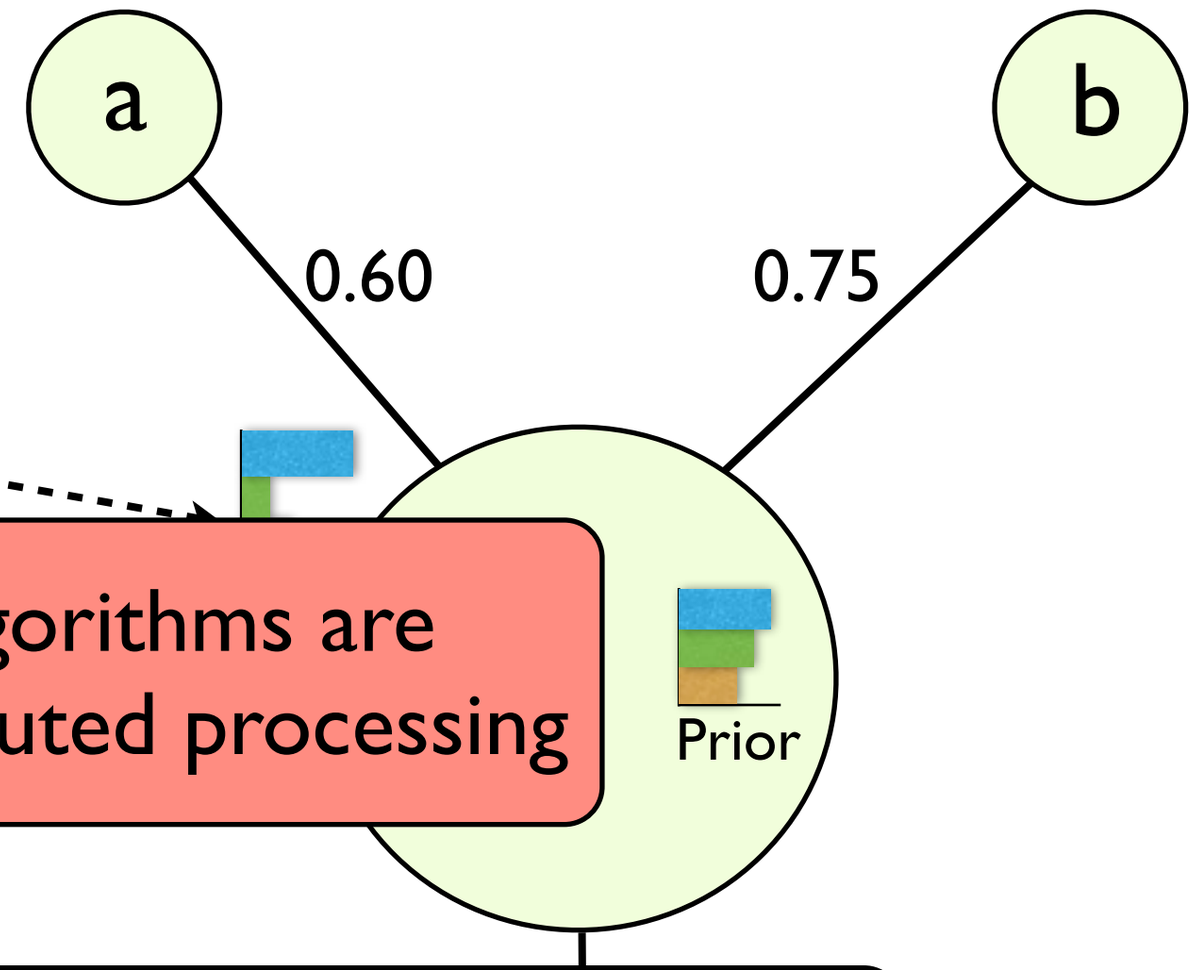

New label

Graph-based algorithms are amenable to distributed processing

Code in Junto Label Propagation Toolkit

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http://code.google.com/p/junto/

Outline

- Motivation
- Graph Construction
- Inference Methods
- Scalability
- Applications
  - Phone Classification
  - Text Categorization
  - Dialog Act Tagging
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- Training supervised models requires large amounts of labeled data (phone classification in resource-scarce languages)

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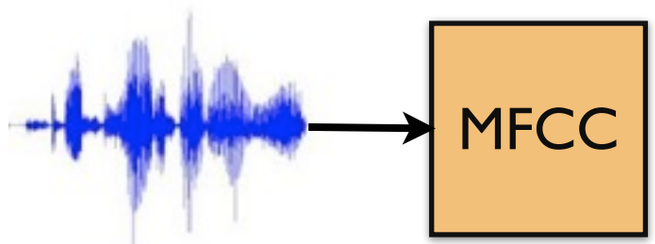
  - further mapped down to 39 phones for scoring

Feature Extraction

Feature Extraction



Hamming
Window
25ms @ 100 Hz

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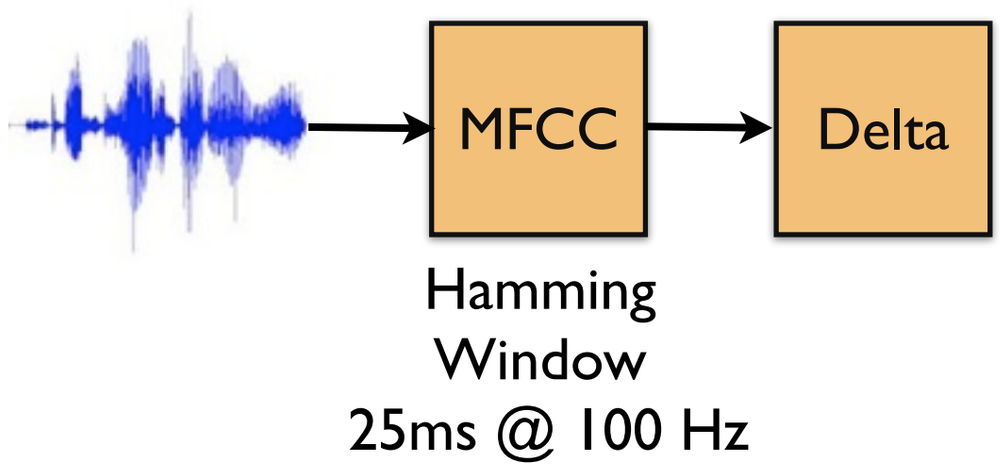

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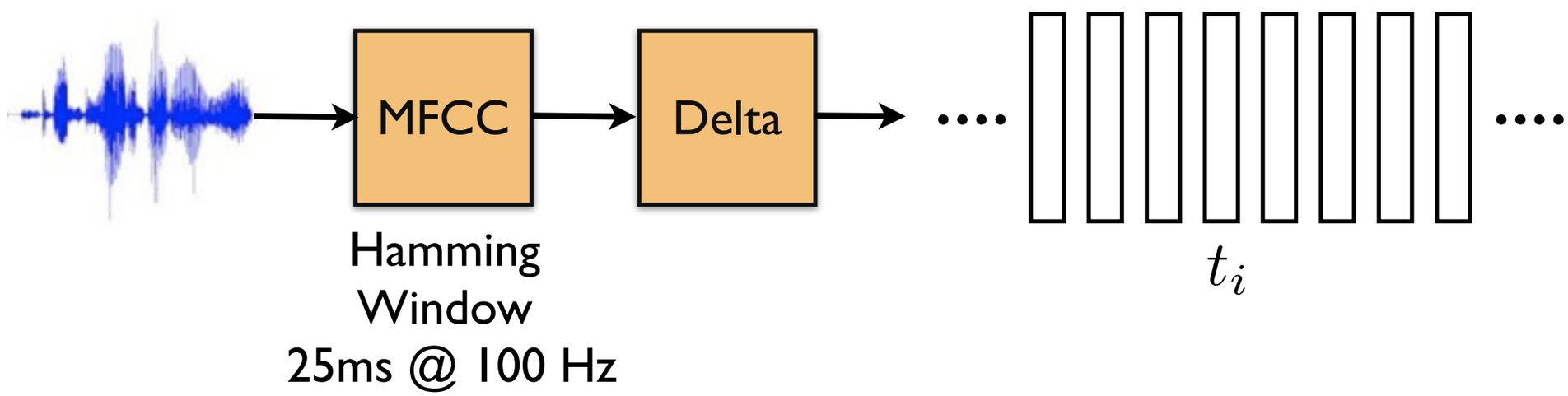

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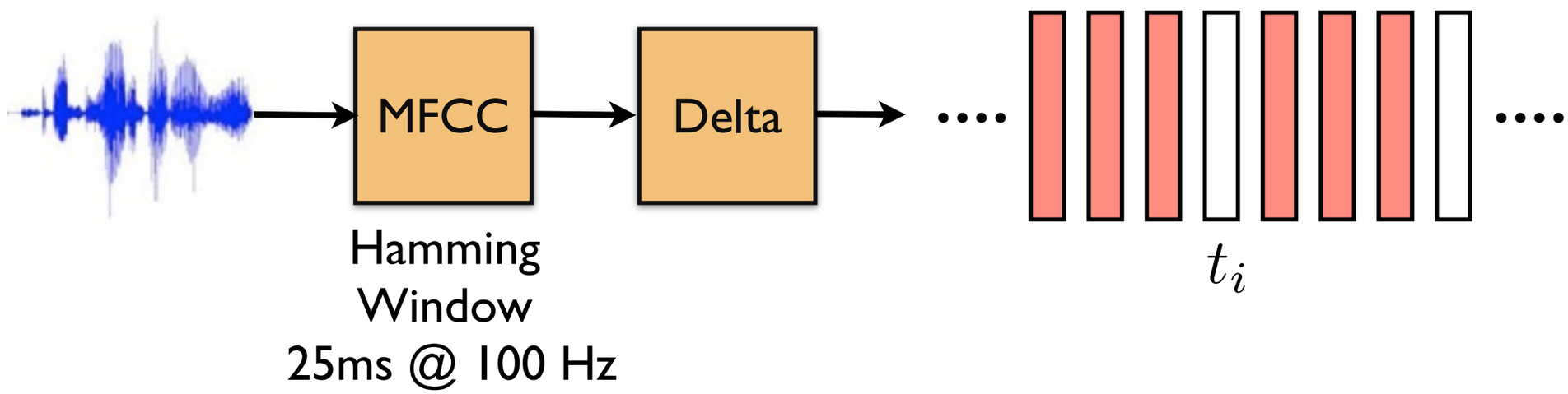

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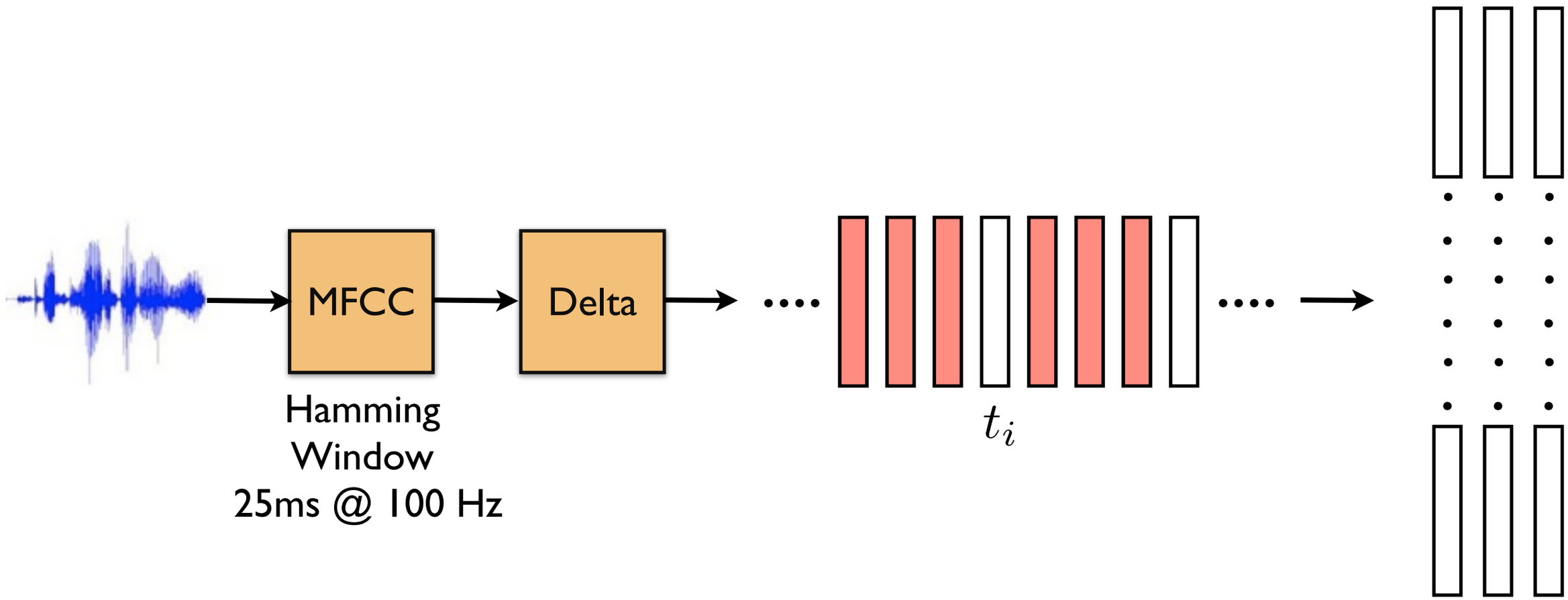

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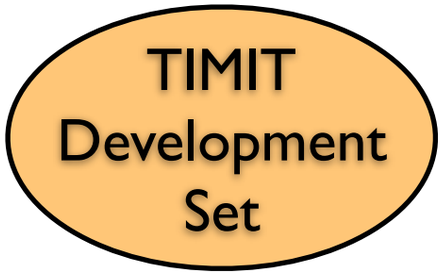

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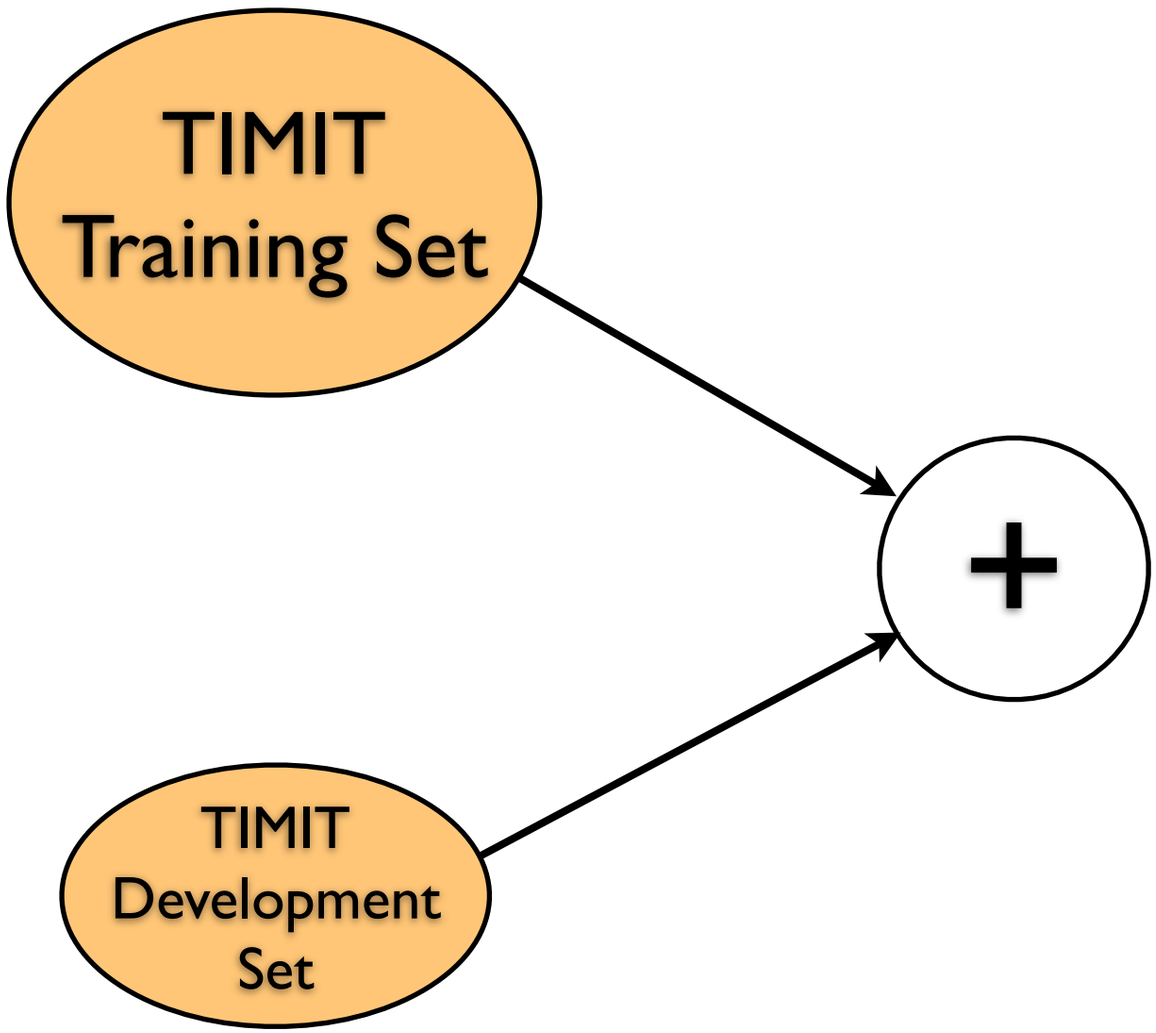
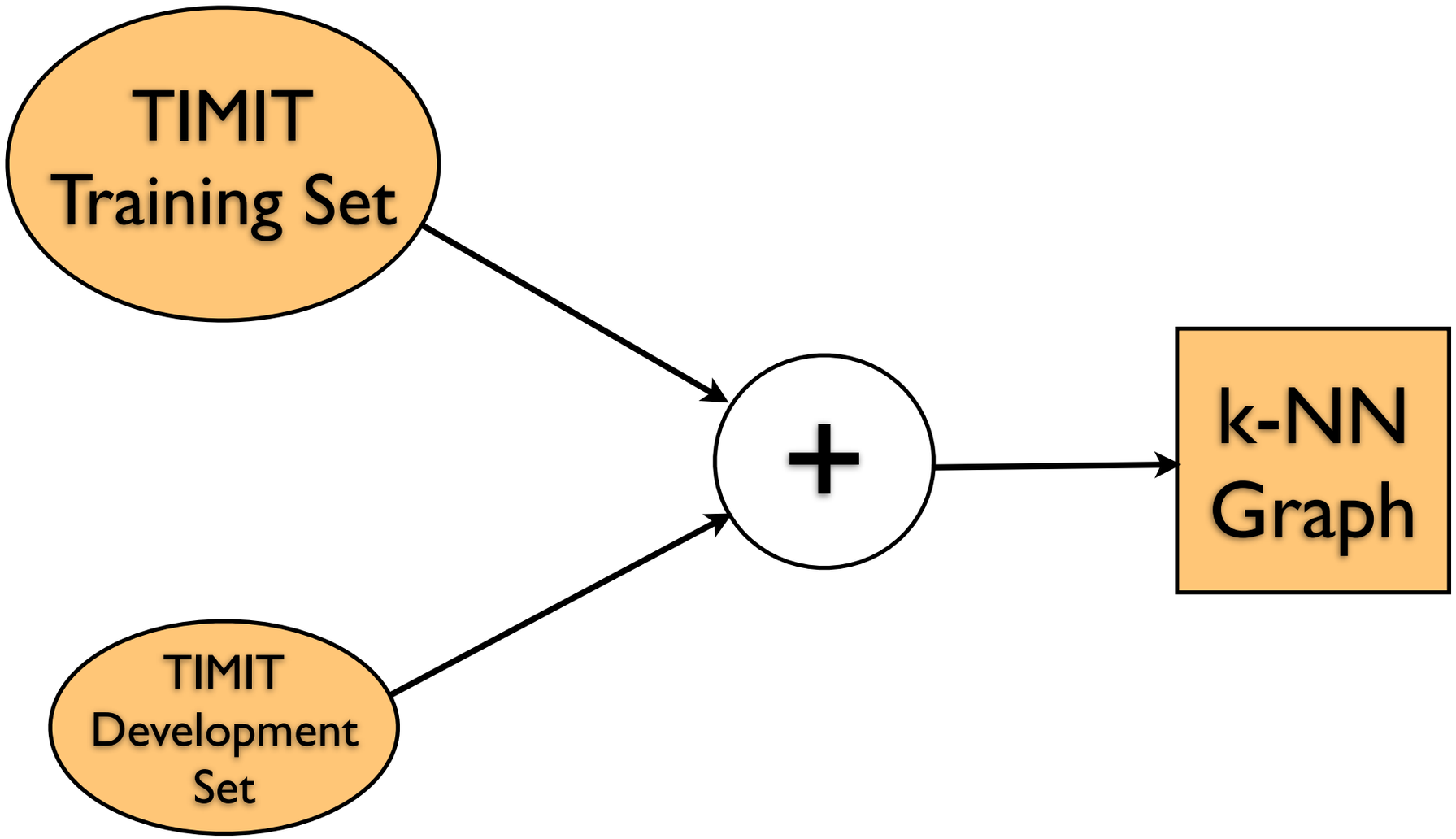
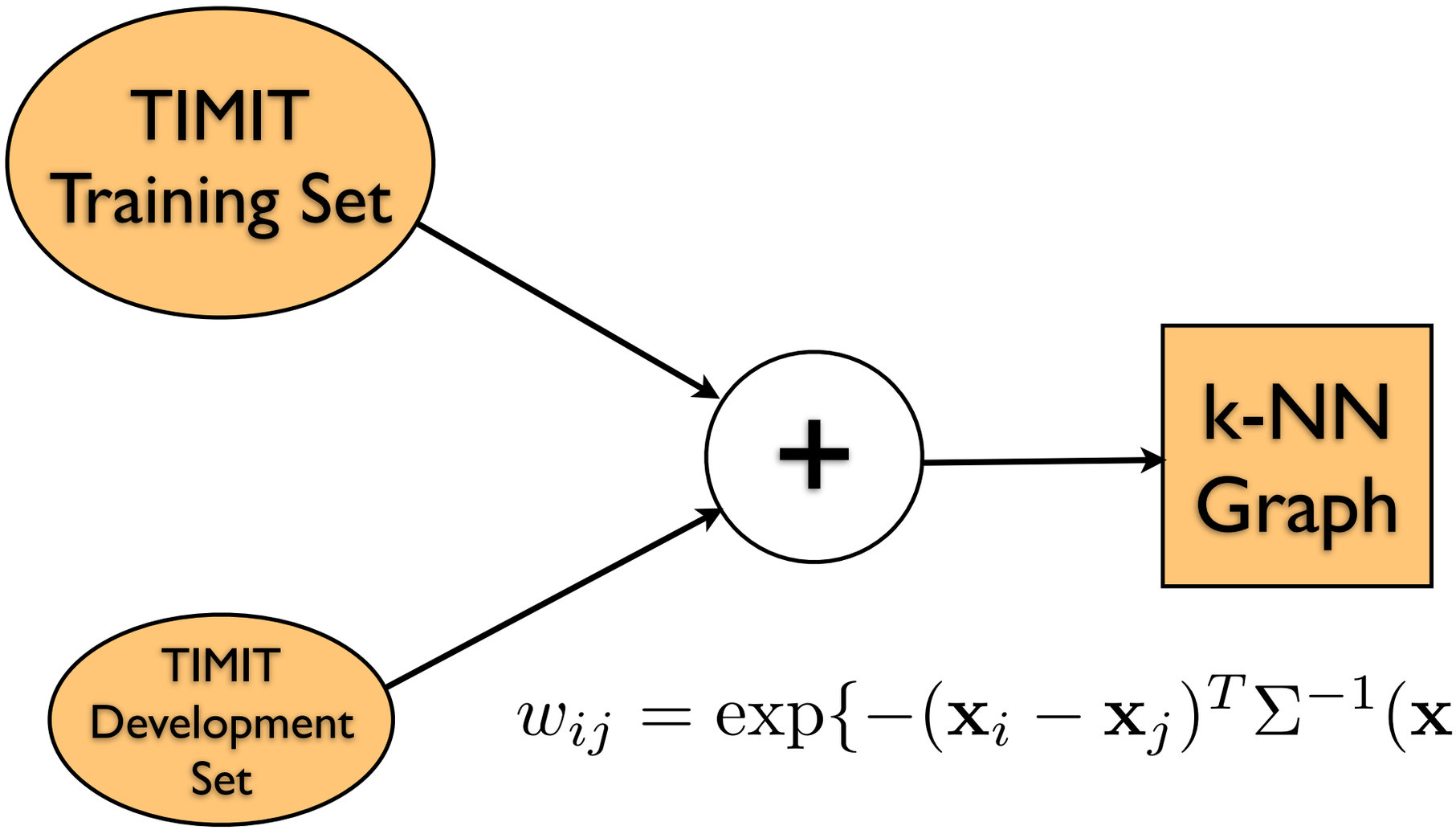


Graph Construction



Graph Construction



$$w_{ij} = \exp\{-(\mathbf{x}_i - \mathbf{x}_j)^T \Sigma^{-1} (\mathbf{x}_i - \mathbf{x}_j)\}$$

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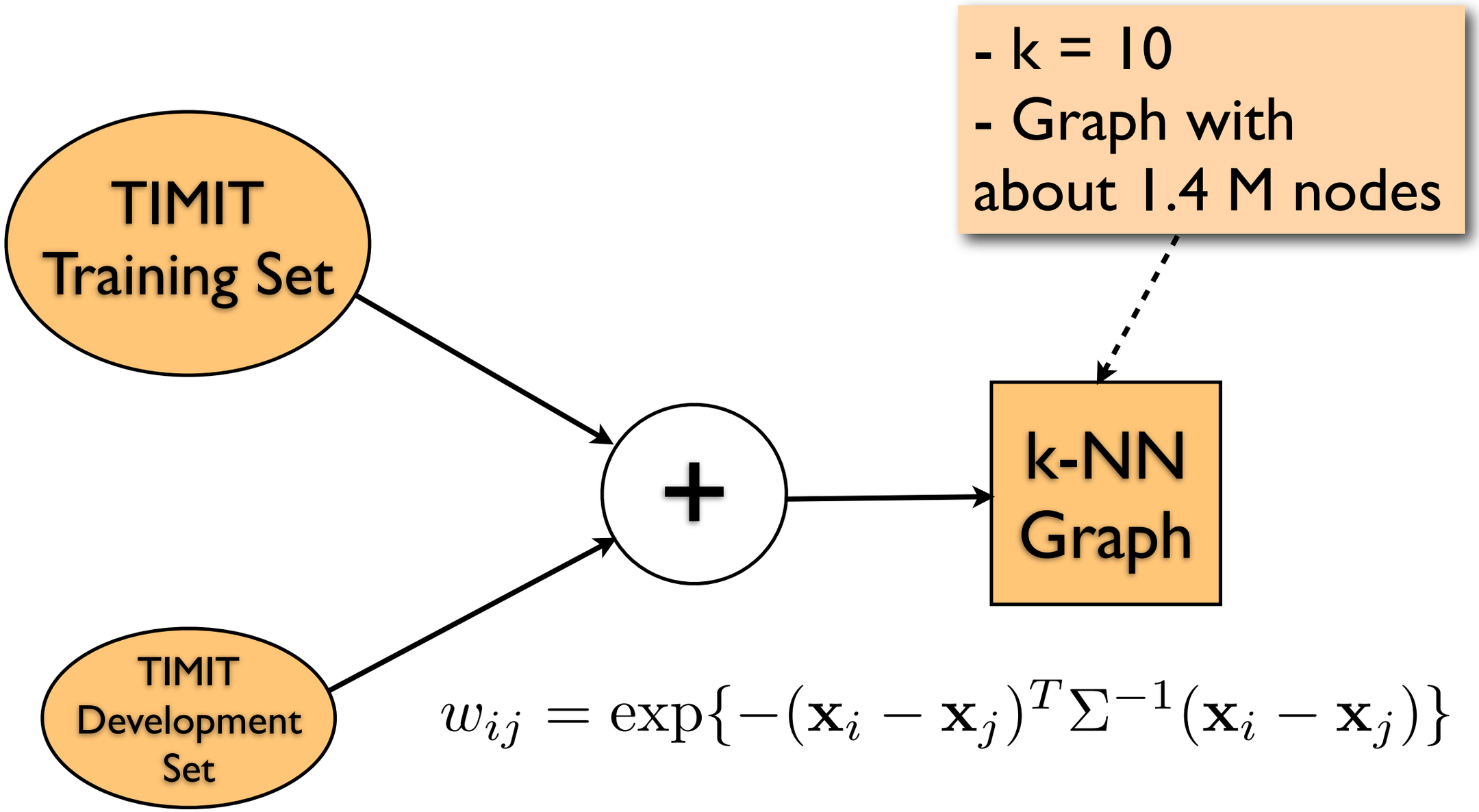

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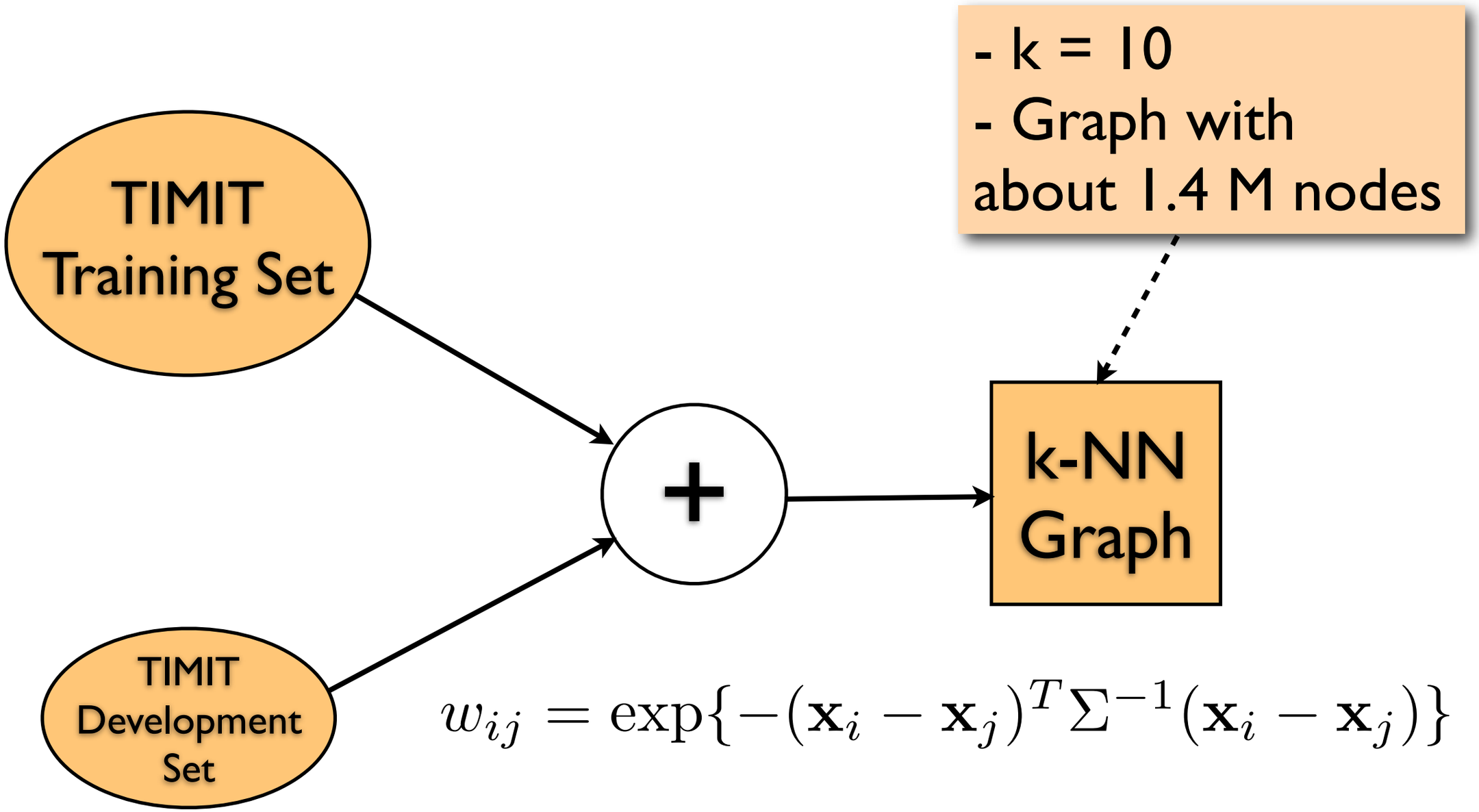

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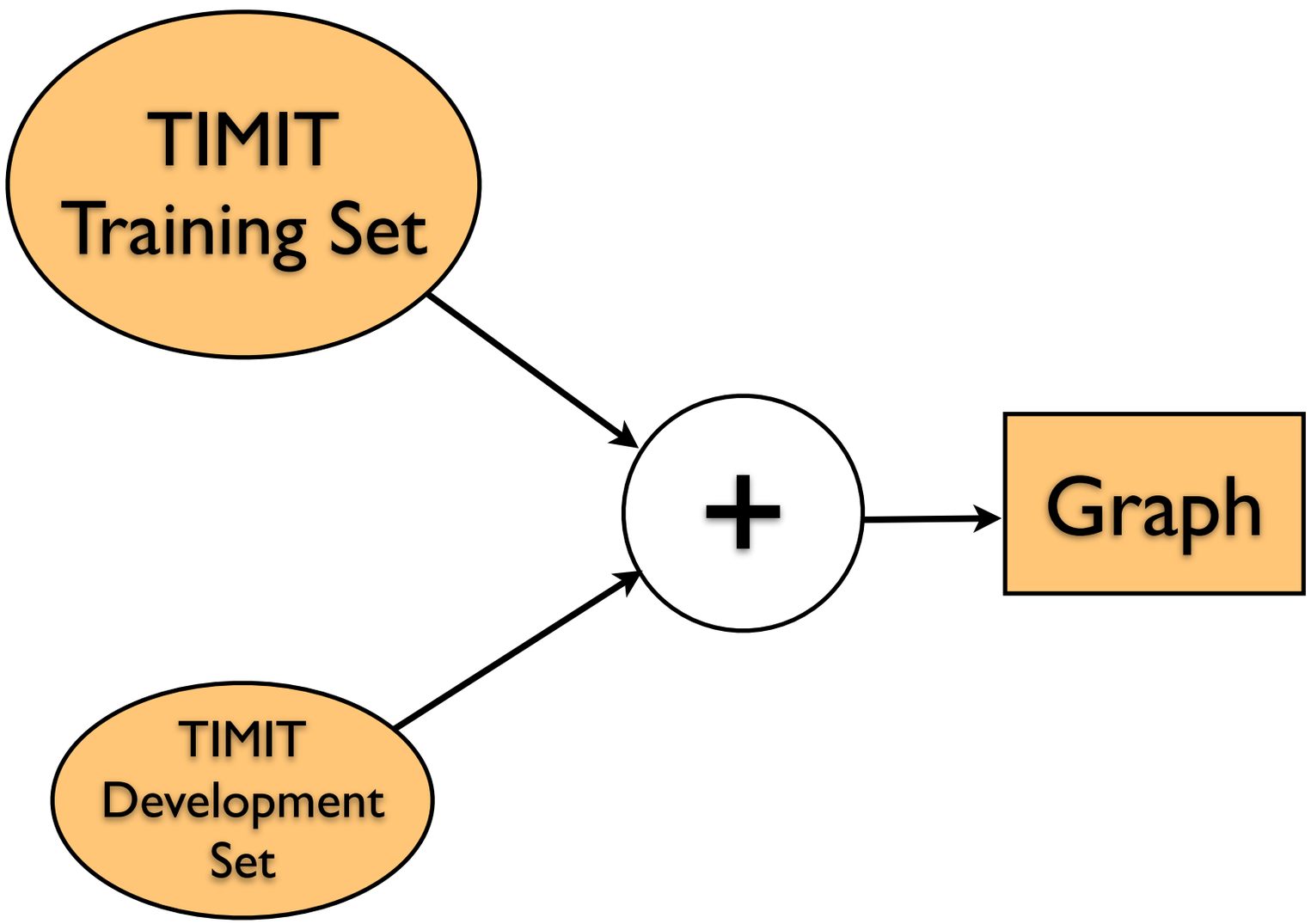
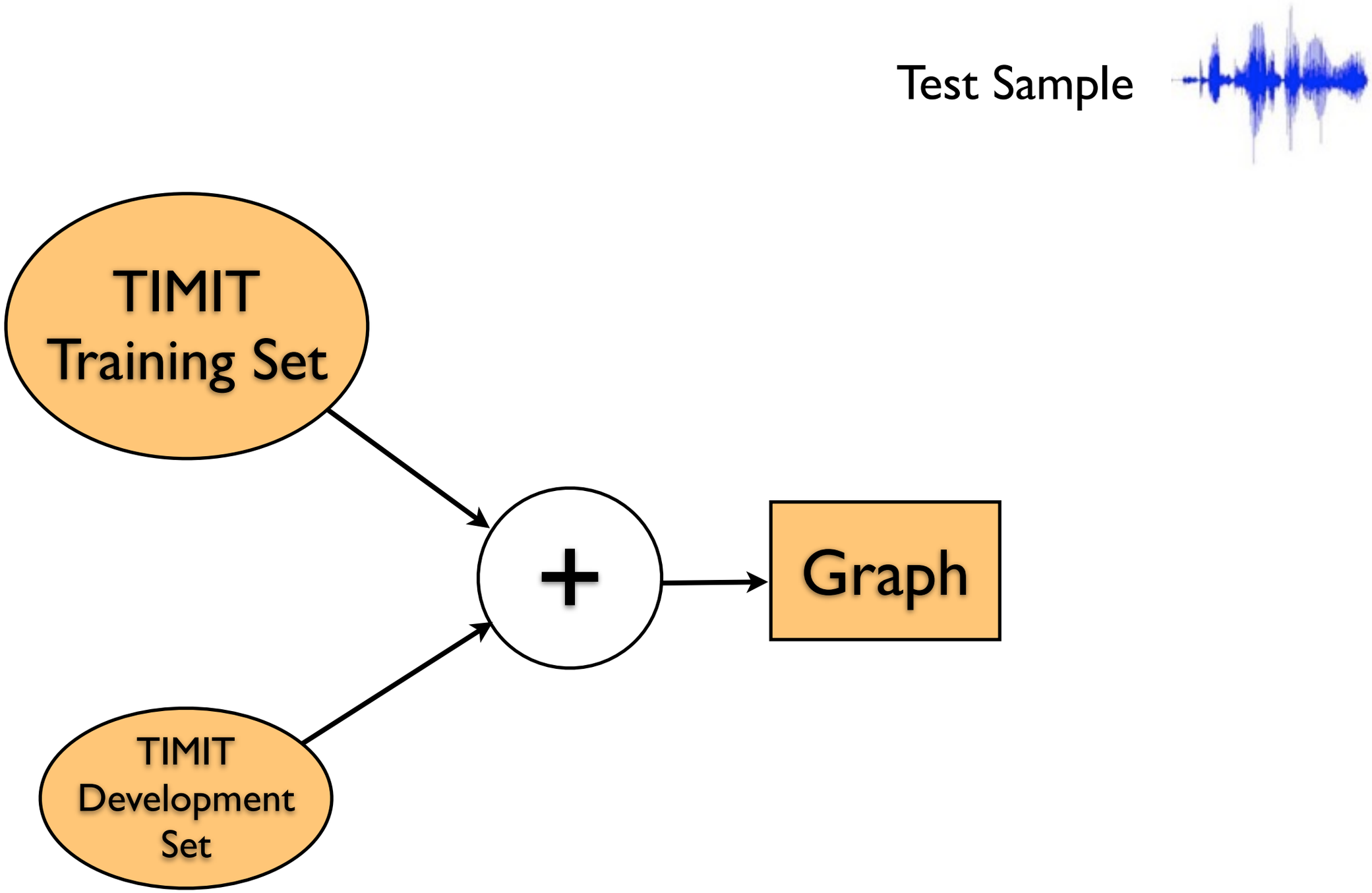
Labels not used during graph construction

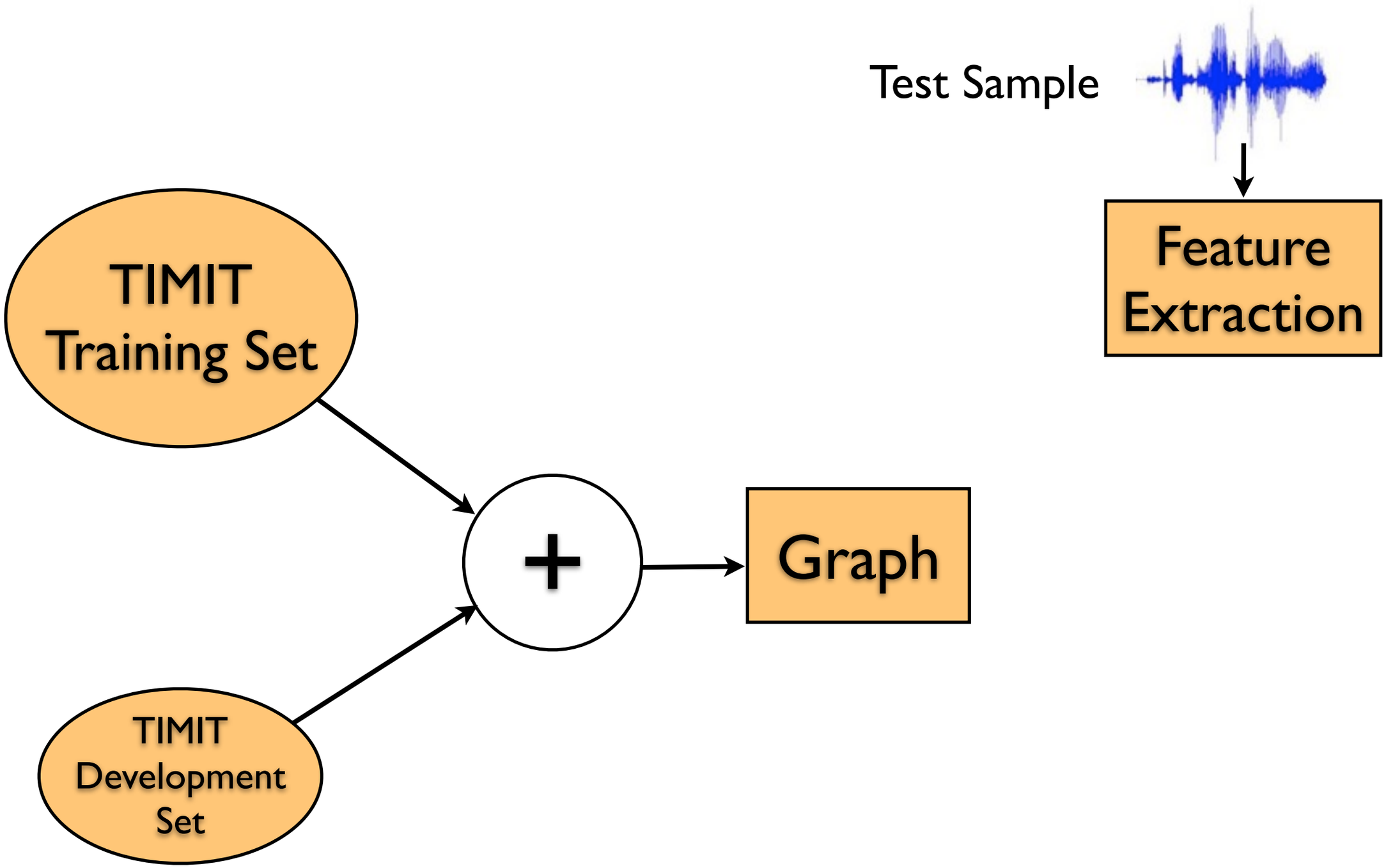
Inductive Extension



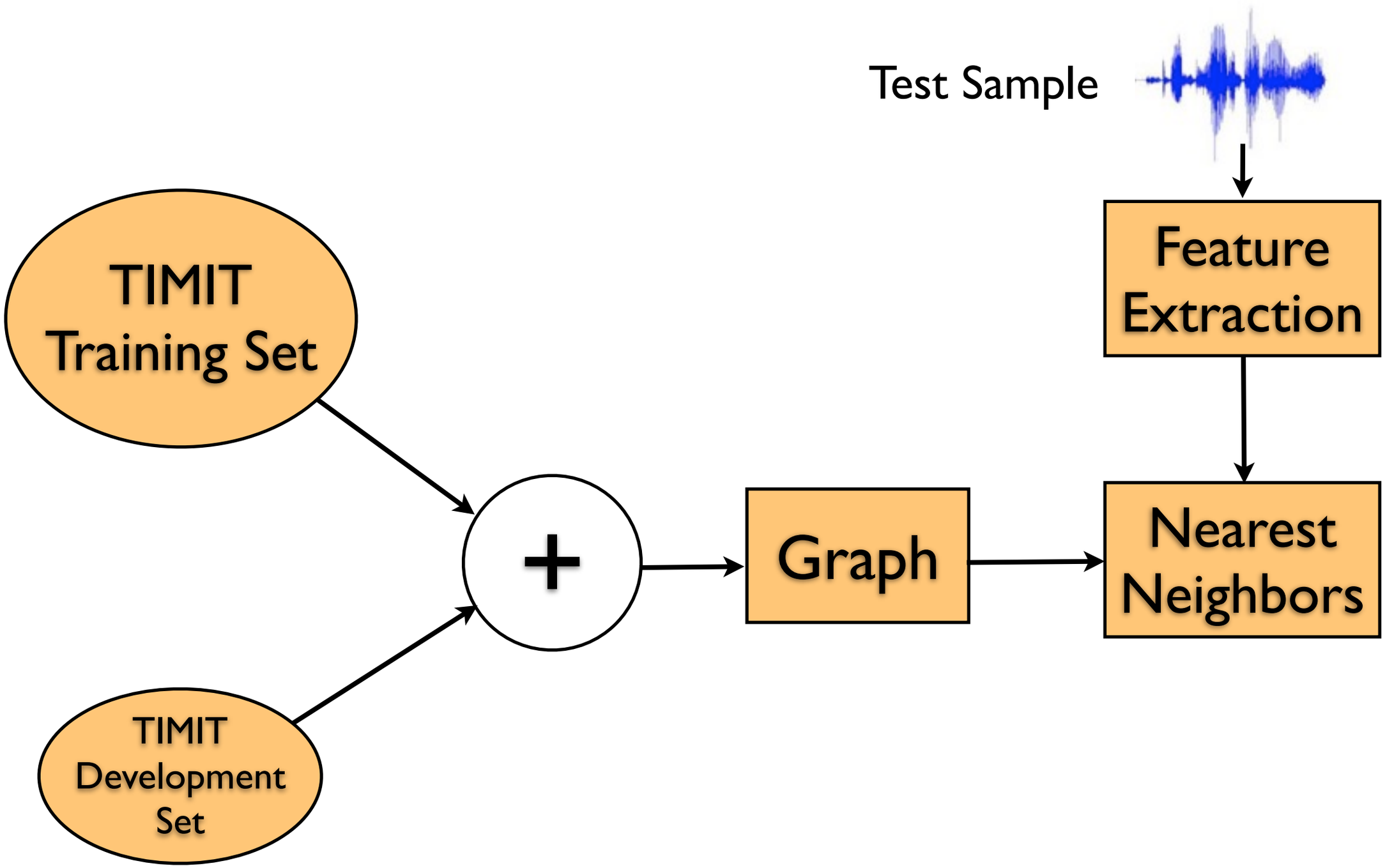
Inductive Extension



Inductive Extension



Inductive Extension



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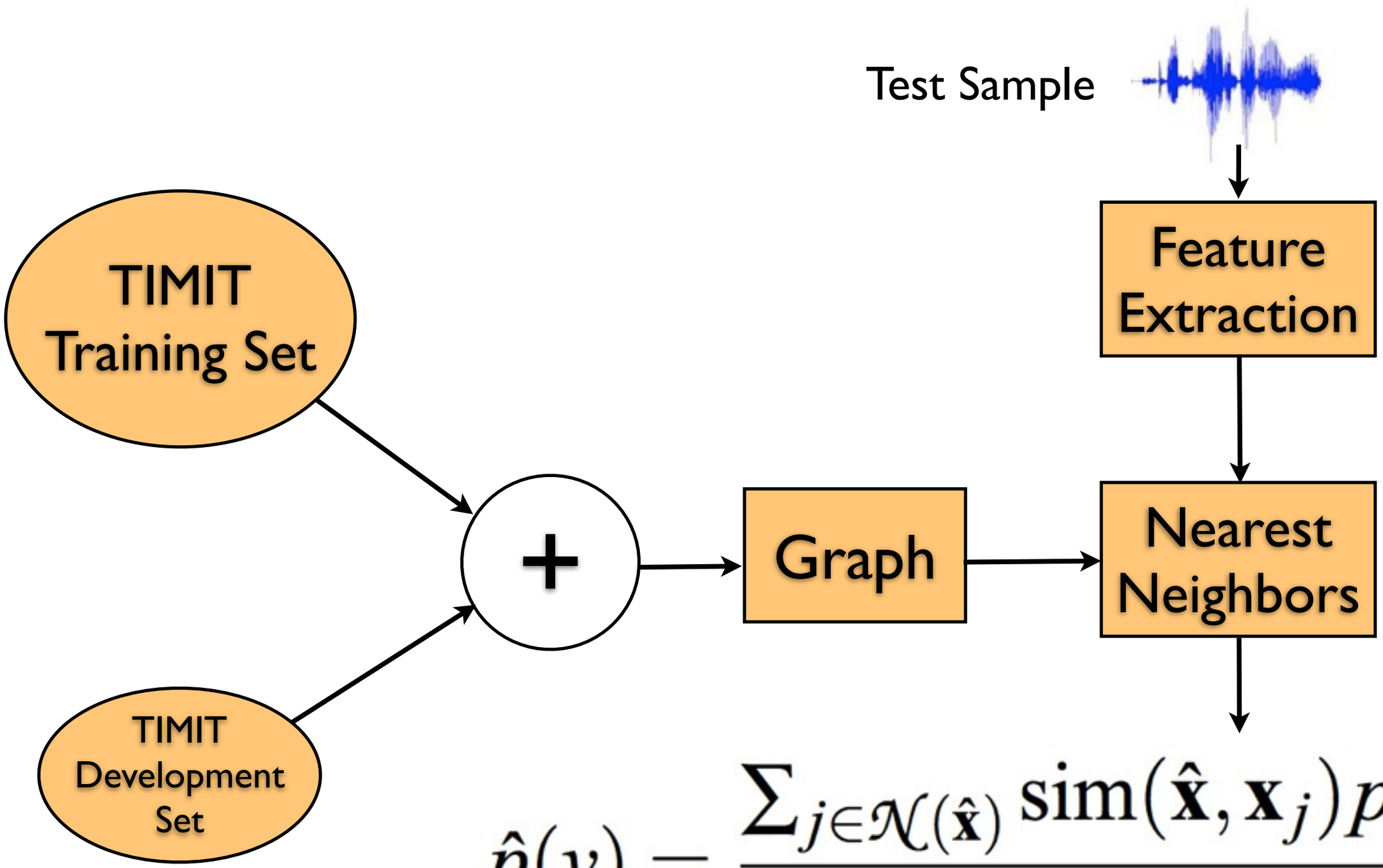

$$\hat{p}(y) = \frac{\sum_{j \in \mathcal{N}(\hat{\mathbf{x}})} \mathrm{sim}(\hat{\mathbf{x}}, \mathbf{x}_j) p_j^*(y)}{\sum_{j \in \mathcal{N}(\hat{\mathbf{x}})} \mathrm{sim}(\hat{\mathbf{x}}, \mathbf{x}_j)}$$

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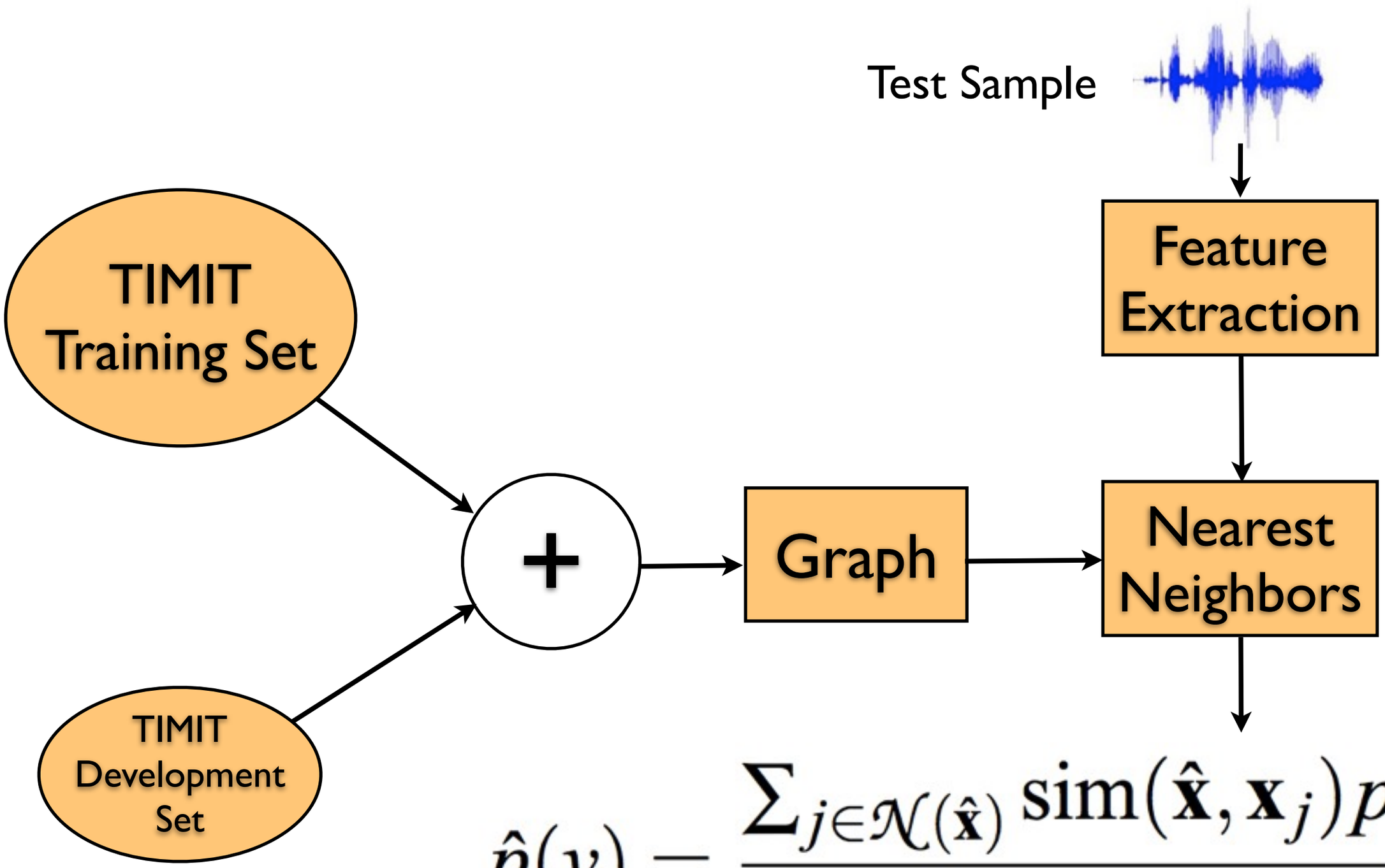

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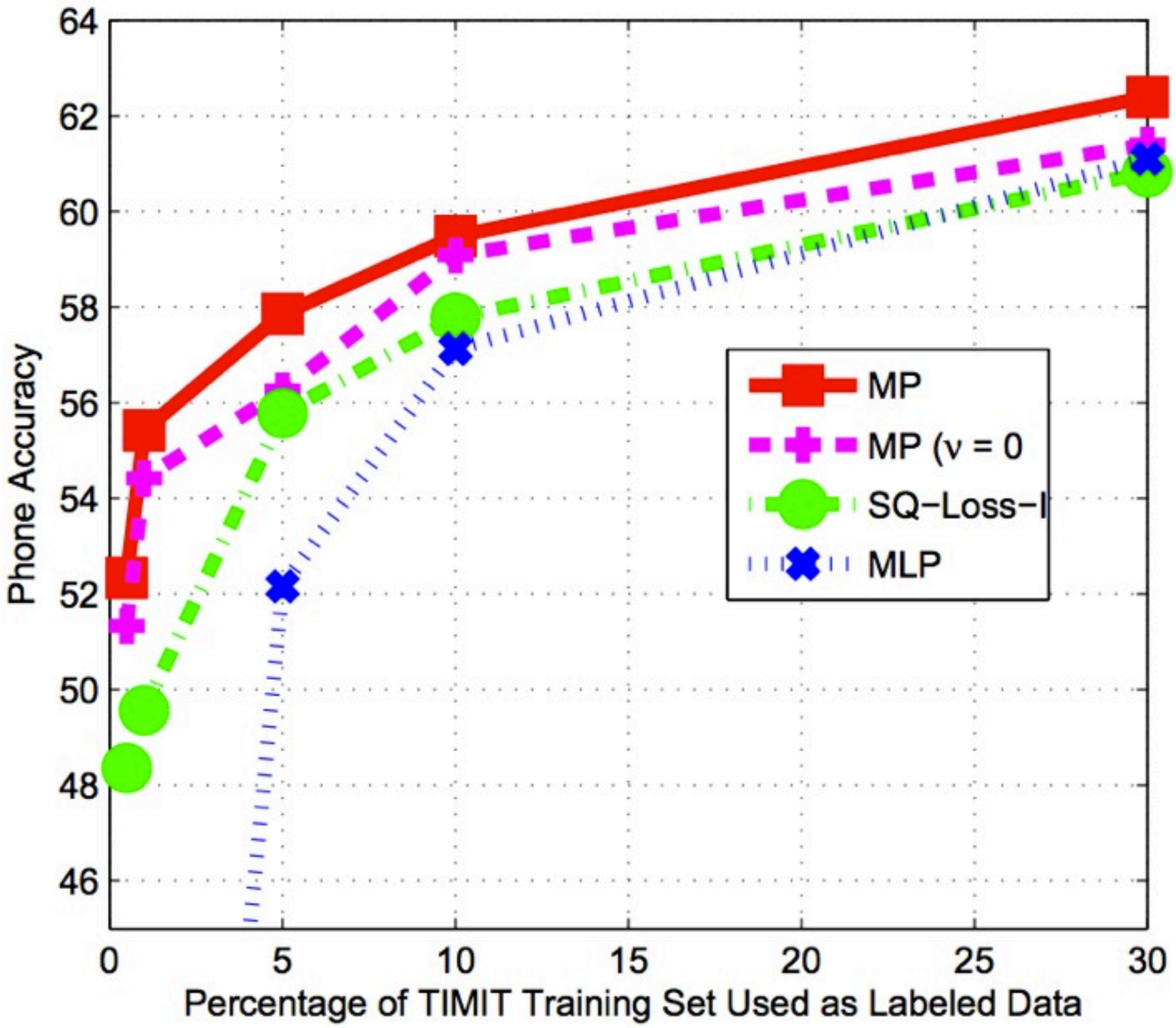
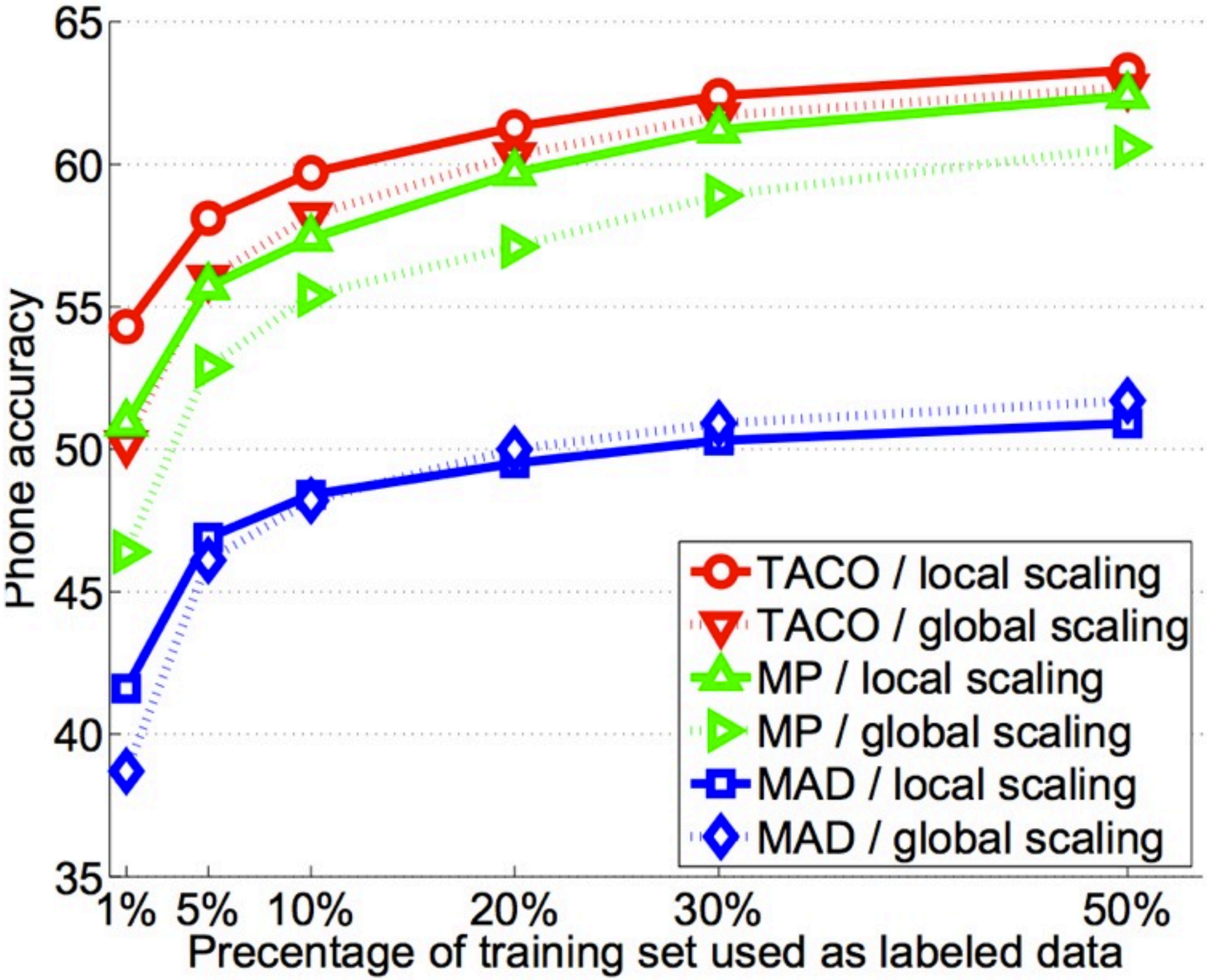
Nadaraya-Watson Estimator

Results (I)


64
62
60
58
56
54
52
50
48
46
Phone Accuracy
0
5
10
15
20
25
30
Percentage of TIMIT Training Set Used as Labeled Data
MP
MP (ν = 0
SQ−Loss−I
MLP

Results (II)


65
60
55
50
45
40
35
Phone accuracy
1%
5%
10%
20%
30%
50%
Precentage of training set used as labeled data
TACO / local scaling
TACO / global scaling
MP / local scaling
MP / global scaling
MAD / local scaling
MAD / global scaling

“Labeled” Graph Construction

[Alexandrescu & Kirchoff, ASRU 2007]

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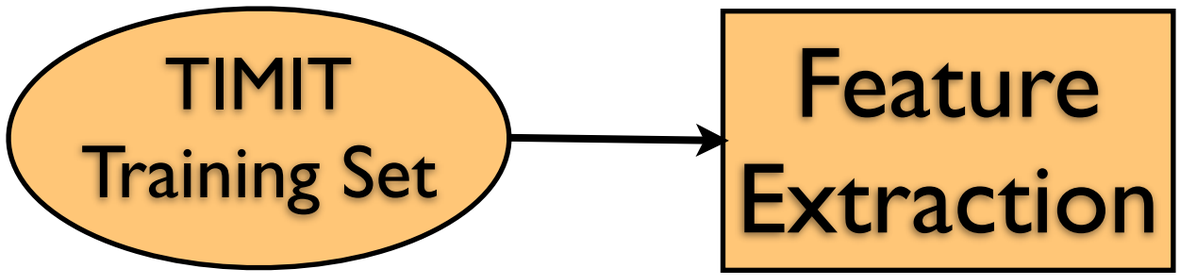

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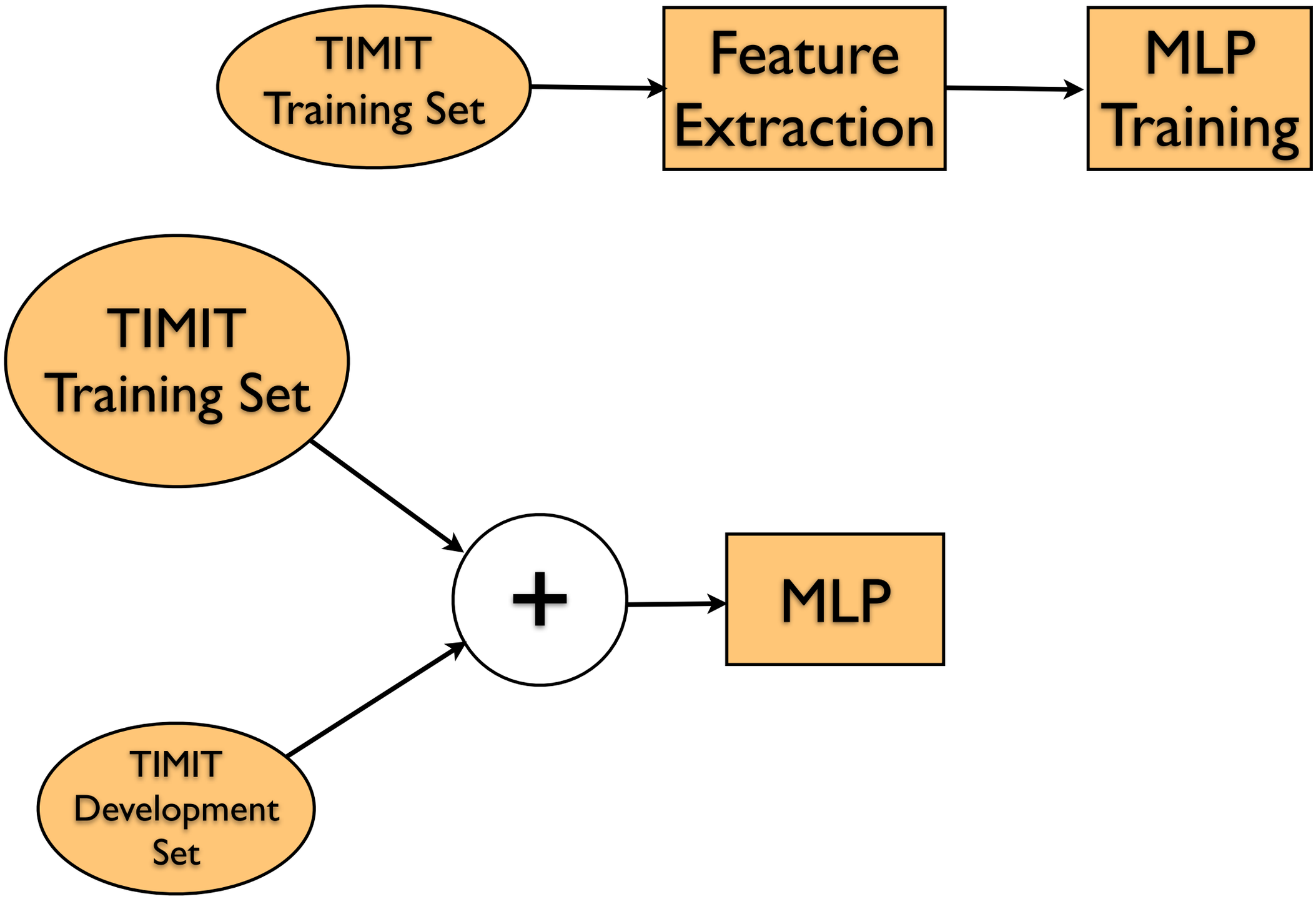

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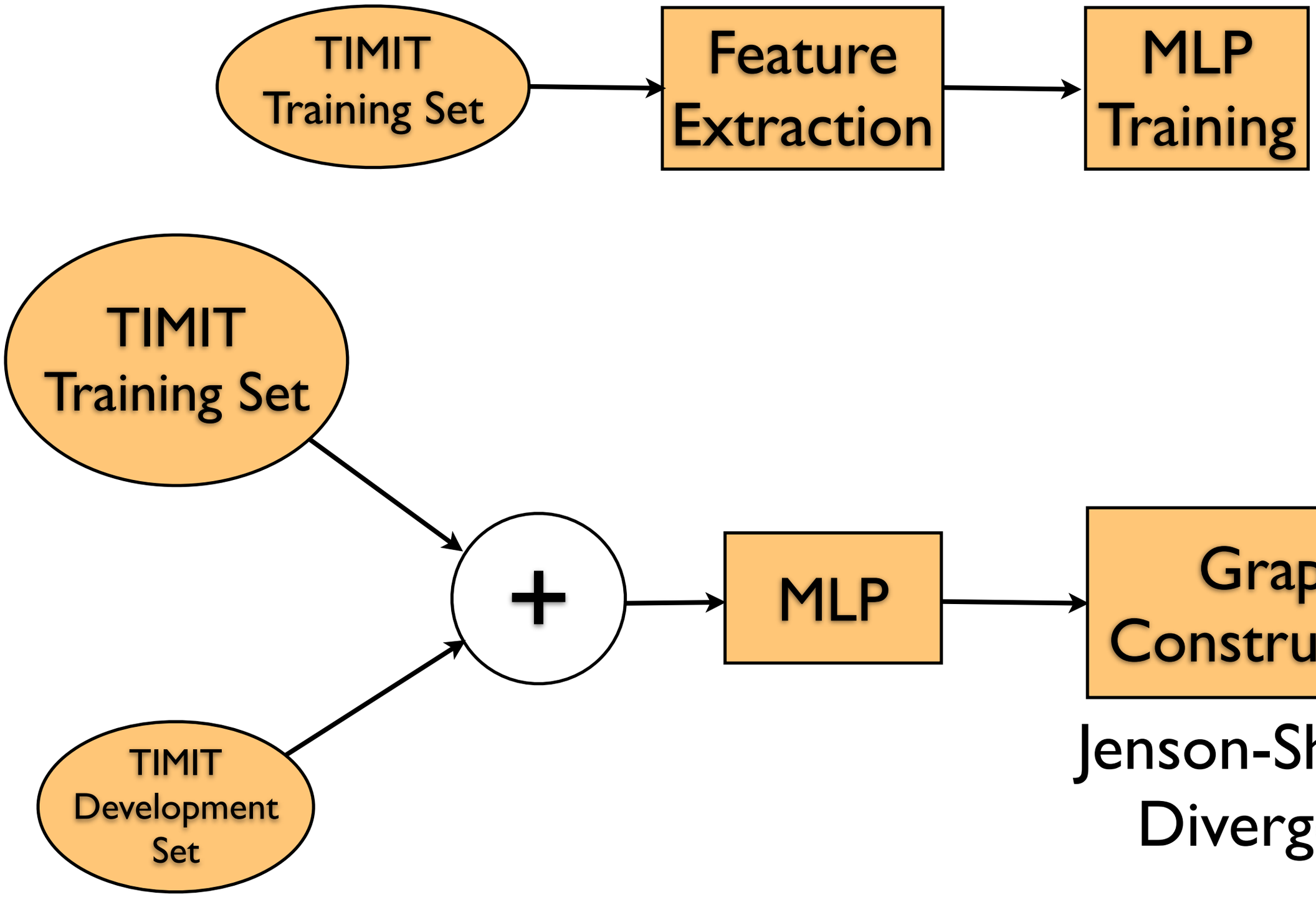
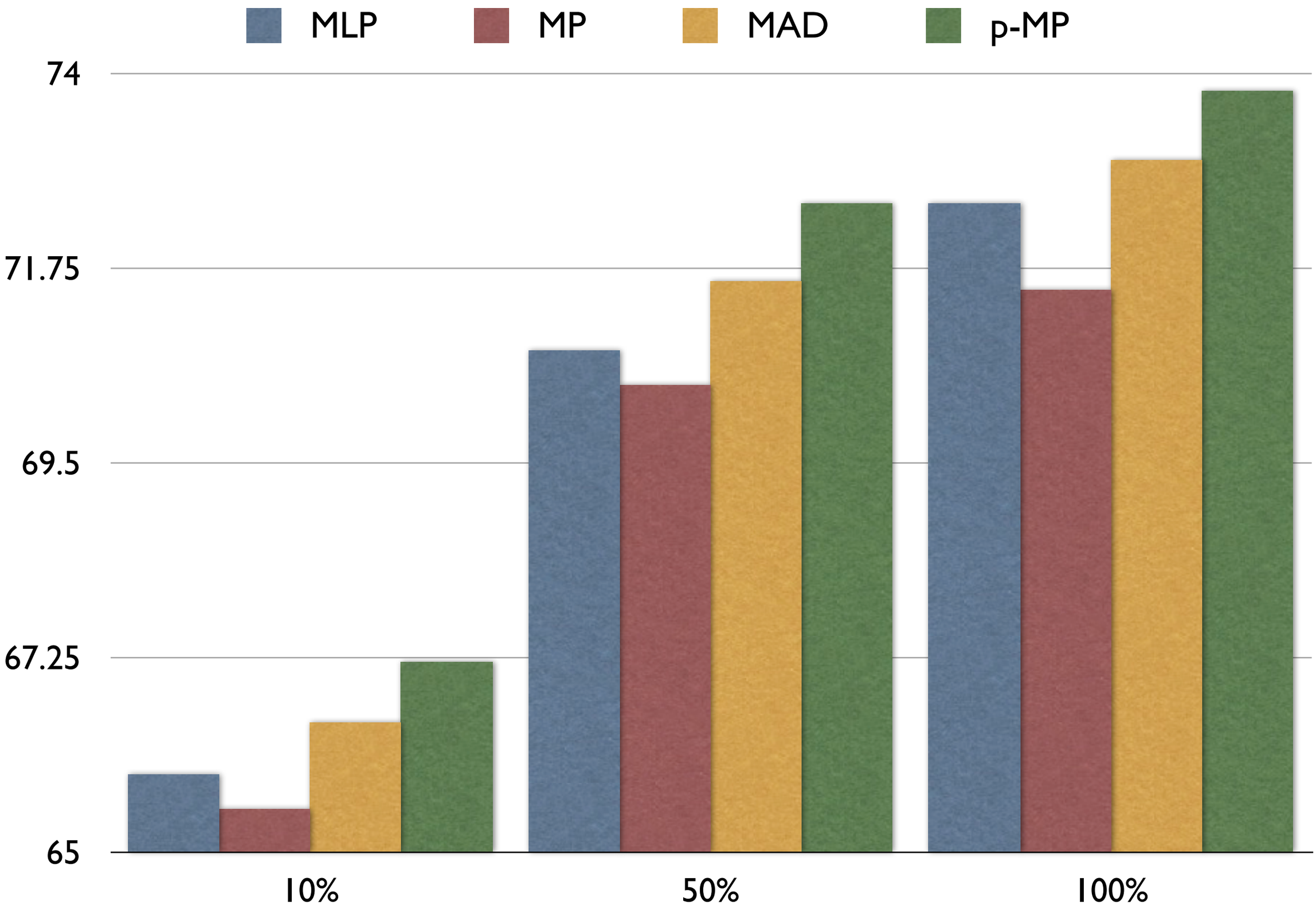


Jenson-Shannon
Divergence

[Alexandrescu & Kirchoff, ASRU 2007]

Results (III)



[Liu & Kirchoff, UW-EE TR 2012]

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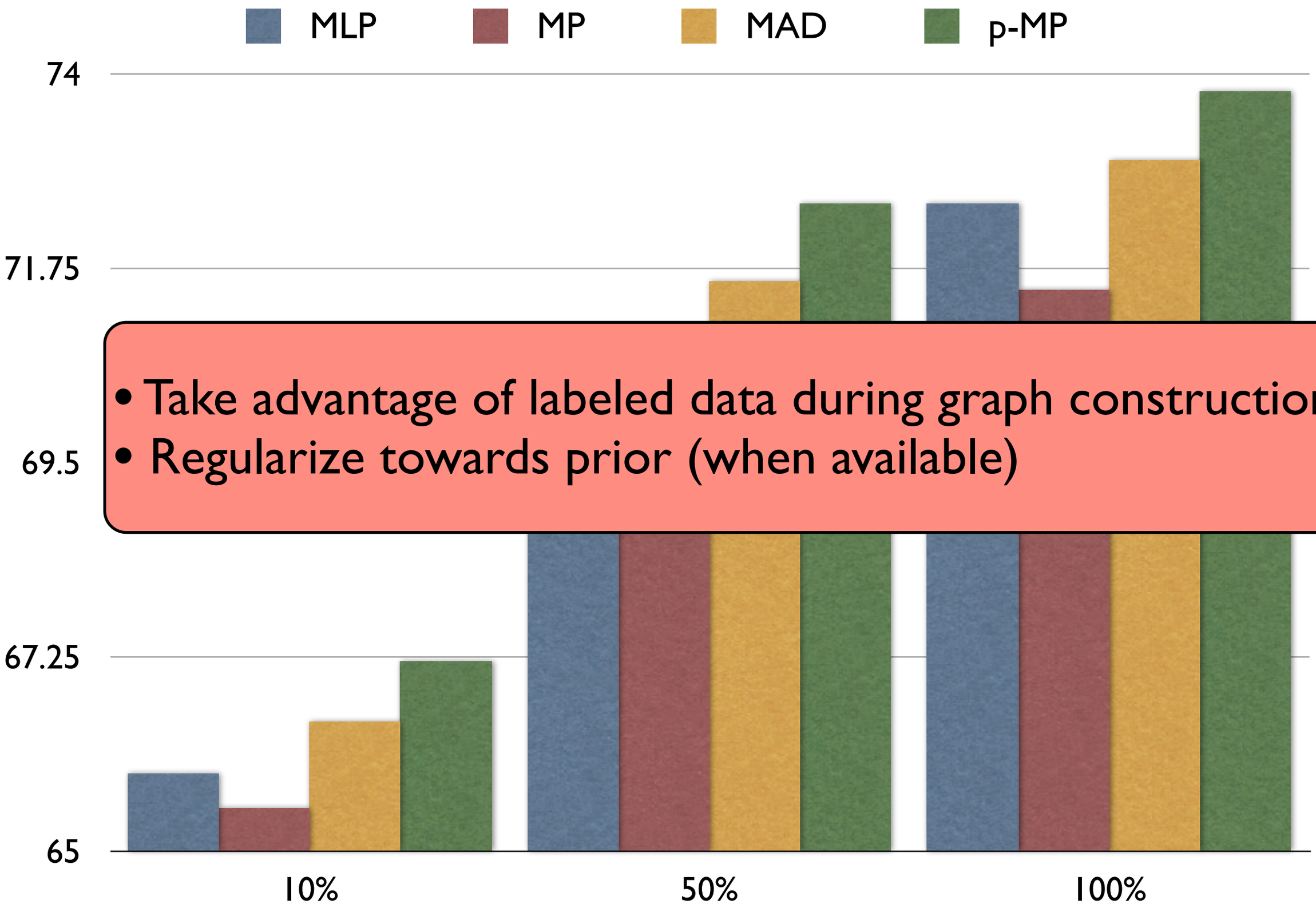

- Take advantage of labeled data during graph construction
- Regularize towards prior (when available)

[Liu & Kirchoff, UW-EE TR 2012]

Switchboard Phonetic Annotation

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- Switchboard corpus consists of about 300 hours of conversational speech.
  - Less reliable automatically generated phone-level annotations [Deshmukh et al., 1998]

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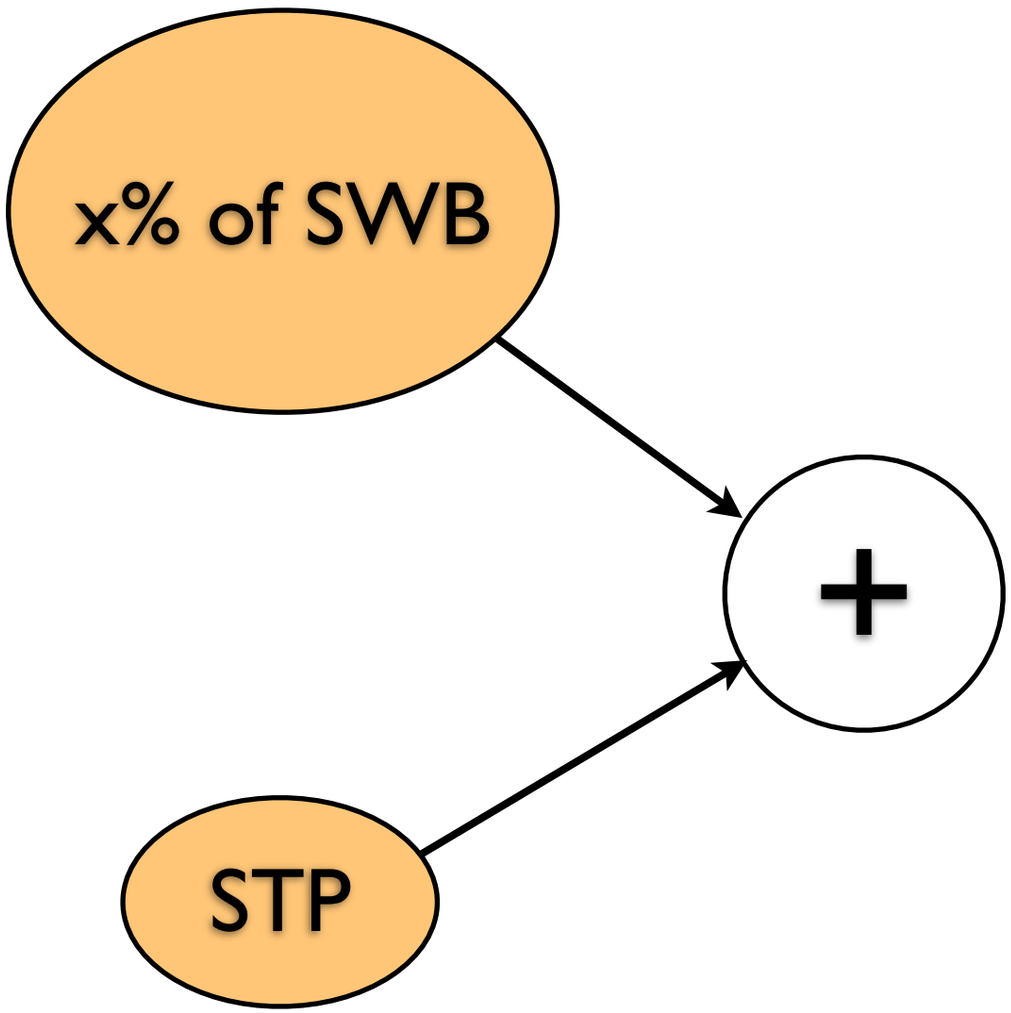
  - Less reliable automatically generated phone-level annotations [Deshmukh et al., 1998]
- **Switchboard transcription project** (STP) [Greenberg, 1995]
  - Manual phonetic annotation
  - Only about 75 minutes of data annotated

Graph Construction

Graph Construction



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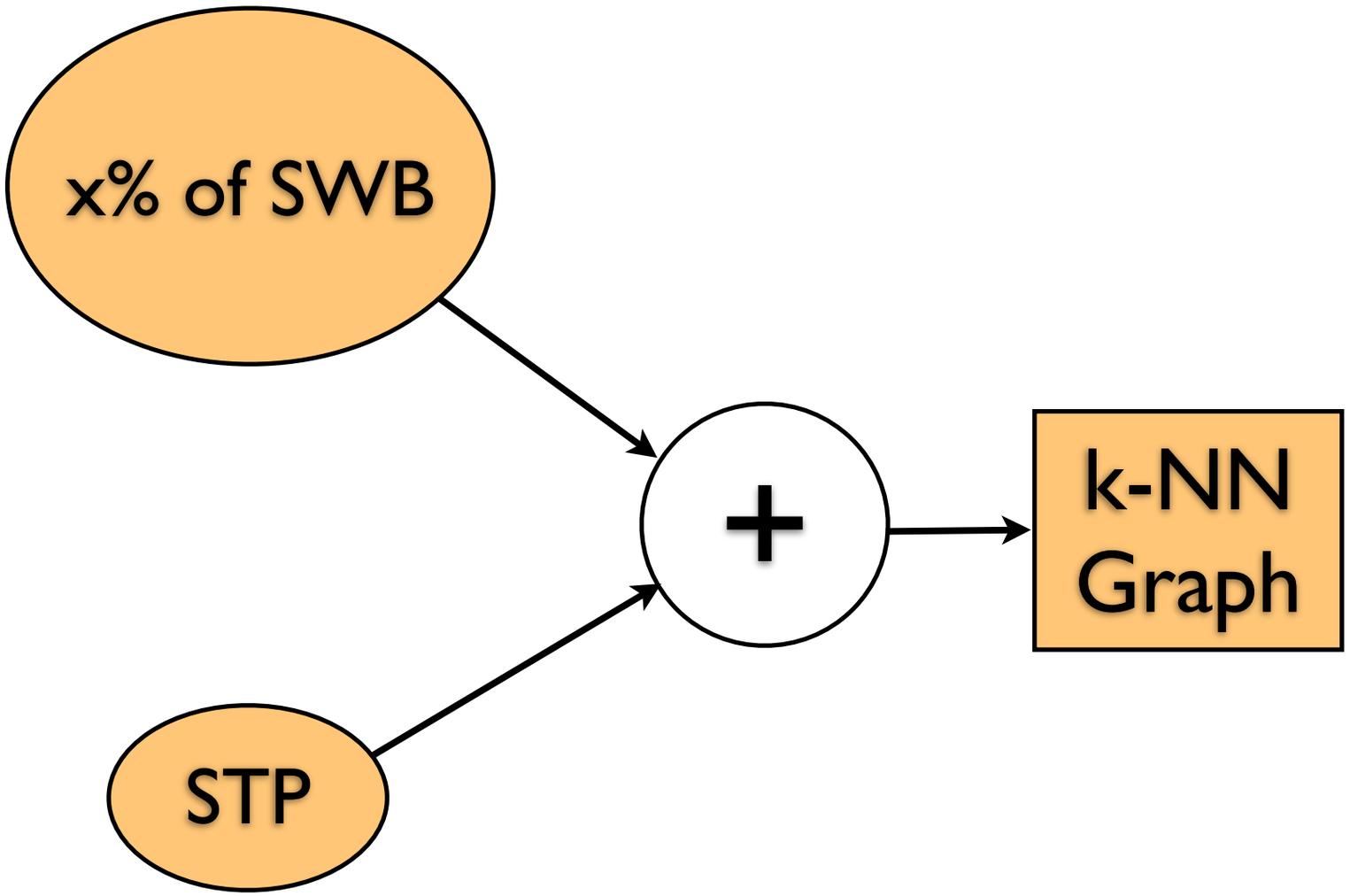

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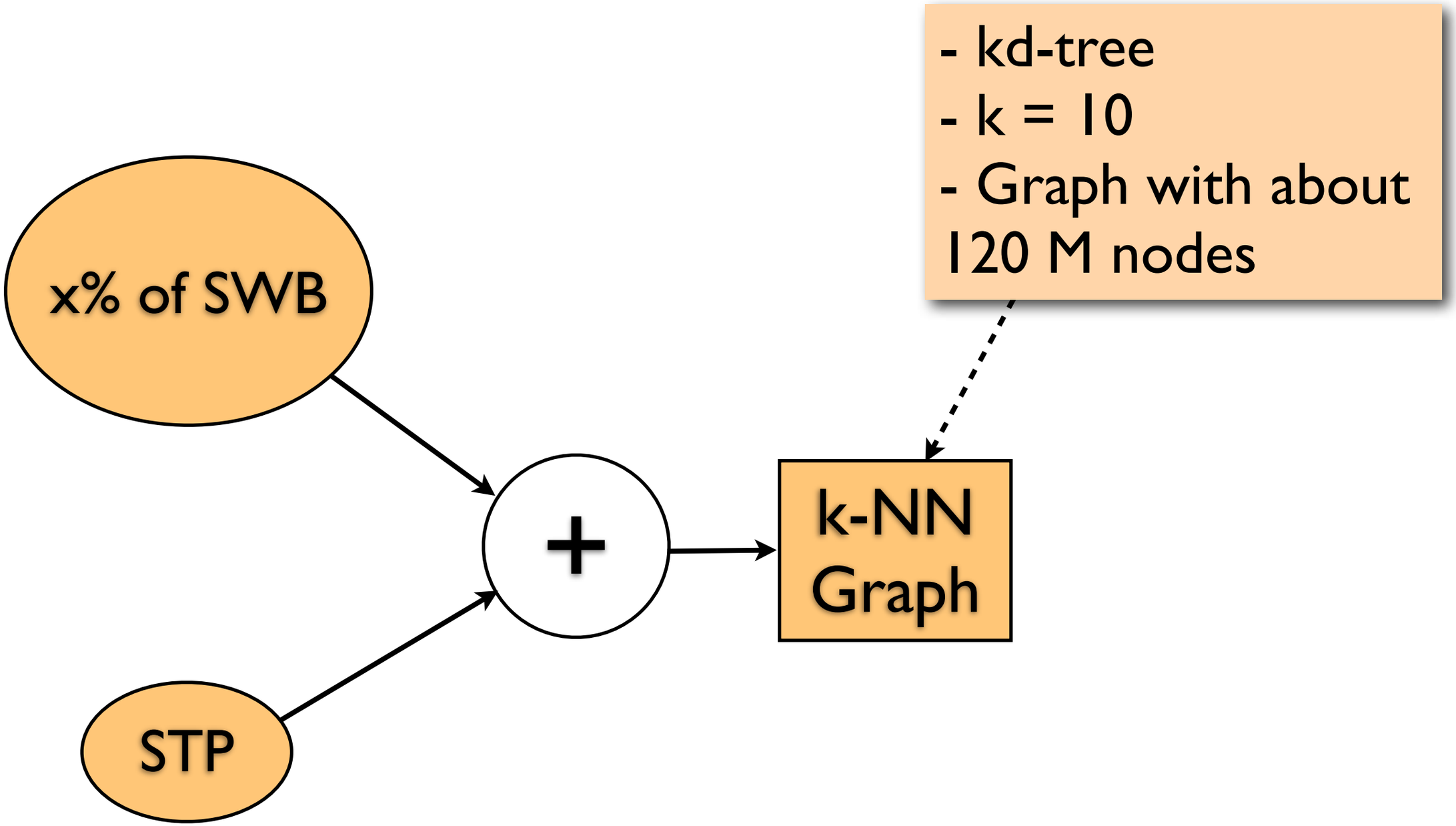

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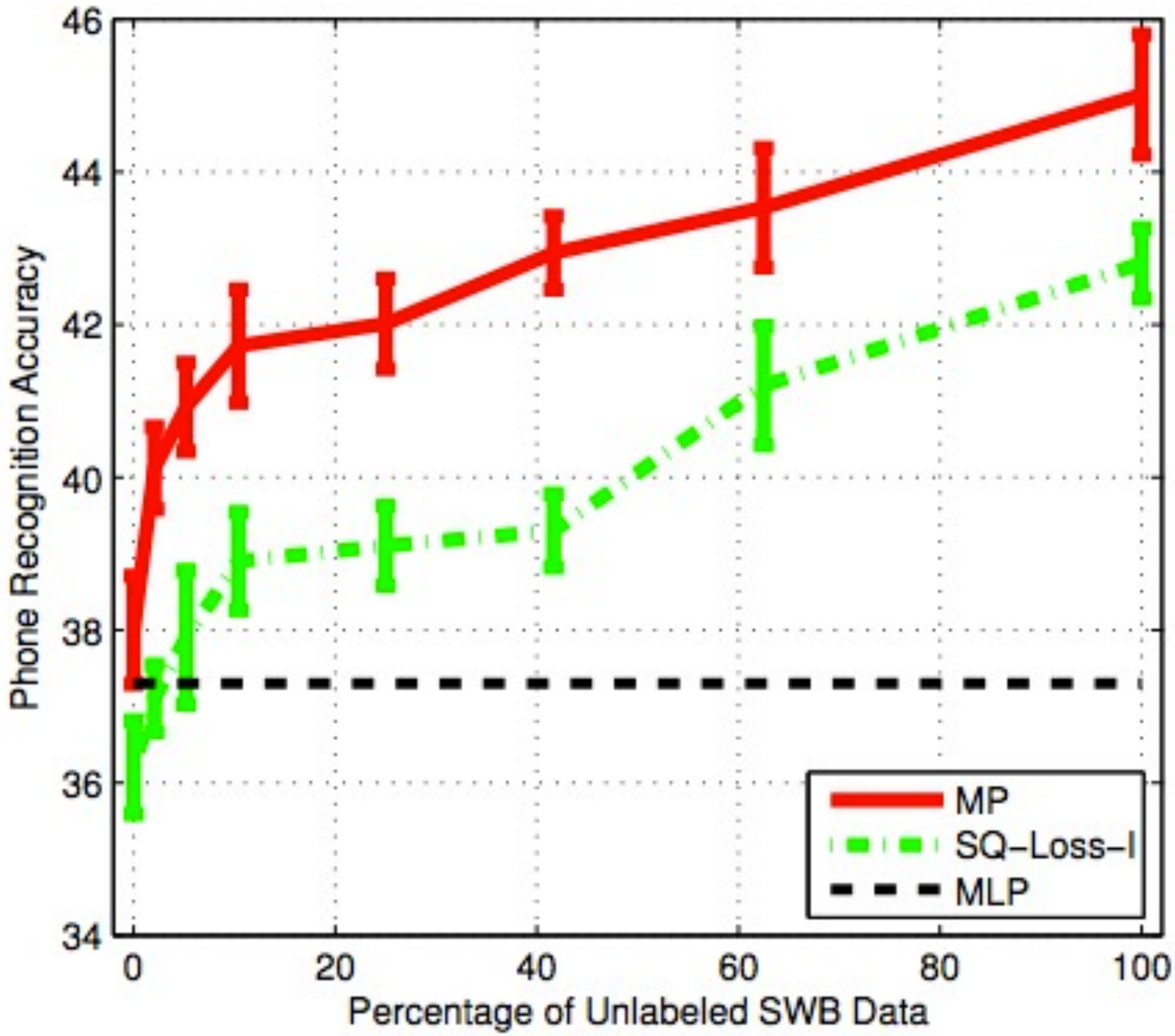

[Subramanya & Bilmes, JMLR 2011]

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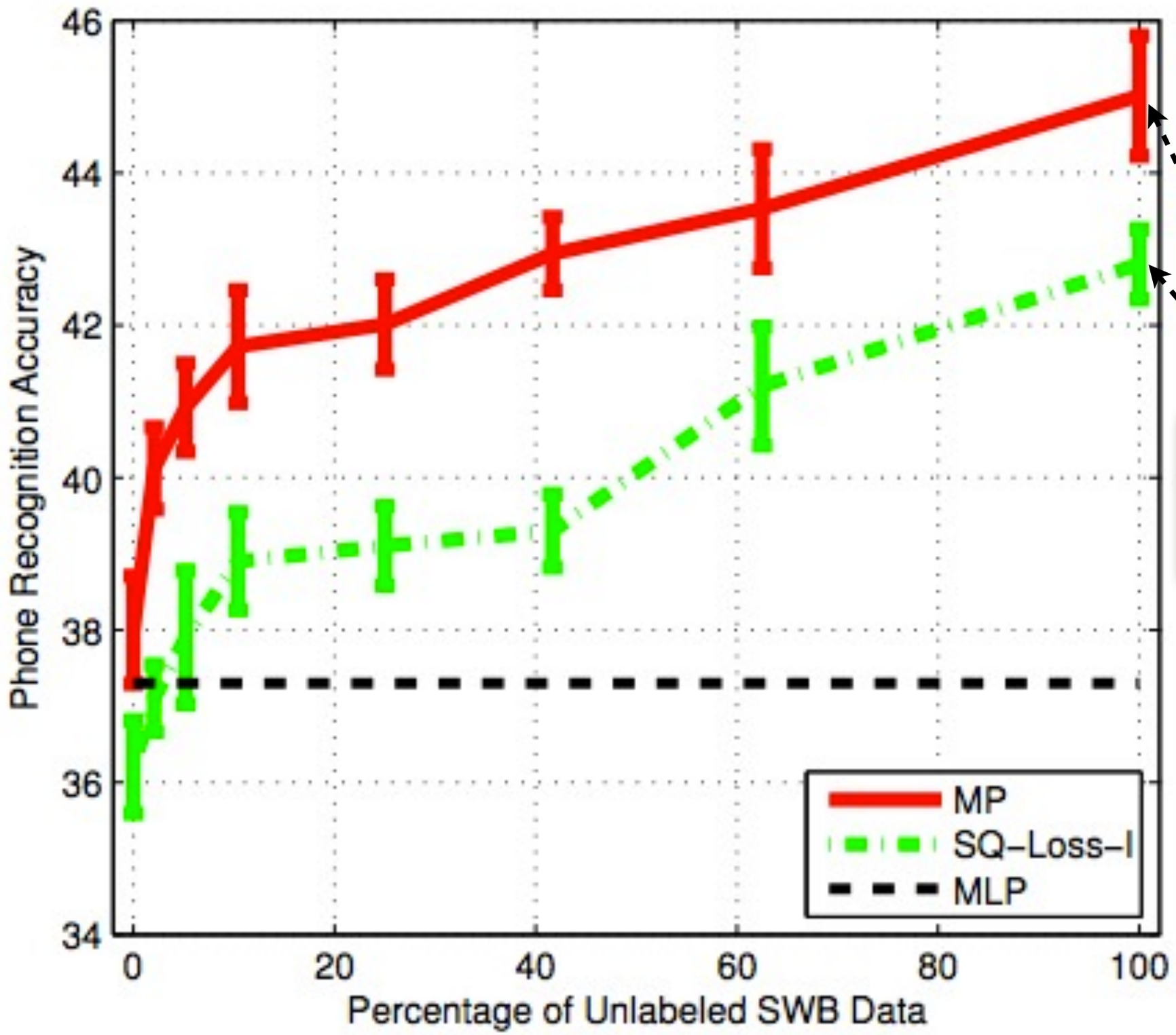

Graph with **120m** vertices

[Subramanya & Bilmes, JMLR 2011]

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- Given a document (e.g., web page, news article), assign it to a fixed number of semantic categories (e.g., sports, politics, entertainment)
- Multi-label problem
- Training supervised models requires large amounts of labeled data [Dumais et al., 1998]

Corpora

- **Reuters** [Lewis, et al., 1978]
  - Newswire
  - About 20K document with 135 categories. Use top 10 categories (e.g., “earnings”, “acquistions”, “wheat”, “interest”) and label the remaining as “other”

Corpora

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- **WebKB** [Bekkerman, et al., 2003]
  - 8K webpages from 4 academic domains
  - Categories include “course”, “department”, “faculty” and “project”

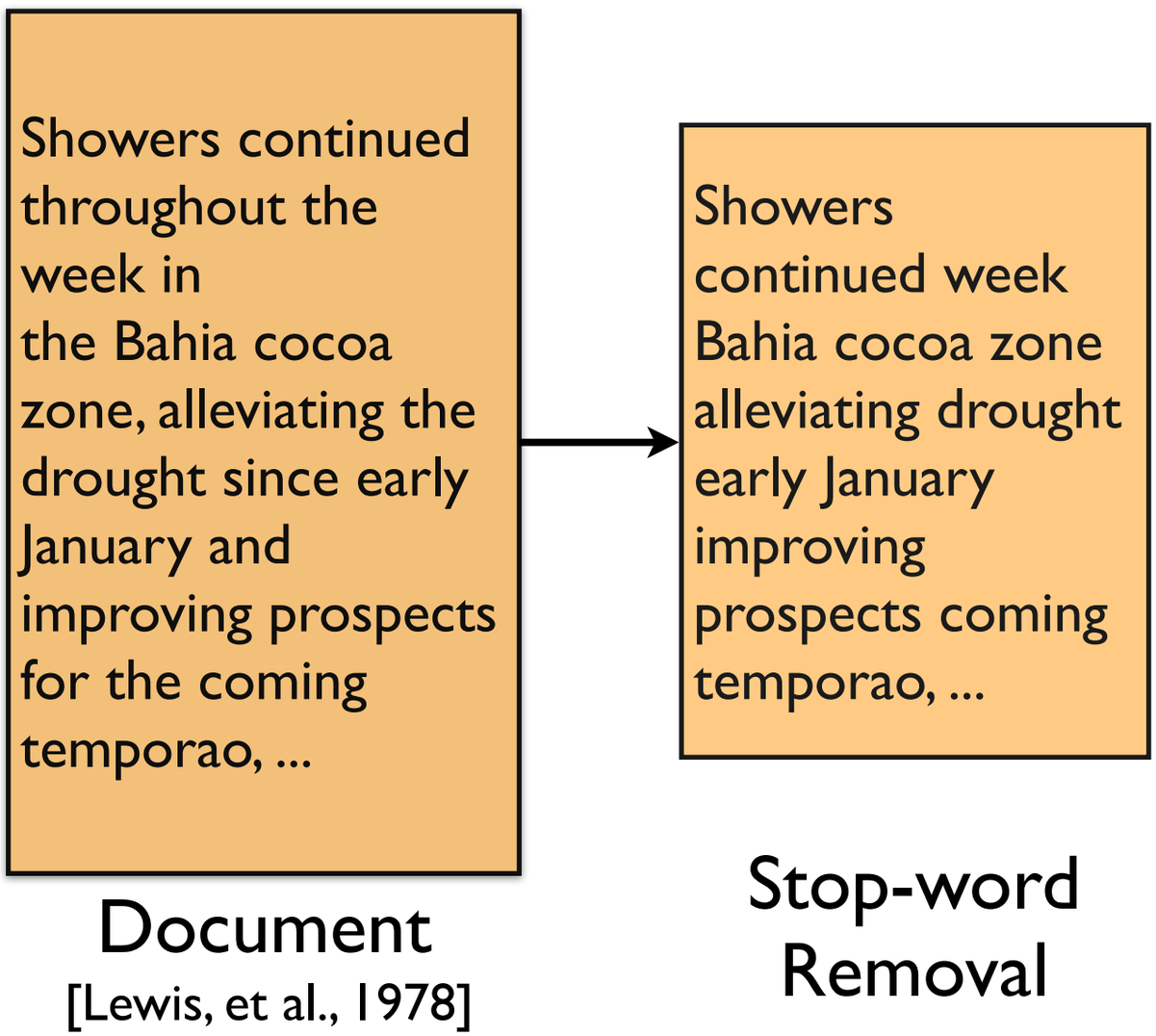
Feature Extraction

Showers continued throughout the week in
the Bahia cocoa zone, alleviating the drought since early January and improving prospects for the coming temporao, ...

Document

[Lewis, et al., 1978]

Feature Extraction



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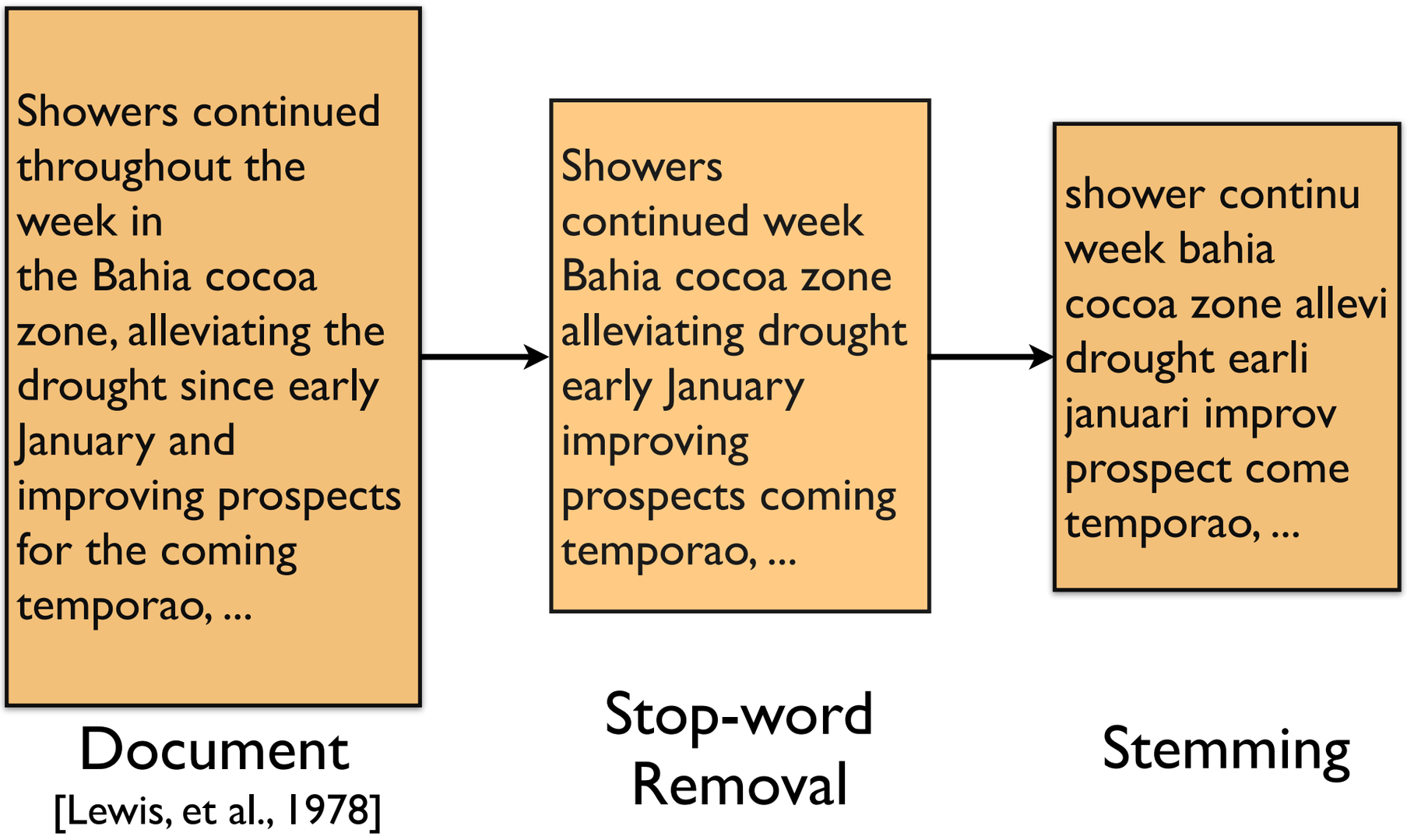

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Document
[Lewis, et al., 1978]

Showers continued week Bahia cocoa zone alleviating drought early January improving prospects coming temporao, ...

Stop-word Removal

shower continu week bahia cocoa zone allevi drought earli januari improv prospect come temporao, ...

Stemming

shower	0.01
bahia	0.34
cocoa	0.33
.	.
.	.
.	.
.	.

TFIDF weighted Bag-of-words

Results

Average PRBEP	SVM	TSVM	SGT	LP	MP	MAD
Reuters	48.9	59.3	60.3	59.7	**66.3**	-
WebKB	23.0	29.2	36.8	41.2	51.9	**53.7**

Precision-recall break even point (PRBEP)

Results

Support Vector Machine (Supervised)

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Precision-recall break even point (PRBEP)

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Transductive SVM [Joachims 1999]

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Precision-recall break even point (PRBEP)

Results

Support Vector Machine (Supervised)

Transductive SVM [Joachims 1999]

Spectral Graph Transduction (SGT) [Joachims 2003]

Label Propagation [Zhu & Ghahramani 2002]

Measure Propagation [Subramanya & Bilmes 2008]

Modified Adsorption [Talukdar & Crammer 2009]

Average PRBEP	SVM	TSVM	SGT	LP	MP	MAD
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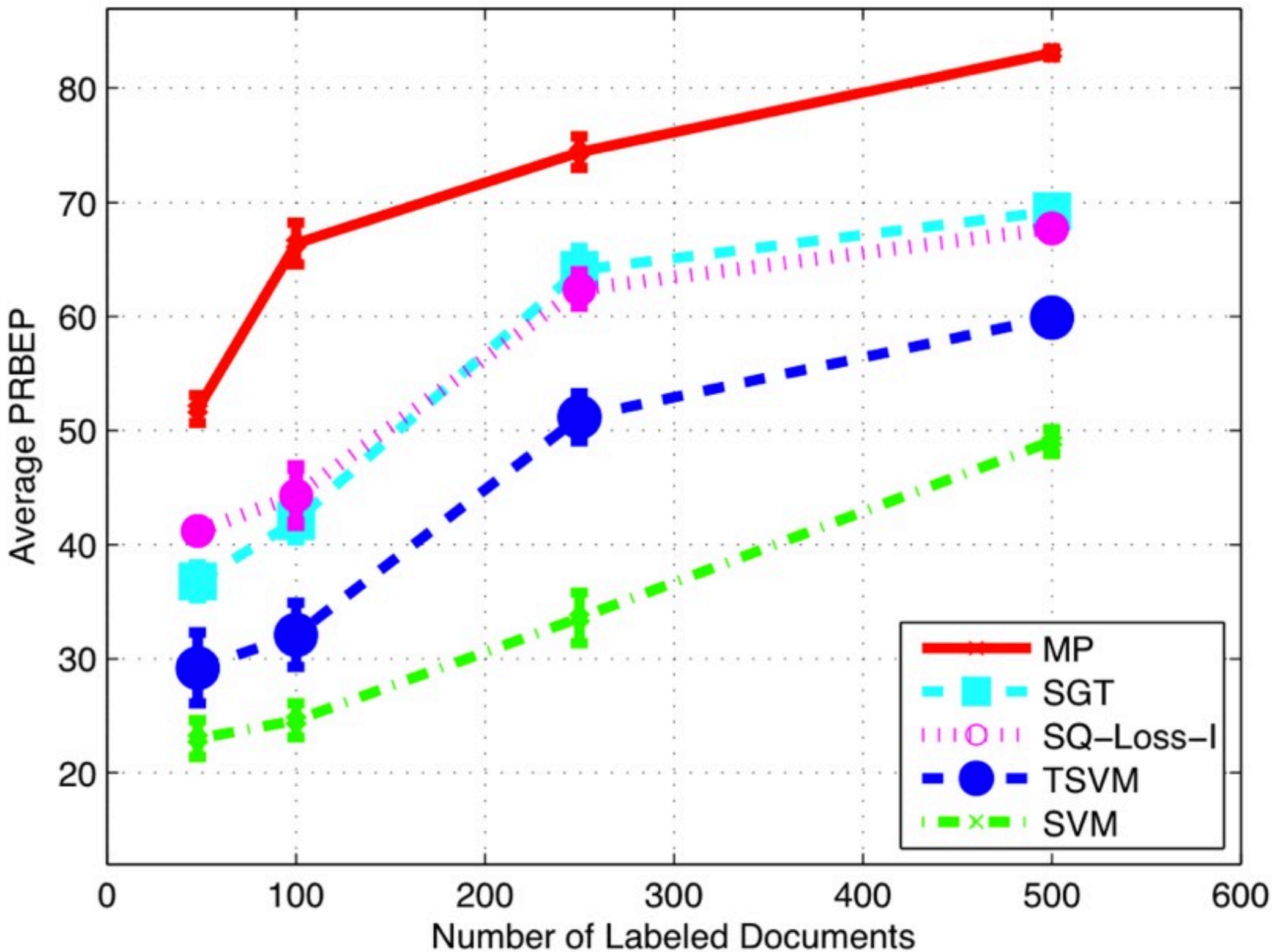
Precision-recall break even point (PRBEP)

Results on WebKB



[Subramanya & Bilmes, EMNLP 2008]

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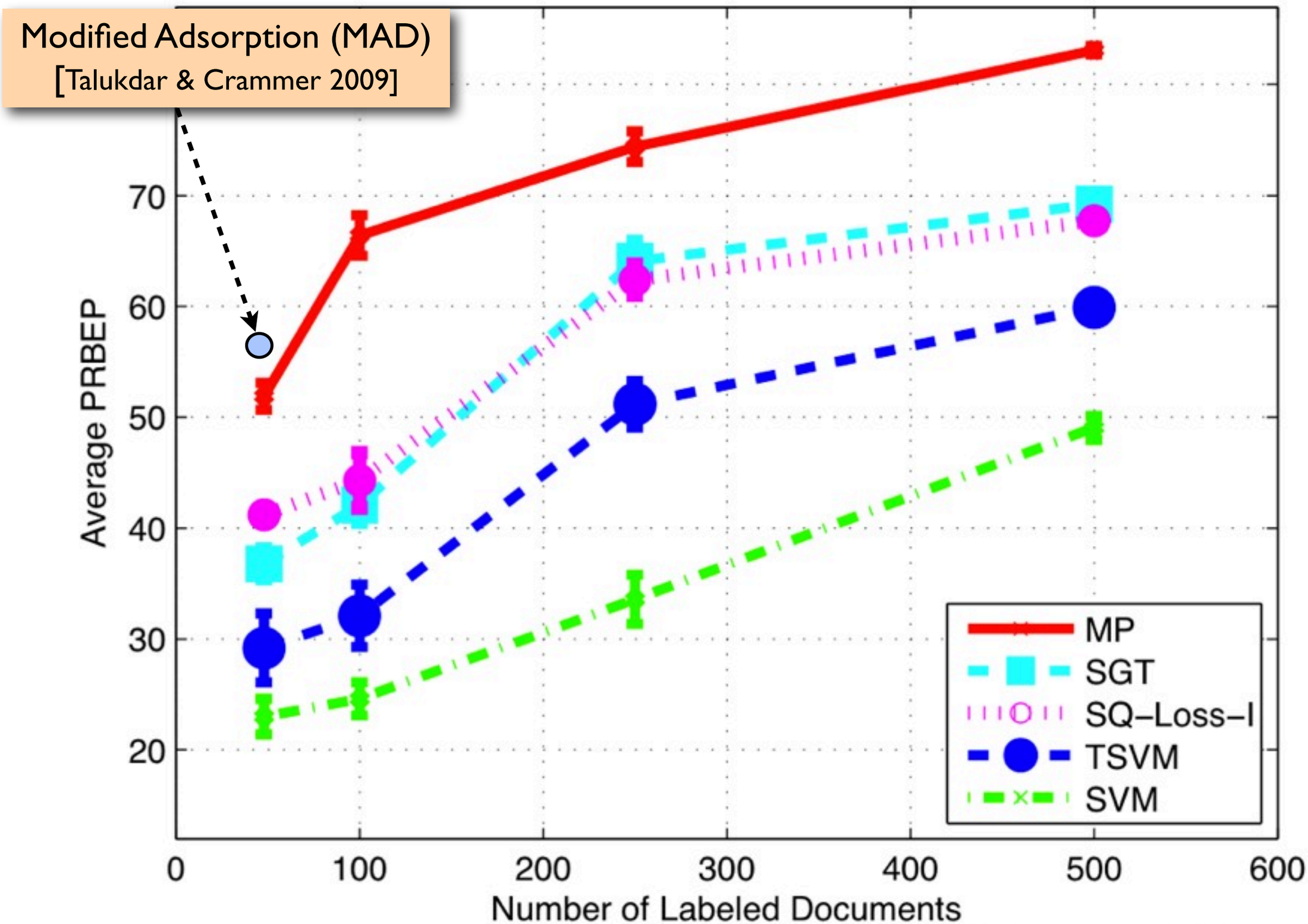

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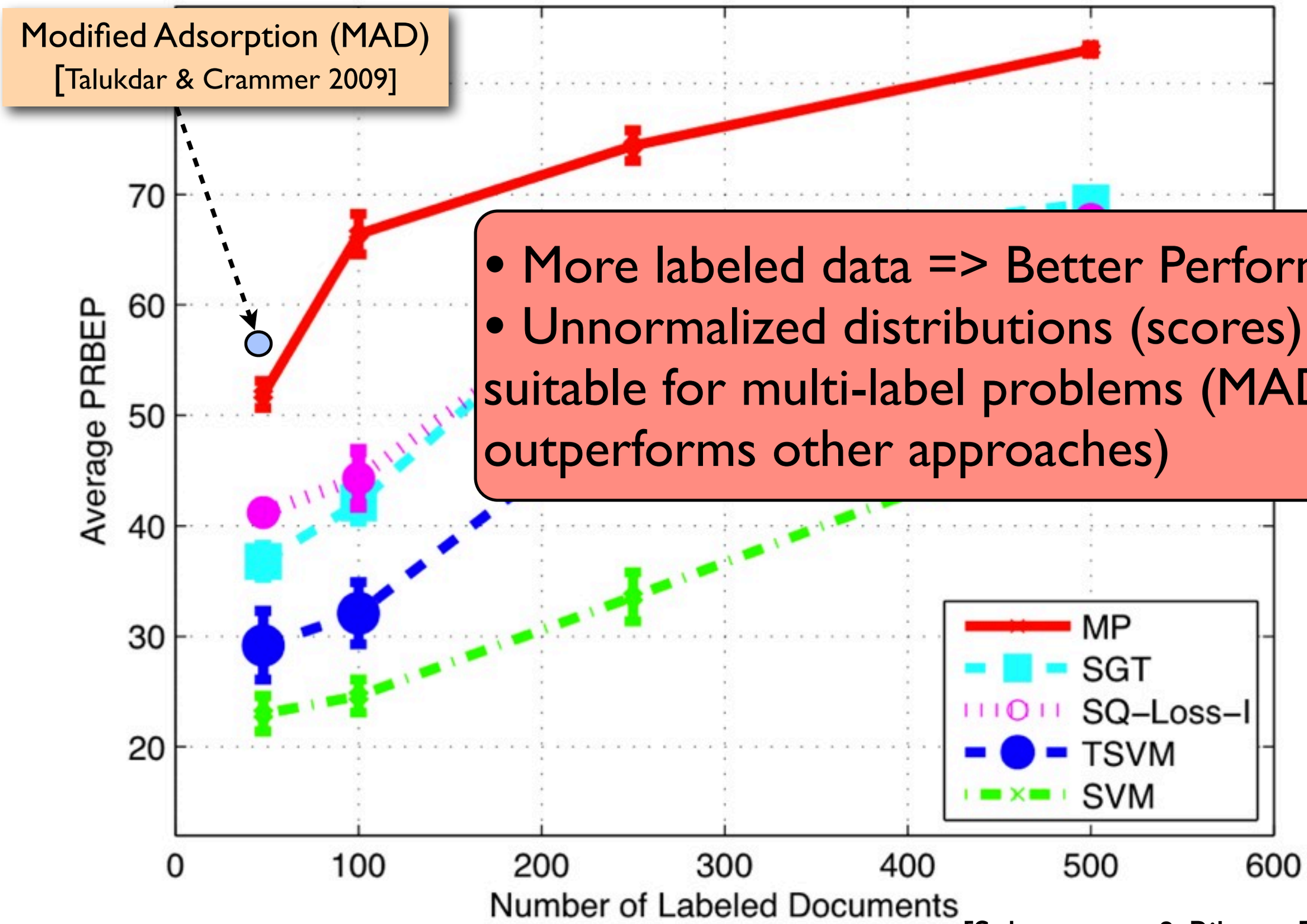

- More labeled data => Better Performance
- Unnormalized distributions (scores) more suitable for multi-label problems (MAD outperforms other approaches)

[Subramanya & Bilmes, EMNLP 2008]

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- Motivation
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Applications:
- Phone Classification
- Text Categorization
- Dialog Act Tagging [Subramanya & Bilmes, JMLR 2011]
- Statistical Machine Translation
- POS Tagging
- MultiLingual POS Tagging

Problem description

- Dialog acts (DA) reflect the function that utterances serve in discourse
- Applications in automatic speech recognition (ASR), machine translation (MT) & natural language processing (NLP)

Switchboard DA Corpus

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- Switchboard dialog act (DA) tagging project [Jurafsky, et al., 1997]

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  - Manually labeled 1155 conversations (about 200k sentences)
  - Example labels: question, answer, backchannel, agreement... (total of 42 different DAs)

Switchboard DA Corpus

- Switchboard dialog act (DA) tagging project [Jurafsky, et al., 1997]
  - Manually labeled 1155 conversations (about 200k sentences)
  - Example labels: question, answer, backchannel, agreement... (total of 42 different DAs)
  - Use top 18 DAs (about 185k sentences)

Graph Construction

Graph Construction

- Bigram & trigram TFIDF

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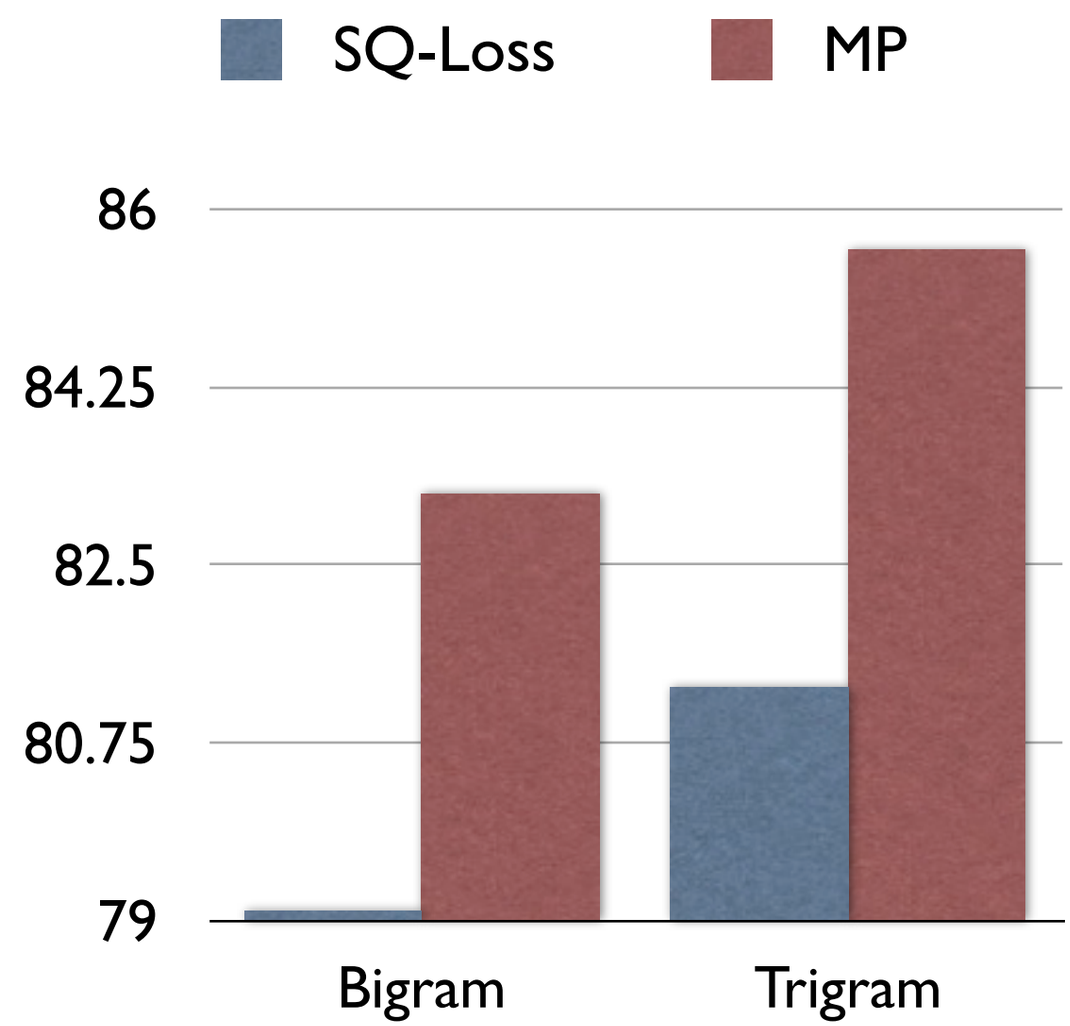
- Bigram & trigram TFIDF
- Cosine similarity

Graph Construction

- Bigram & trigram TFIDF
- Cosine similarity
- k-NN graph

SWB DA Tagging Results

- Baseline performance: 84.2% [Ji & Bilmes, 2005]



[Subramanya & Bilmes, JMLR 2011]

Dihana Corpus

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- Computer-human dialogues
  - answering telephone queries about train services in Spain
  - about 900 dialogues
  - topics include timetables, fares and services

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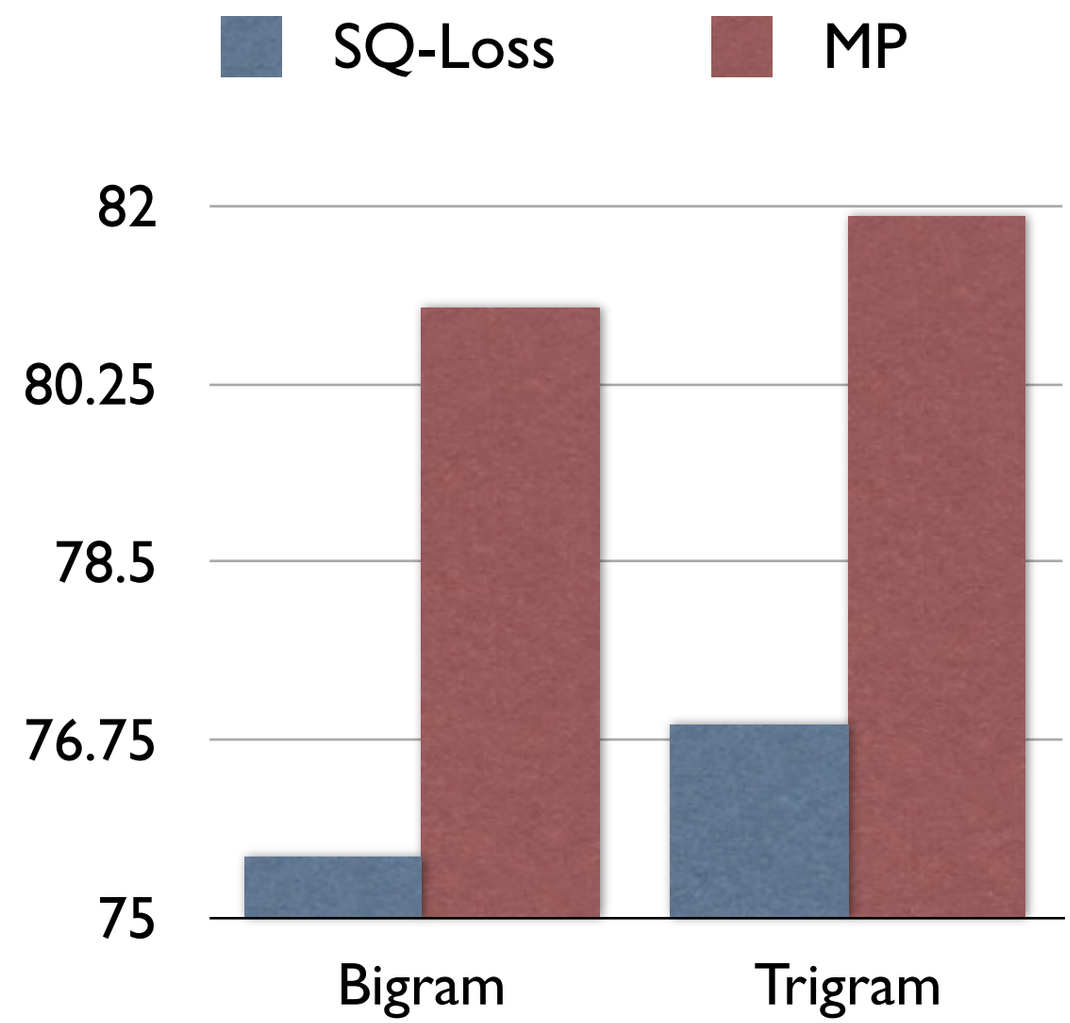
Dihana Corpus

- Computer-human dialogues
  - answering telephone queries about train services in Spain
  - about 900 dialogues
  - topics include timetables, fares and services
- 225 speakers
- Standard train/test set (16k/7.5k sentences)
- Number of labels = 72

Dihana Results

- Results for classifying user turns
- Baseline Performance: 76.4%



[Subramanya & Bilmes, JMLR 2011]

Outline

- Motivation
- Graph Construction
- Inference Methods
- Scalability
- Applications
  - Phone Classification
  - Text Categorization
  - Dialog Act Tagging
  - Statistical Machine Translation [Alexandrescu & Kirchoff, NAACL 2009]
  - POS Tagging
  - MultiLingual POS Tagging
- Conclusion & Future Work

Problem Description

- Phrase-based statistical machine translation (SMT)
- Sentences are translated in isolation

Motivation

Motivation

Source Asf lA ymknk *lk hnAk .....

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Source	Asf lA ymknk *lk hnAk .....
1-best	i'm sorry you can't there is a cost

Motivation

Source	Asf lA ymknk *lk hnAk .....
1-best	i'm sorry you can't there is a cost

Source	E*rA lA ymknk t$gyl .....
1-best	excuse me i you turn tv ....

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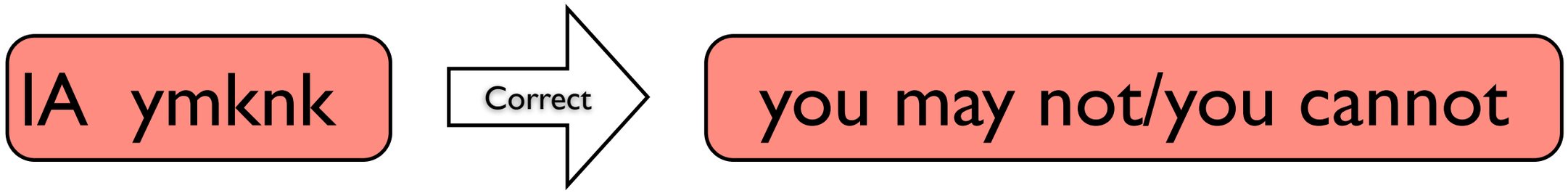

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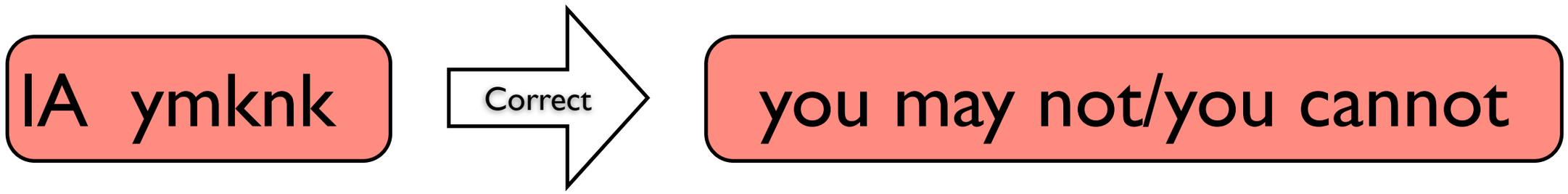

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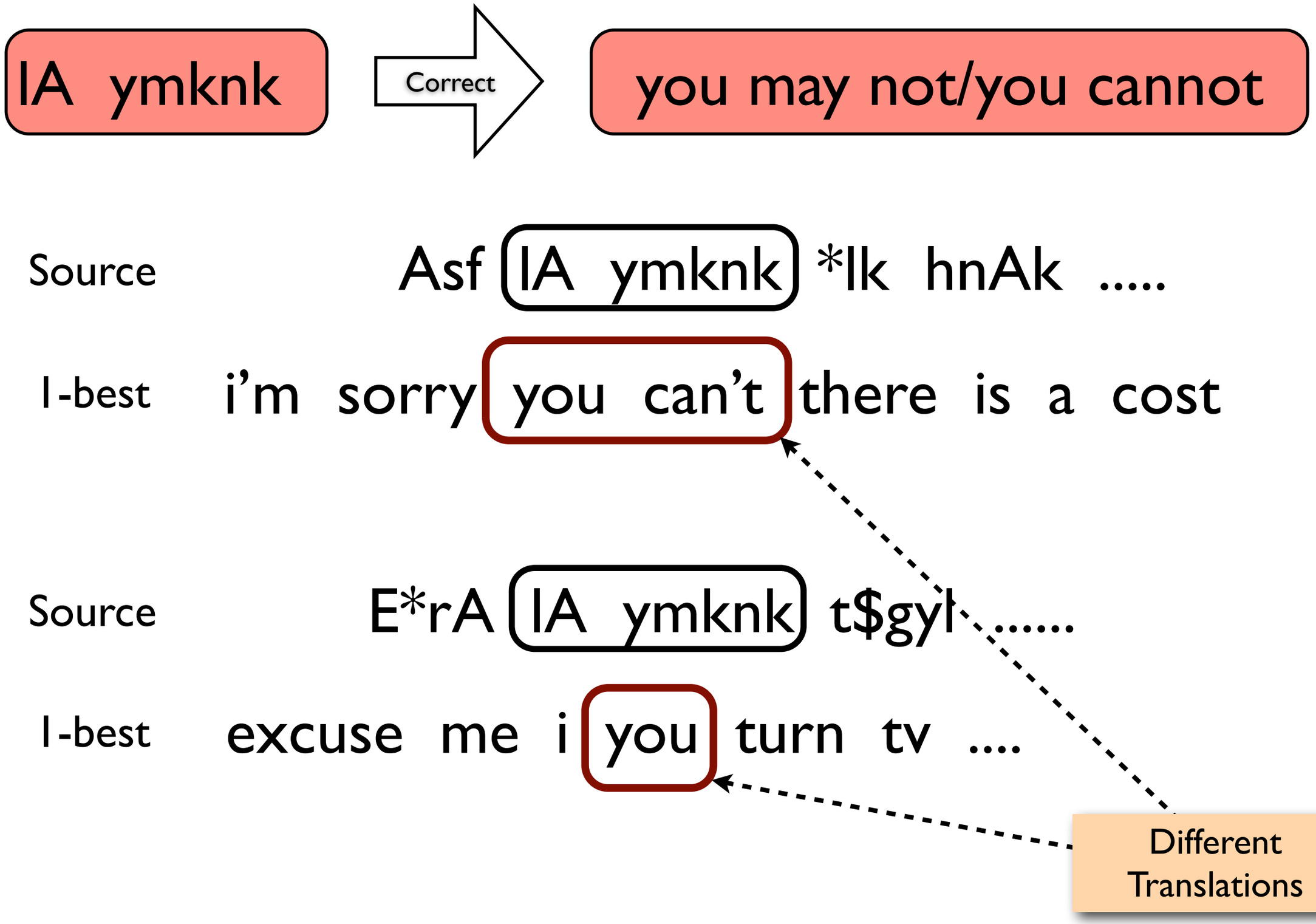
lA ymknk
Correct
you may not/you cannot
Source
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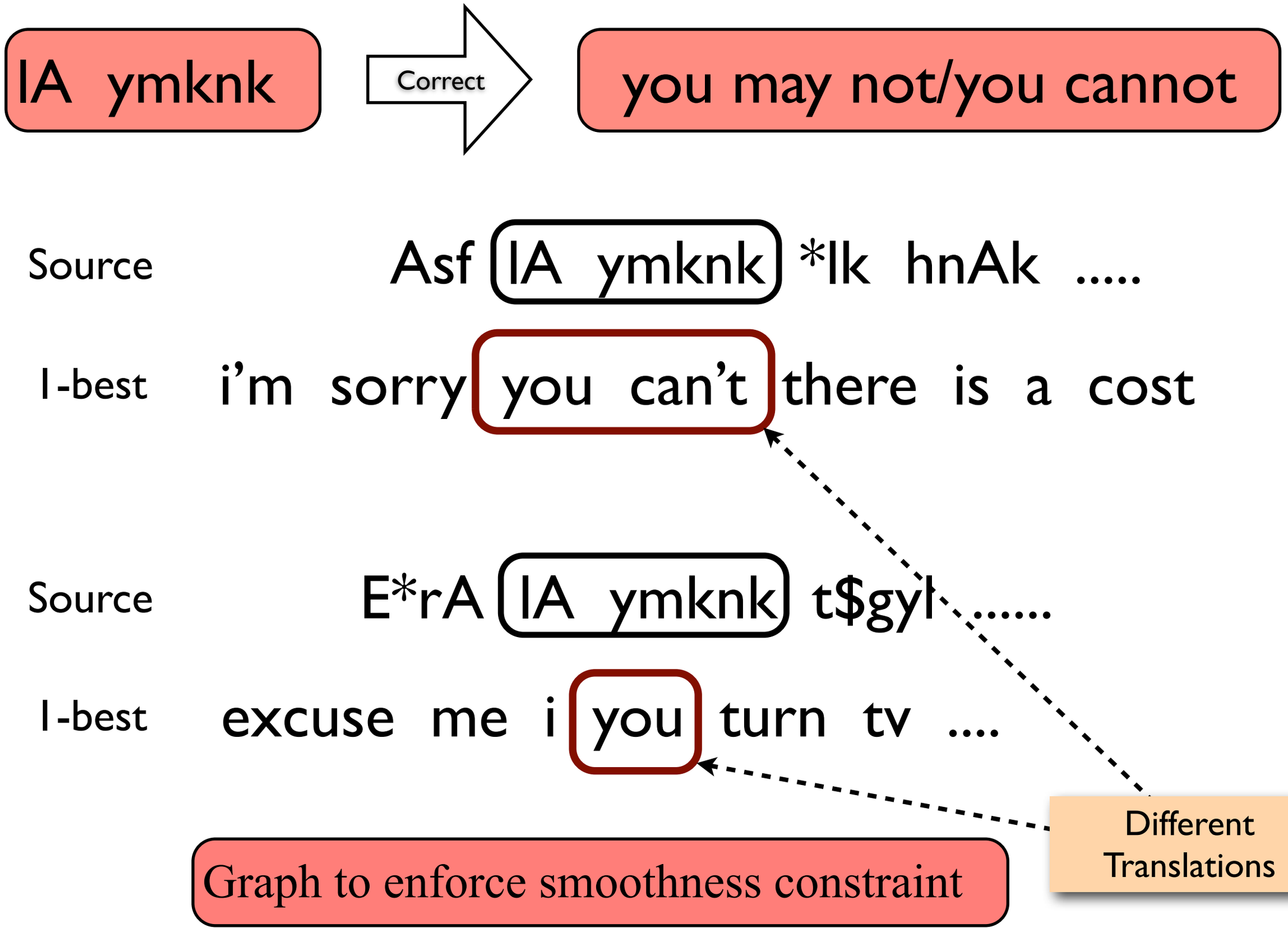
Different Translations

Motivation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i'm sorry you can't there is a cost
Source
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1-best
excuse me i you turn tv ....
Different Translations
Graph to enforce smoothness constraint

Issues

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  - exploit similarity between sentences

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- Input consists of variable-length word strings

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  - exploit similarity between sentences
- Input consists of variable-length word strings
- Output space is structured (number of possible "labels" is very large)

Graph Construction (I)

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Labeled data: $\{(s_1, t_1), \ldots, (s_l, t_l)\}$

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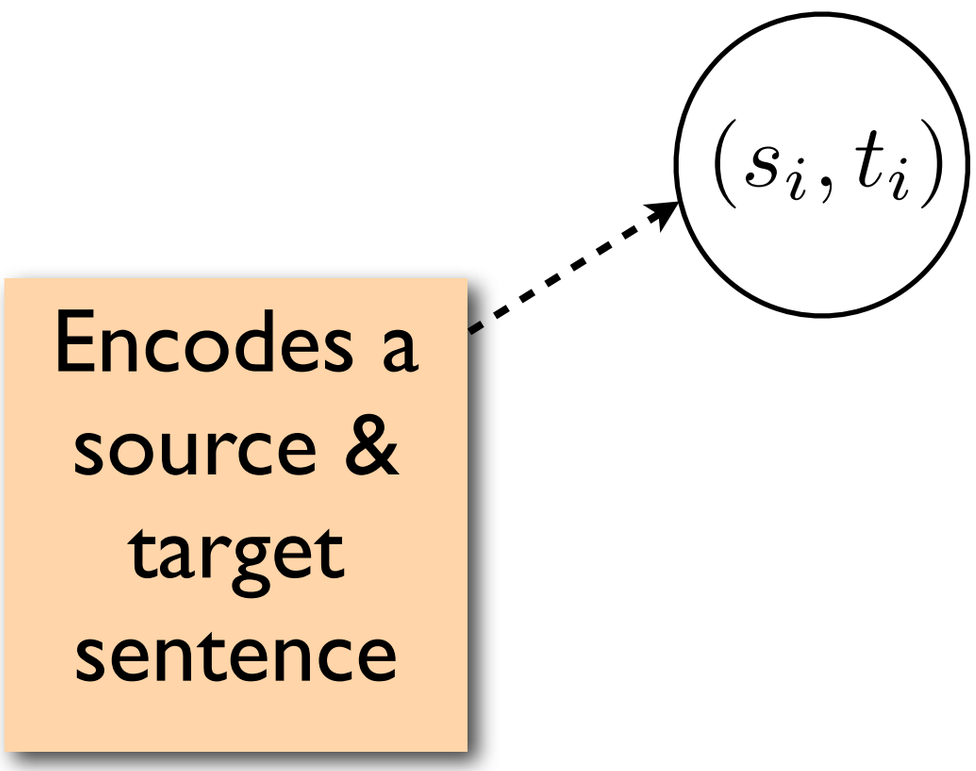

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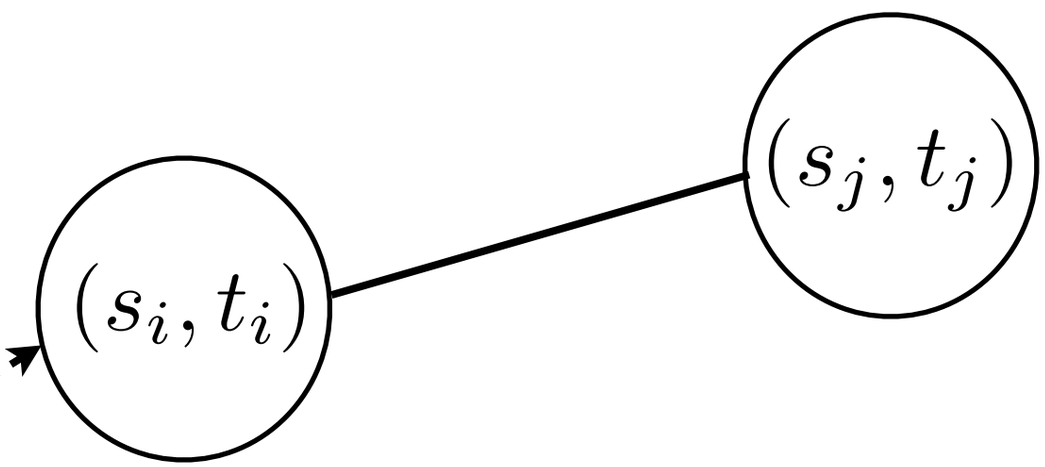

Encodes a source & target sentence

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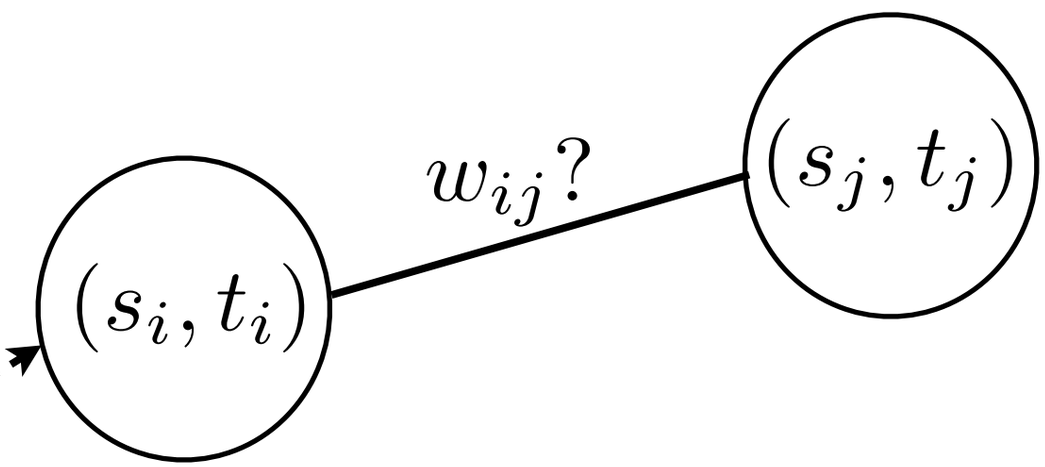

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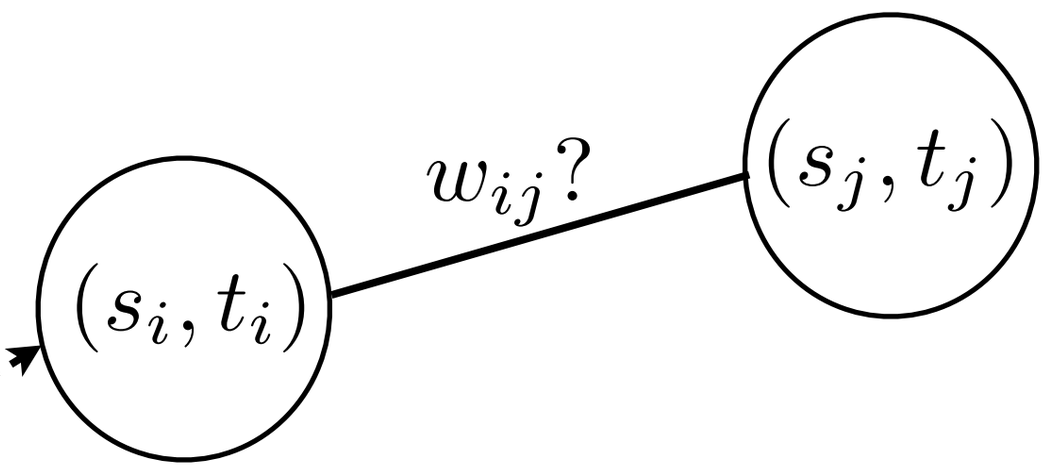

Encodes a source & target sentence

- How do we compute similarity
- What about the test set?

Graph Construction (II)

Labeled data: $\{(s_1, t_1), \ldots, (s_l, t_l)\}$

Unlabeled data (test set): $\{s_{l+1}, \ldots, s_n\}$

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Labeled data: $\{(s_1, t_1), \ldots, (s_l, t_l)\}$

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Unlabeled sentence

s_i

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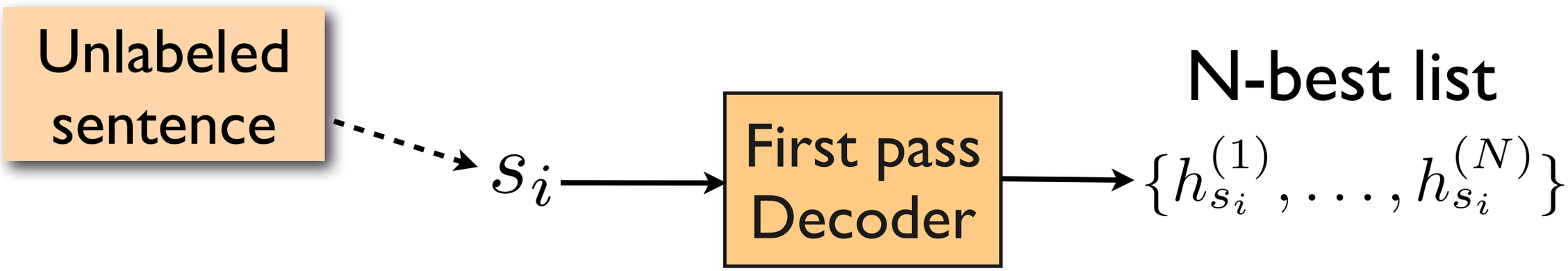

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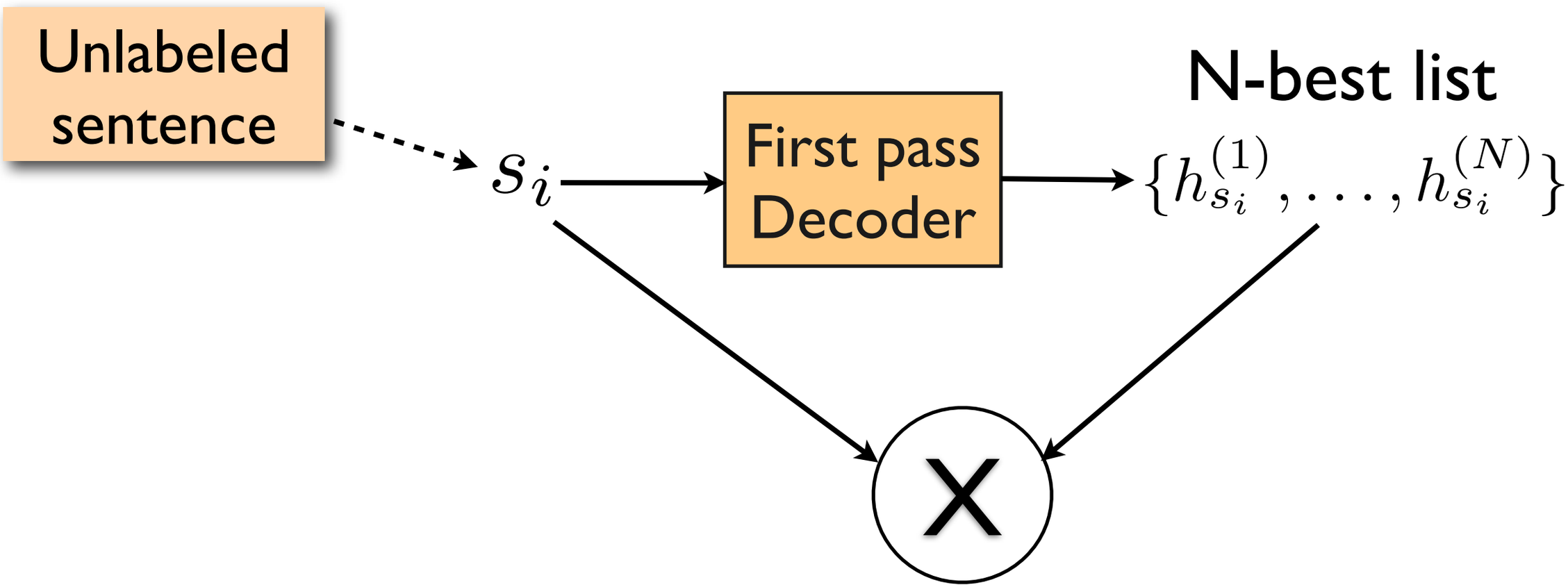

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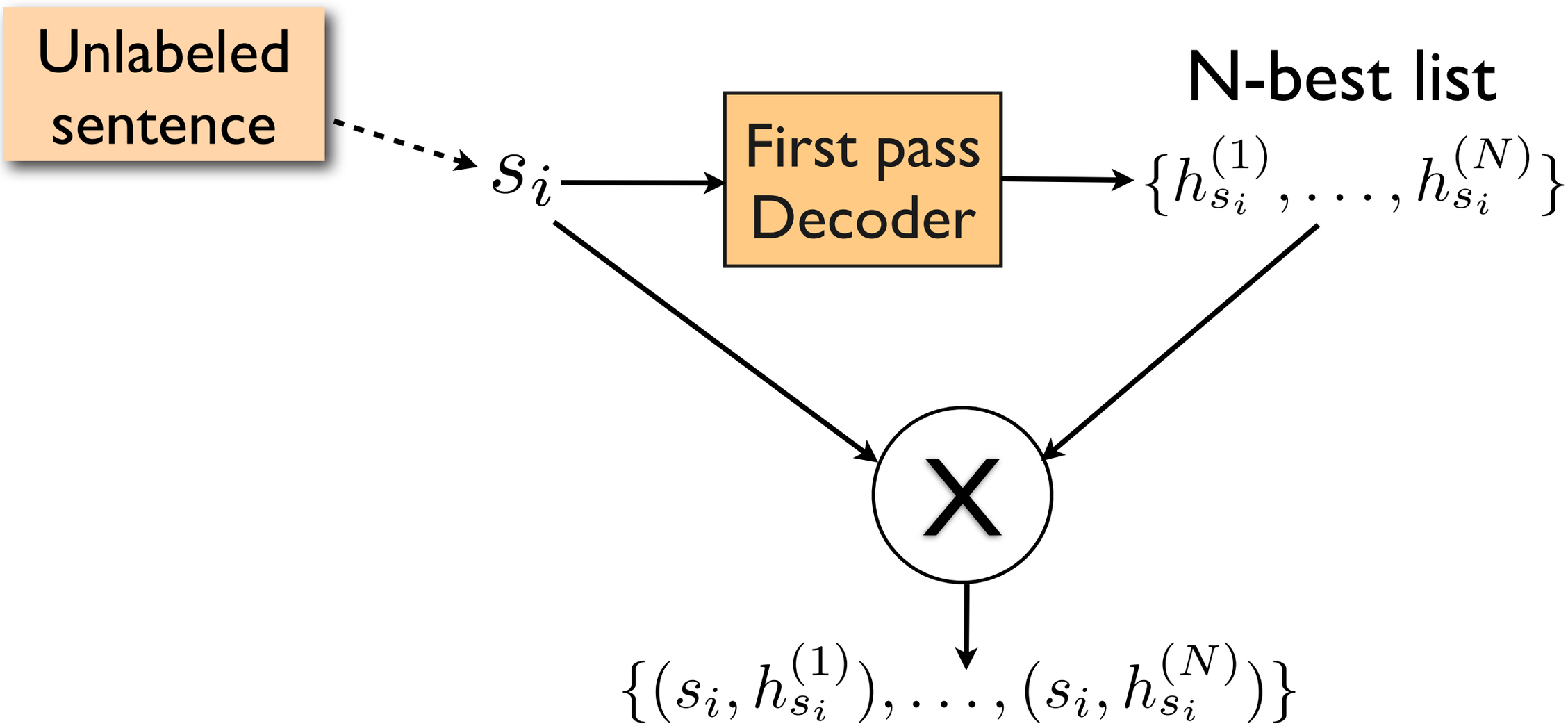
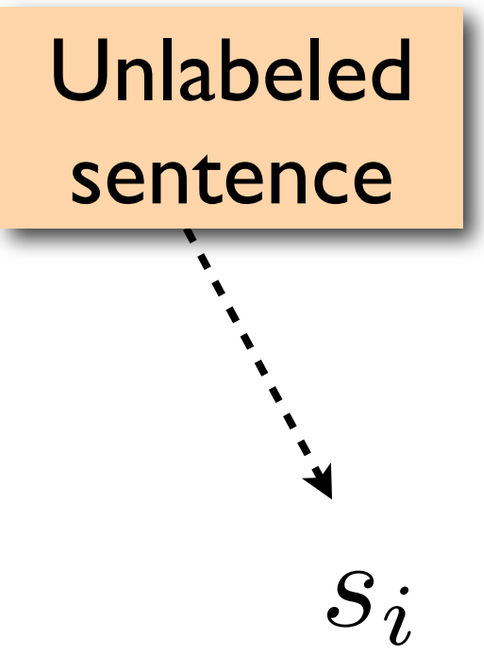


Graph Construction (III)



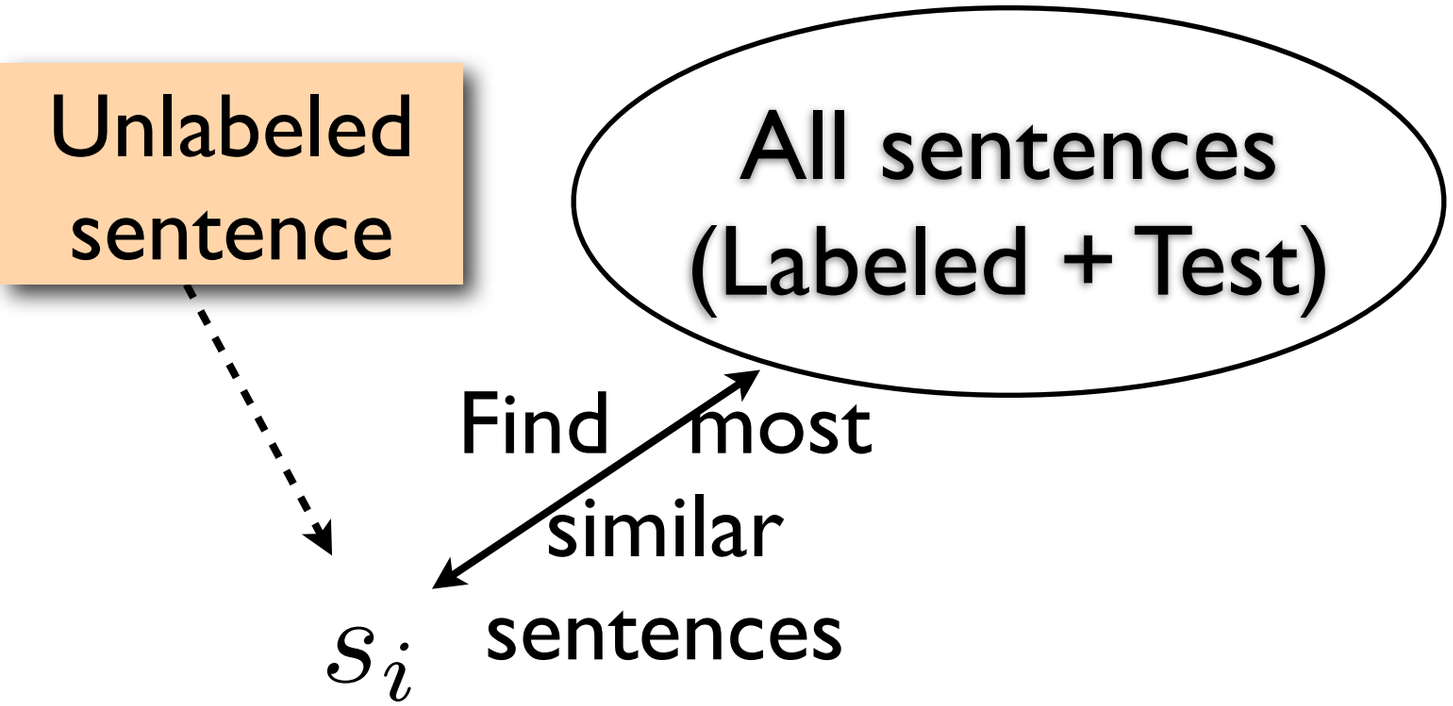
Graph Construction (III)

Unlabeled sentence

All sentences (Labeled + Test)

s_i

Graph Construction (III)



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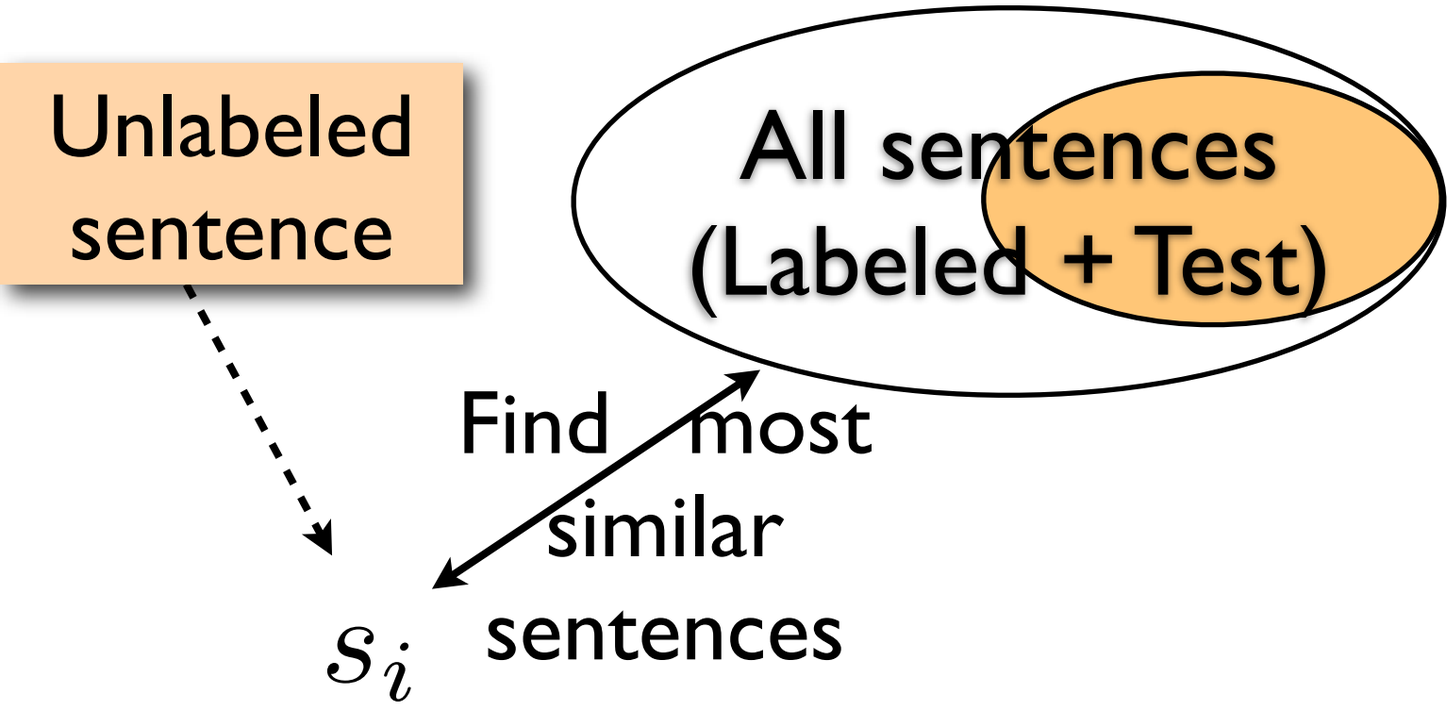

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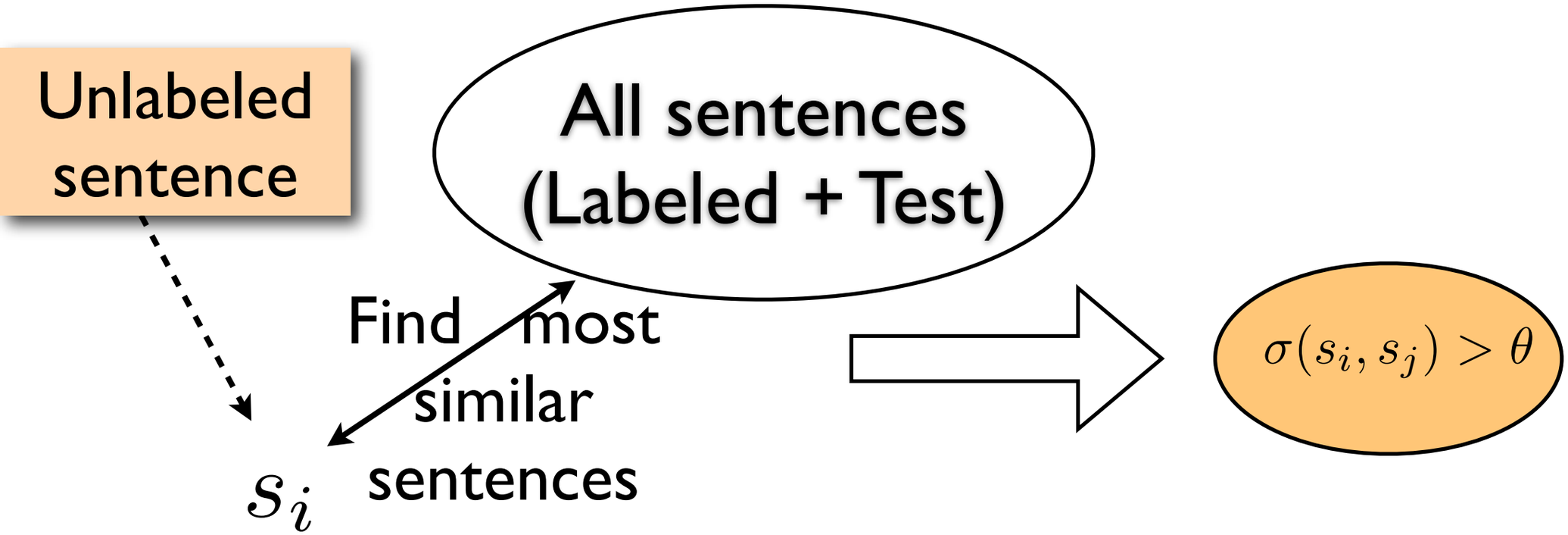

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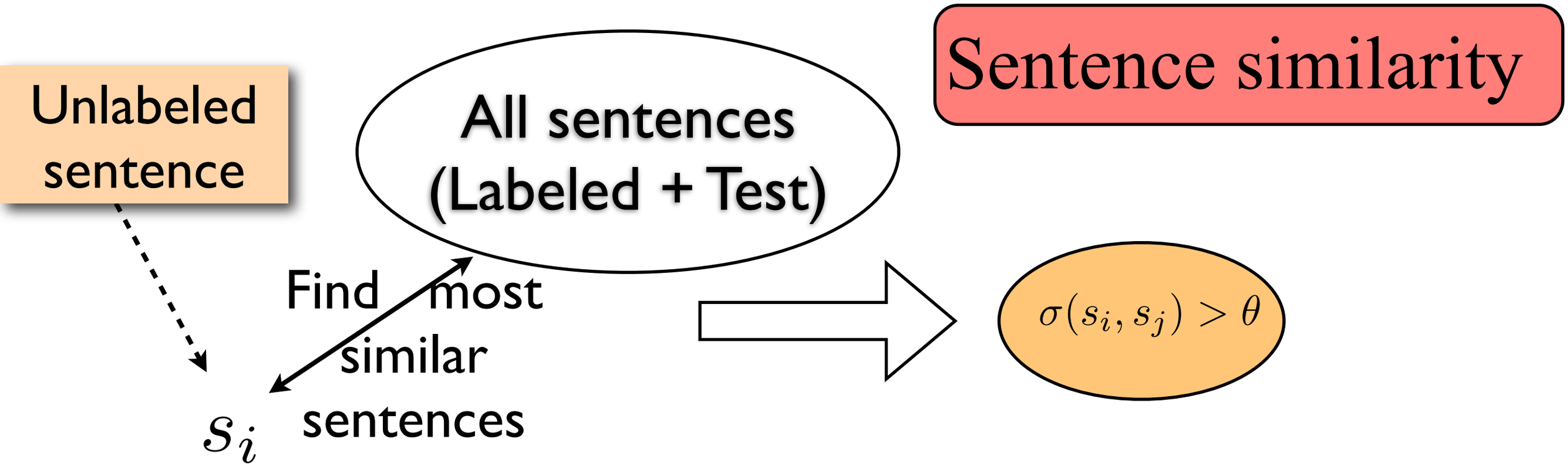

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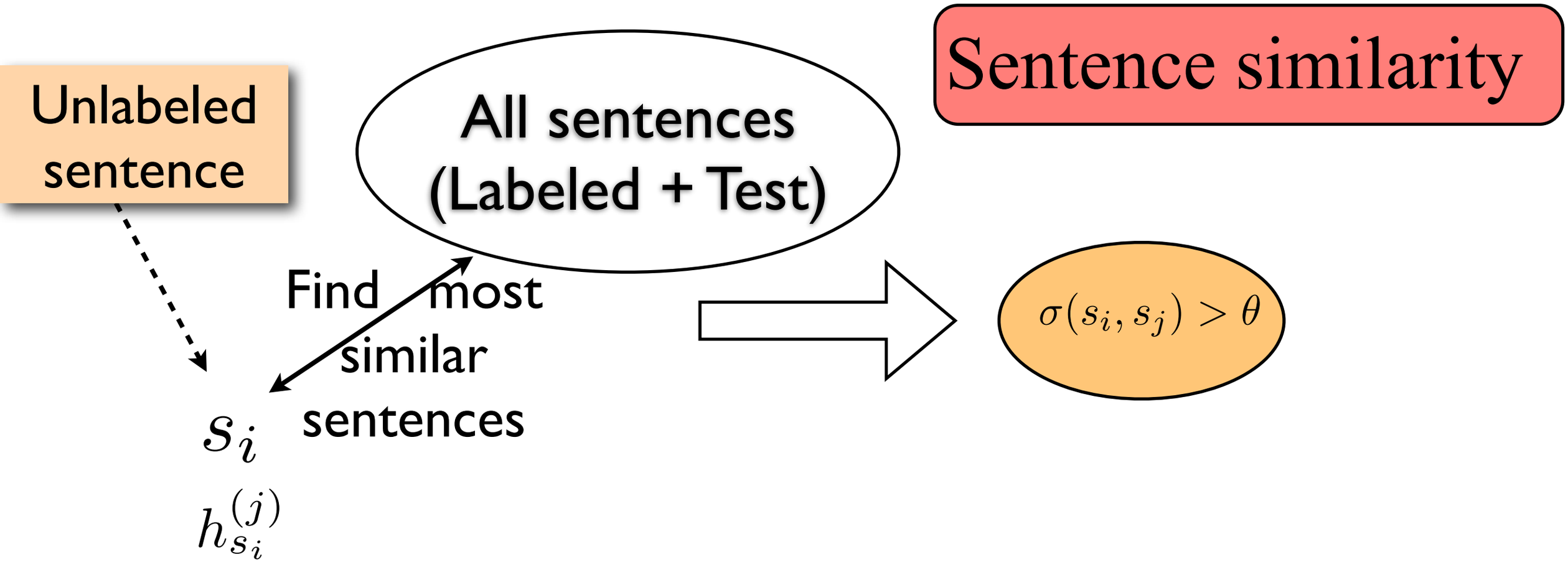

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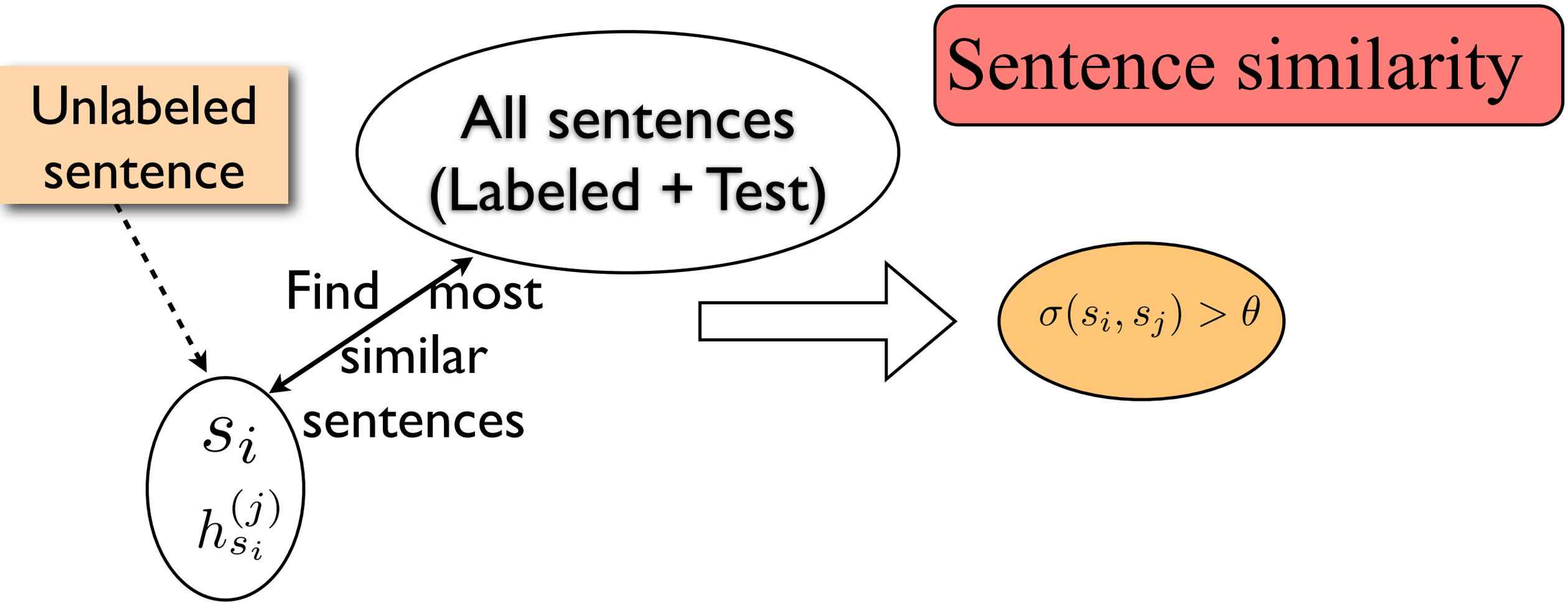

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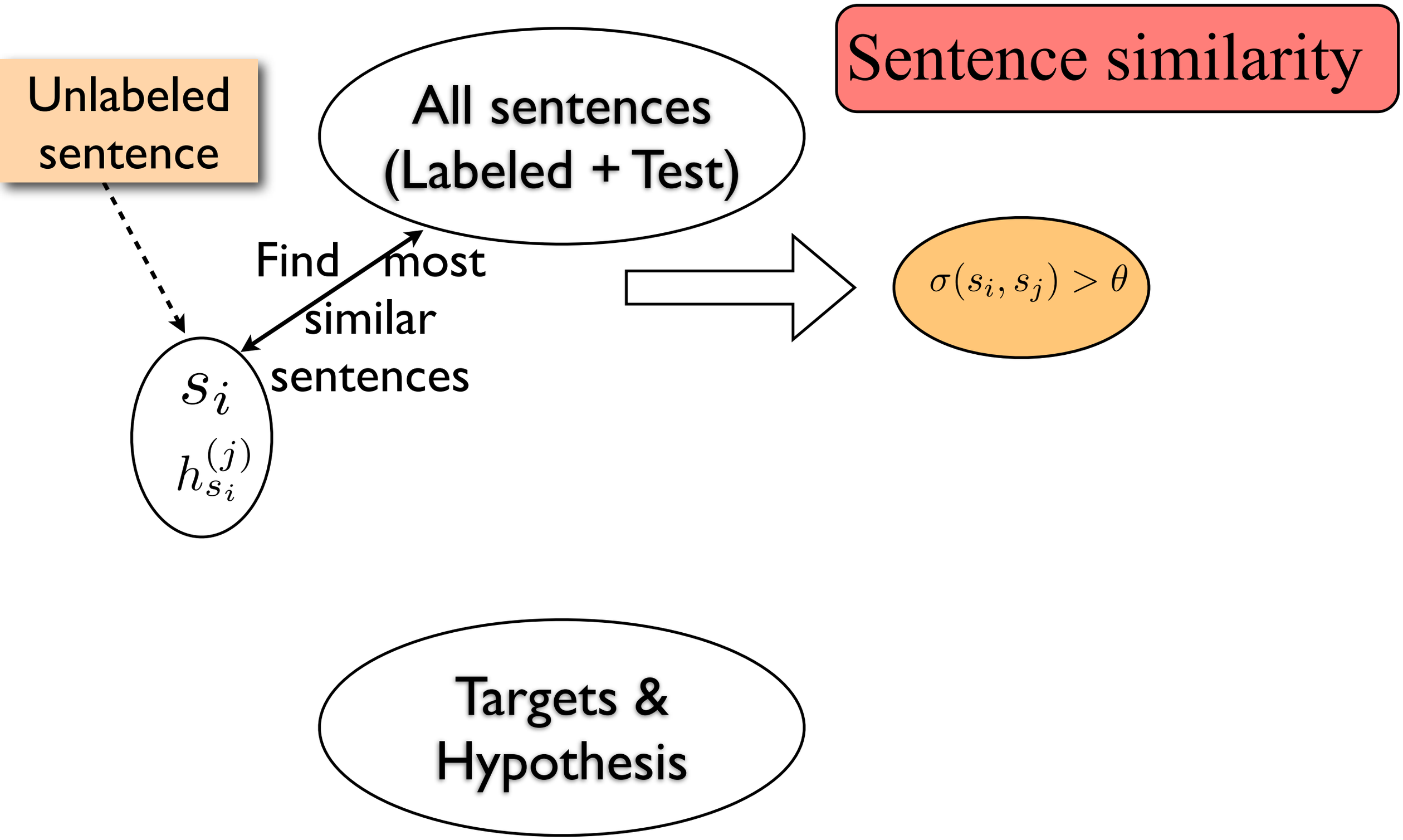

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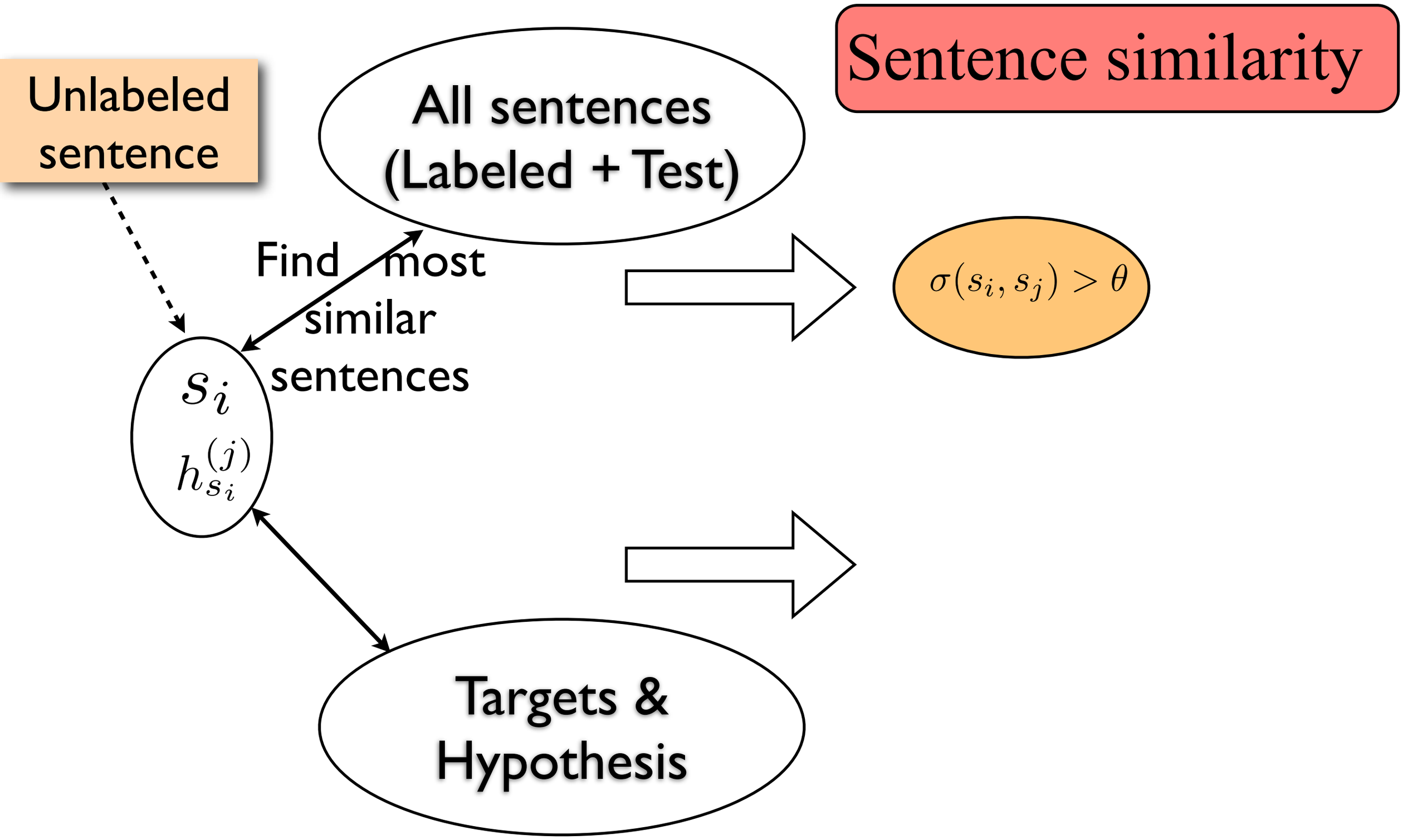

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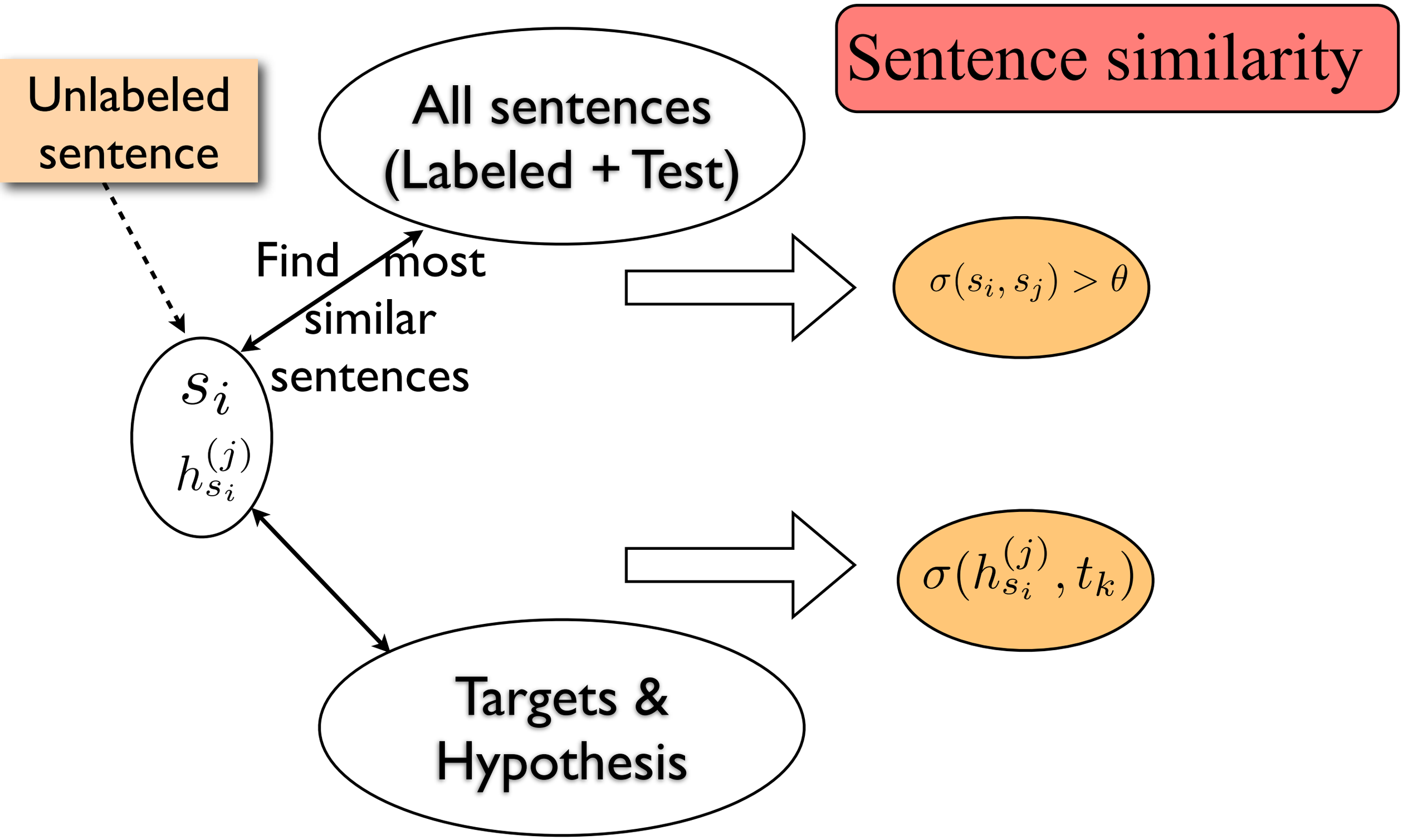

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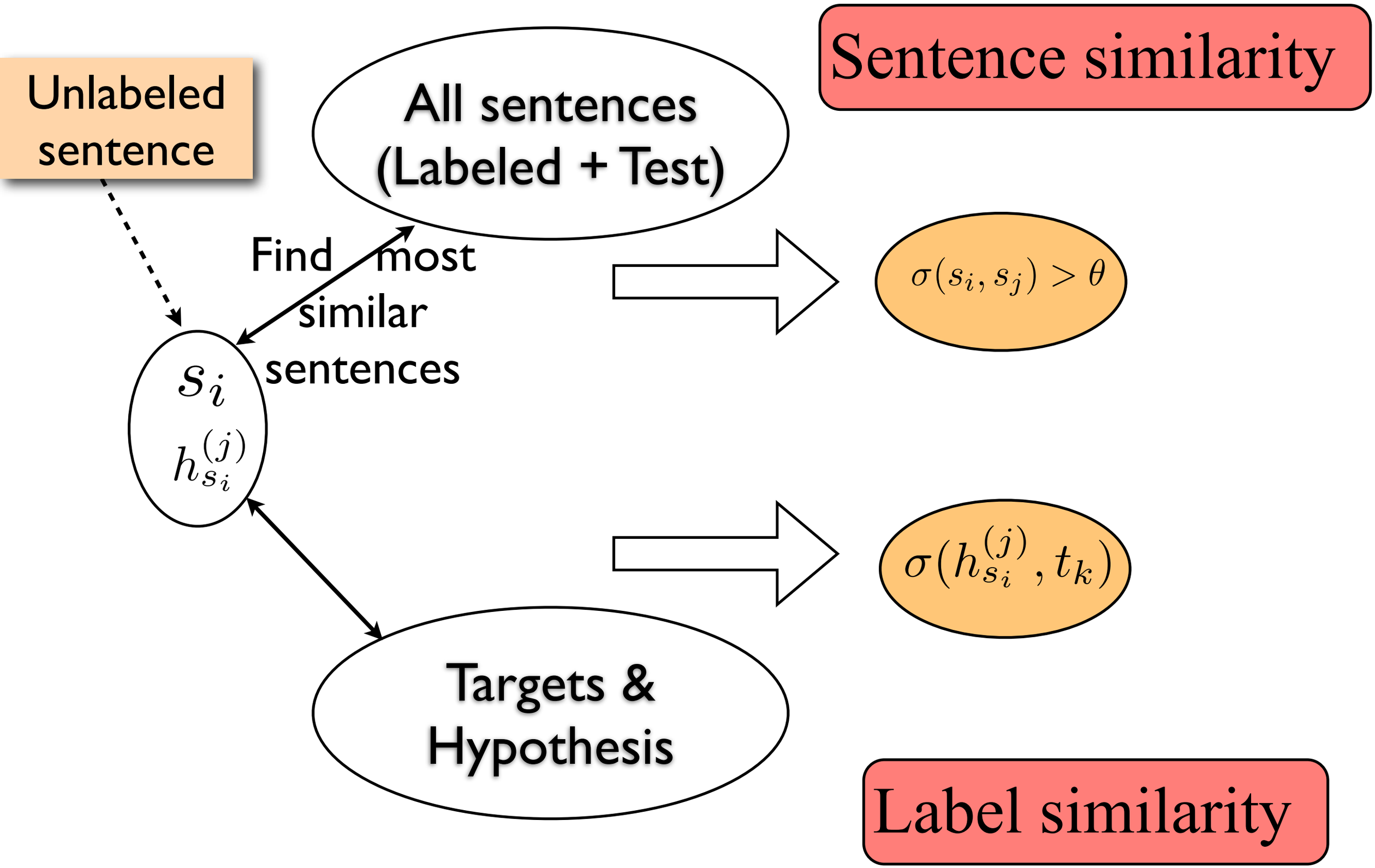

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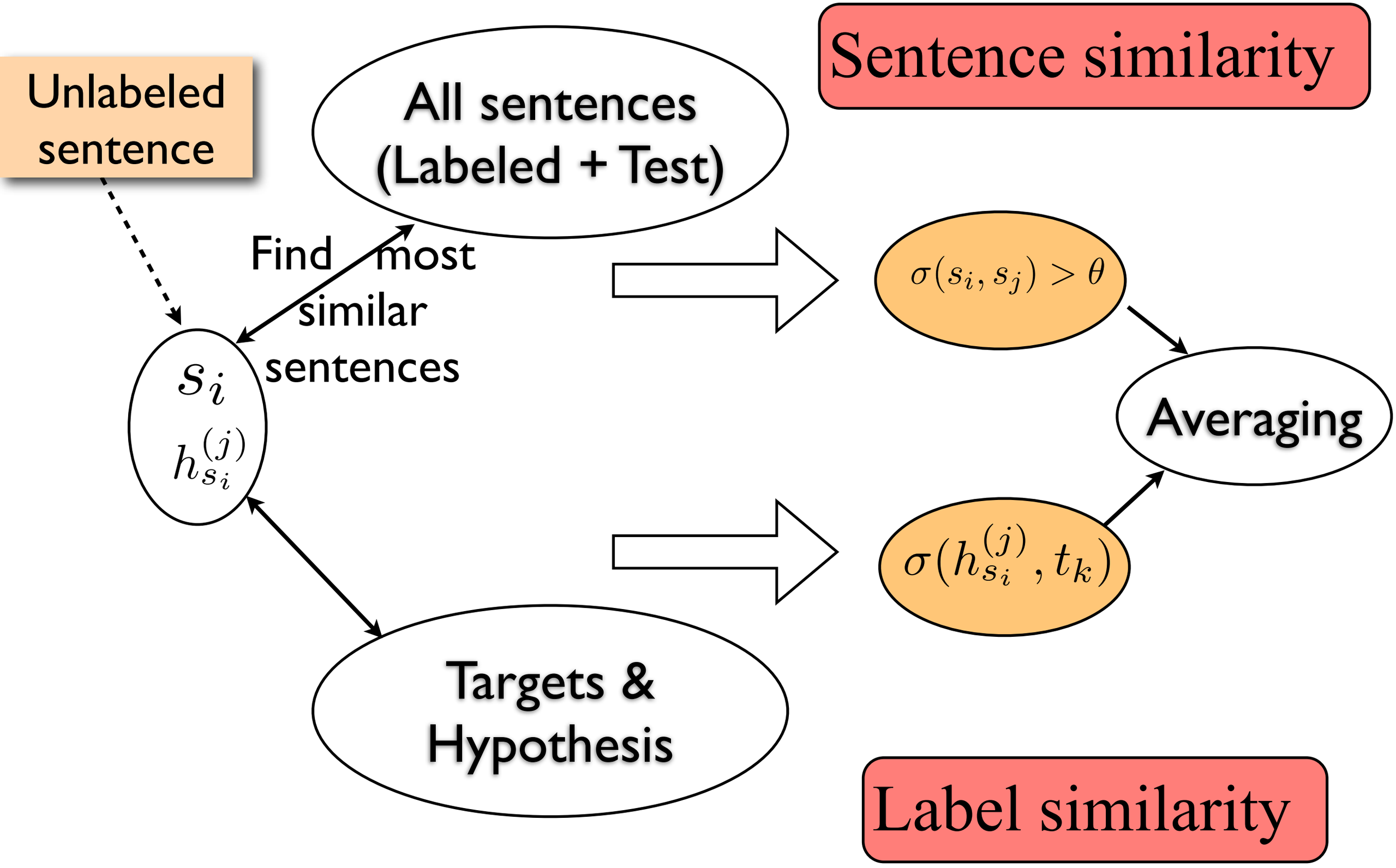
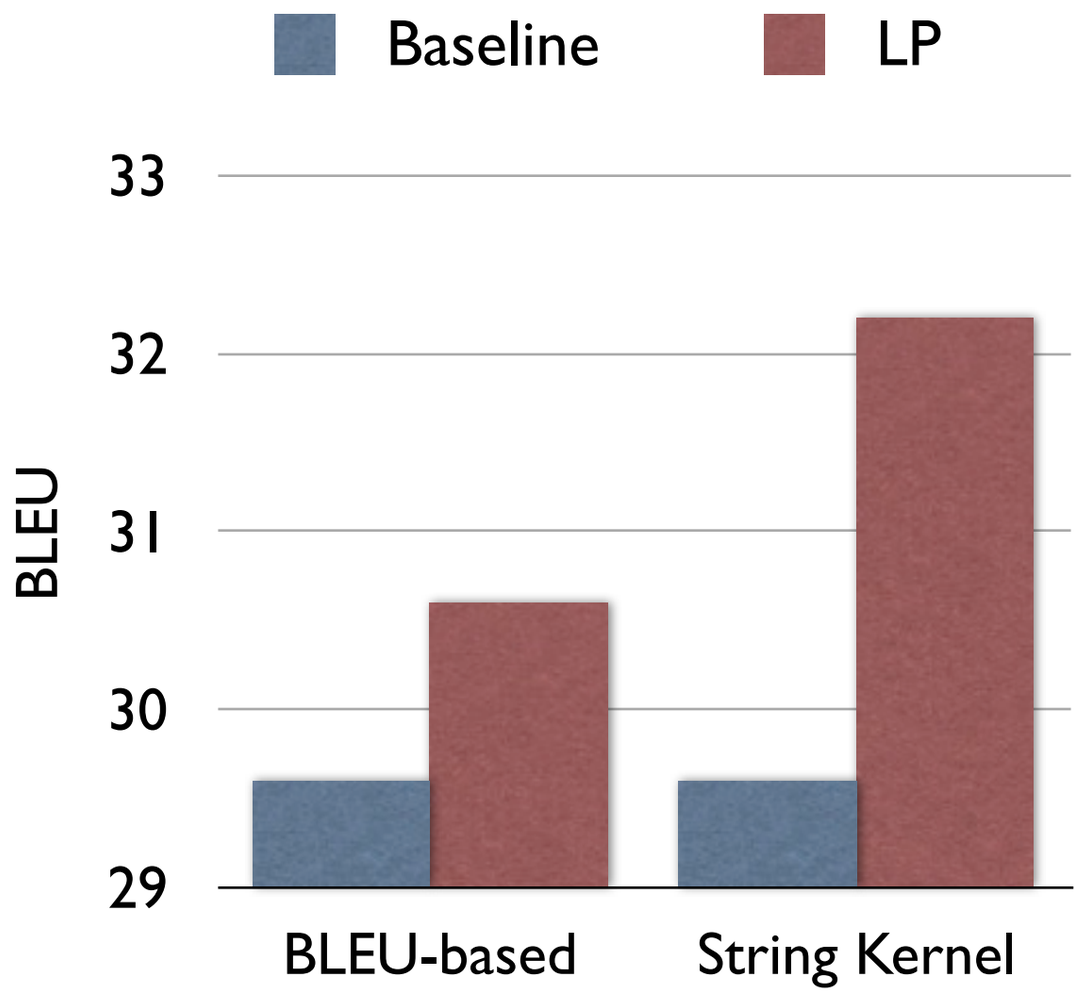


Corpora

- IWSLT 2007 task
  - Italian-to-English (IE) & Arabic-to-English (AE) travel tasks
- Each task has train/dev/eval sets
- Baseline: Standard phrase-based SMT based on a log-linear model. Yields state-of-the-art performance.
- Results are measured using BLEU score and Phrase error rate (PER) [Papineni et al., ACL 2002]

Results (I)



IE Results on eval set

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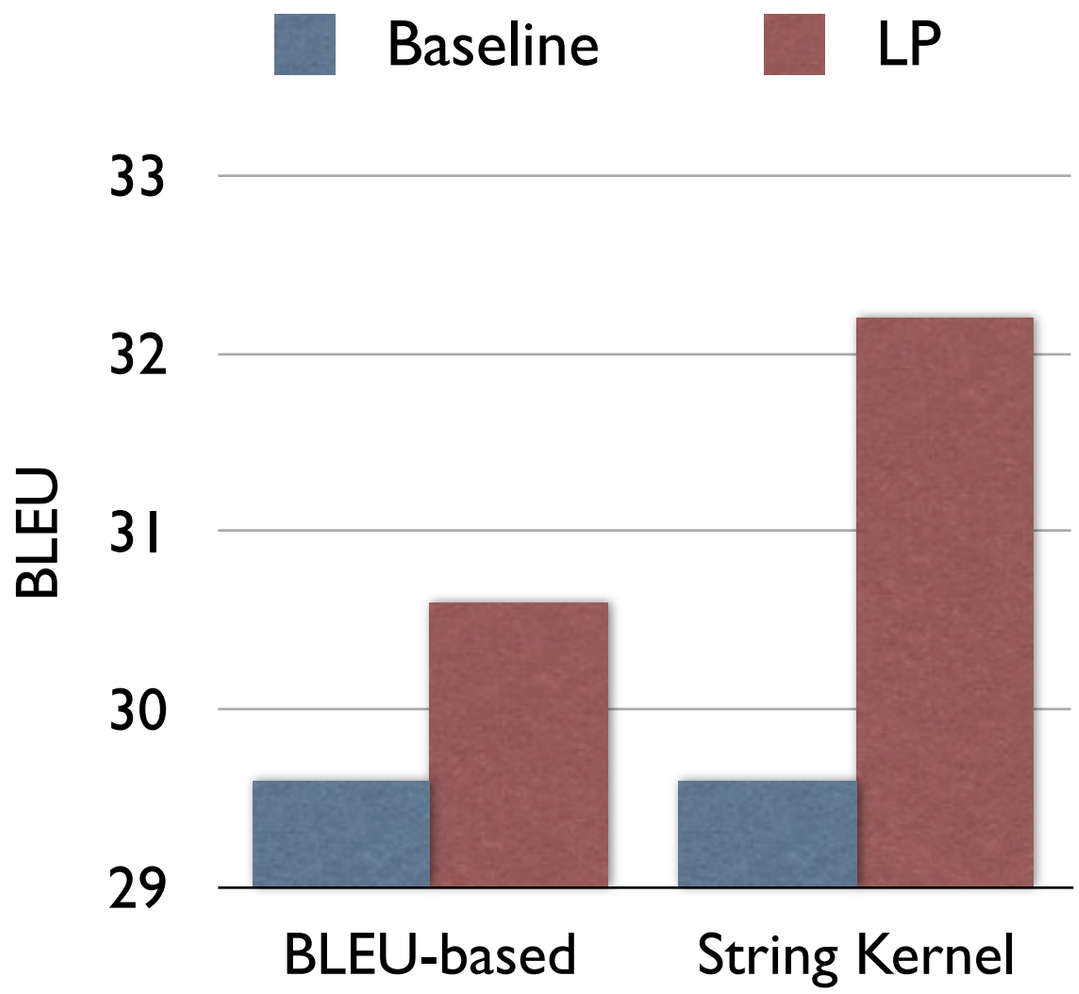
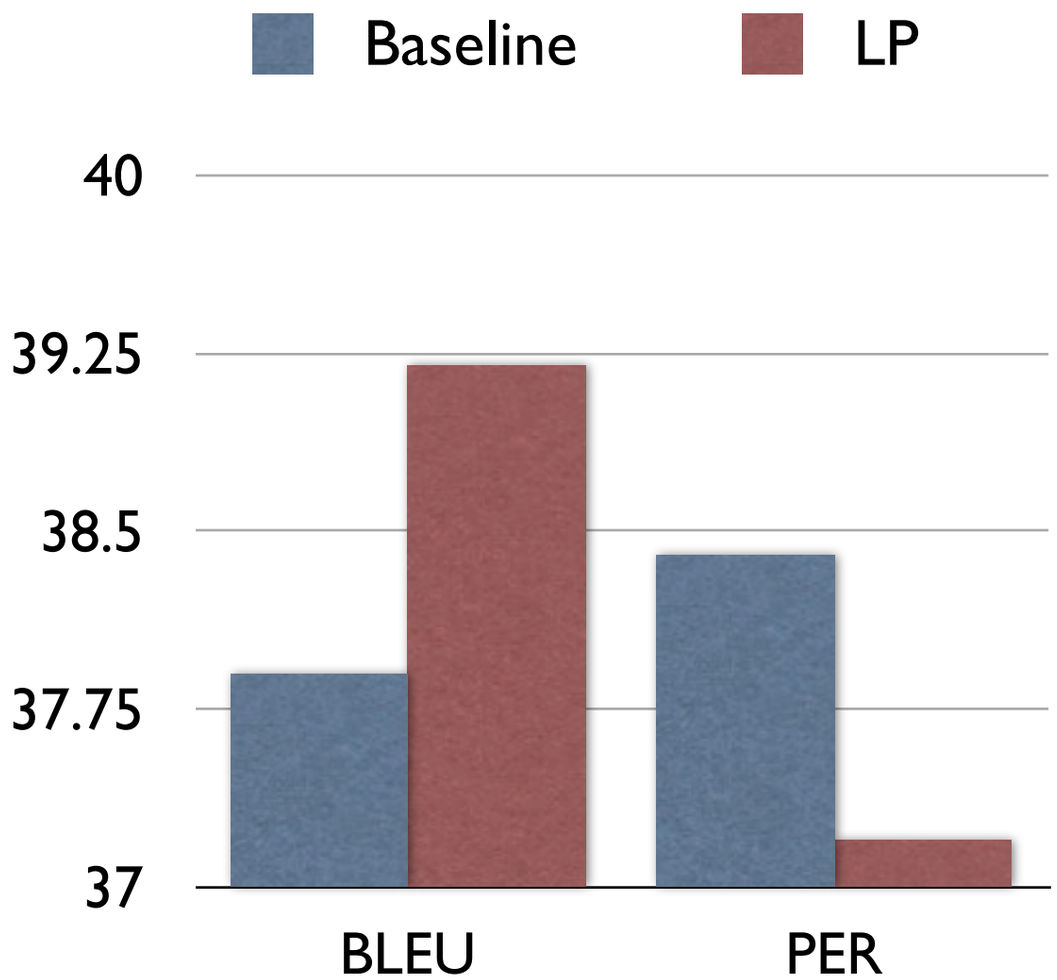


IE Results on eval set

Geometric mean based averaging worked the best

Results (II)



IE Results on eval set

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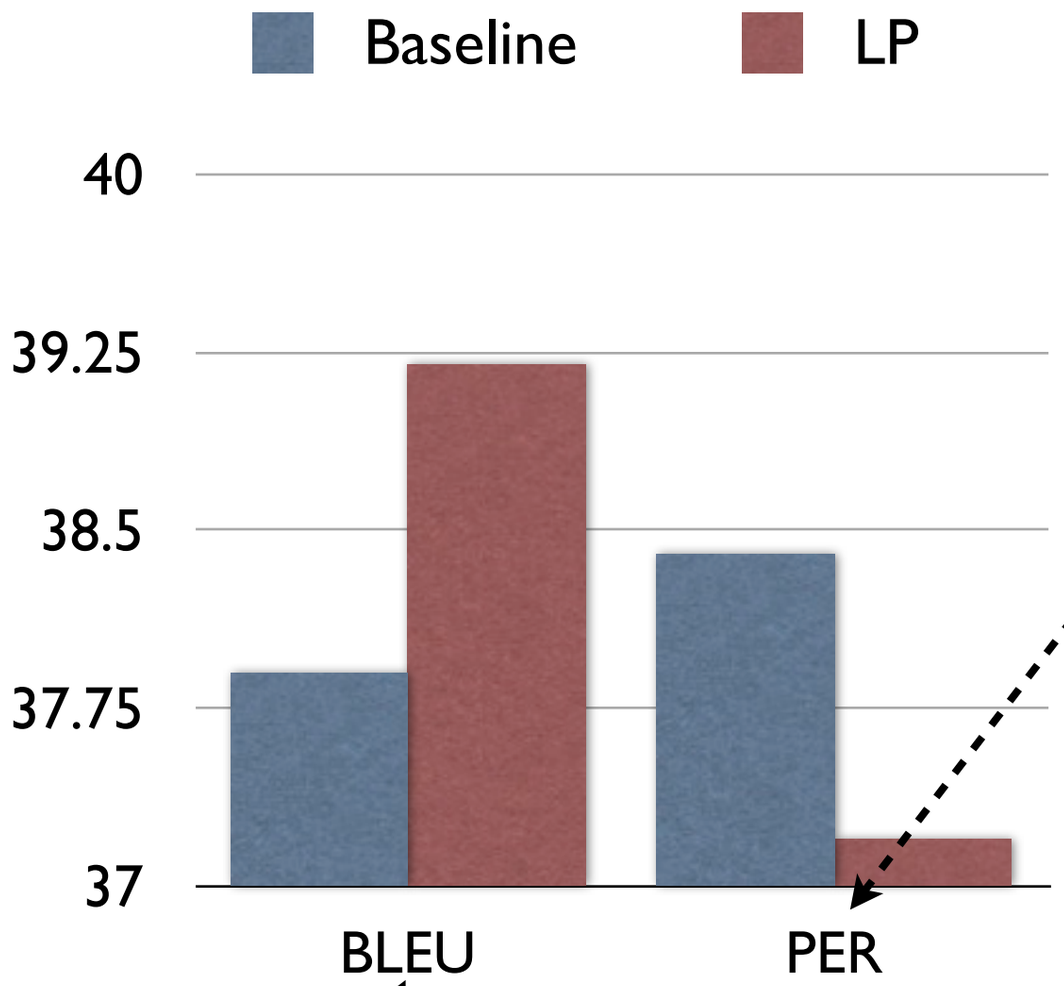

Lower is better

IE Results on eval set

Higher is better

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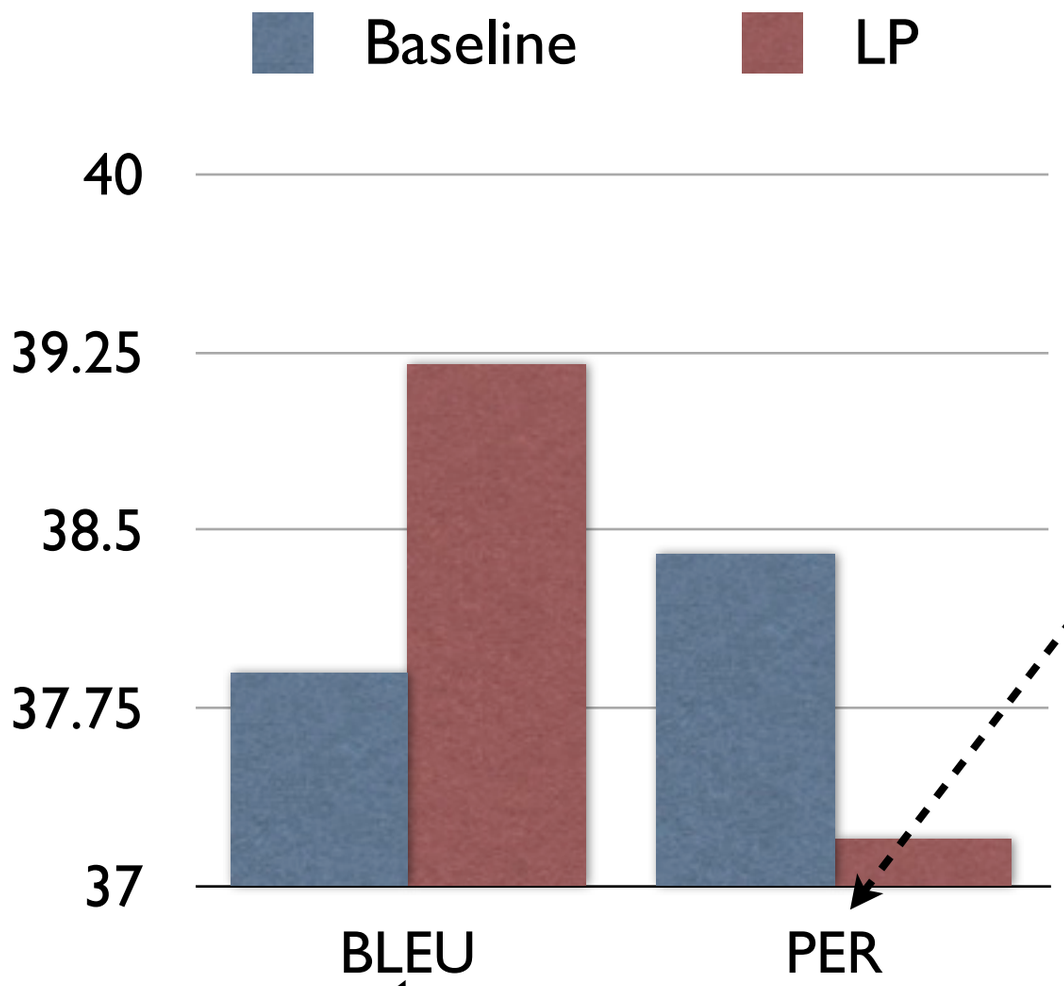

IE Results on eval set

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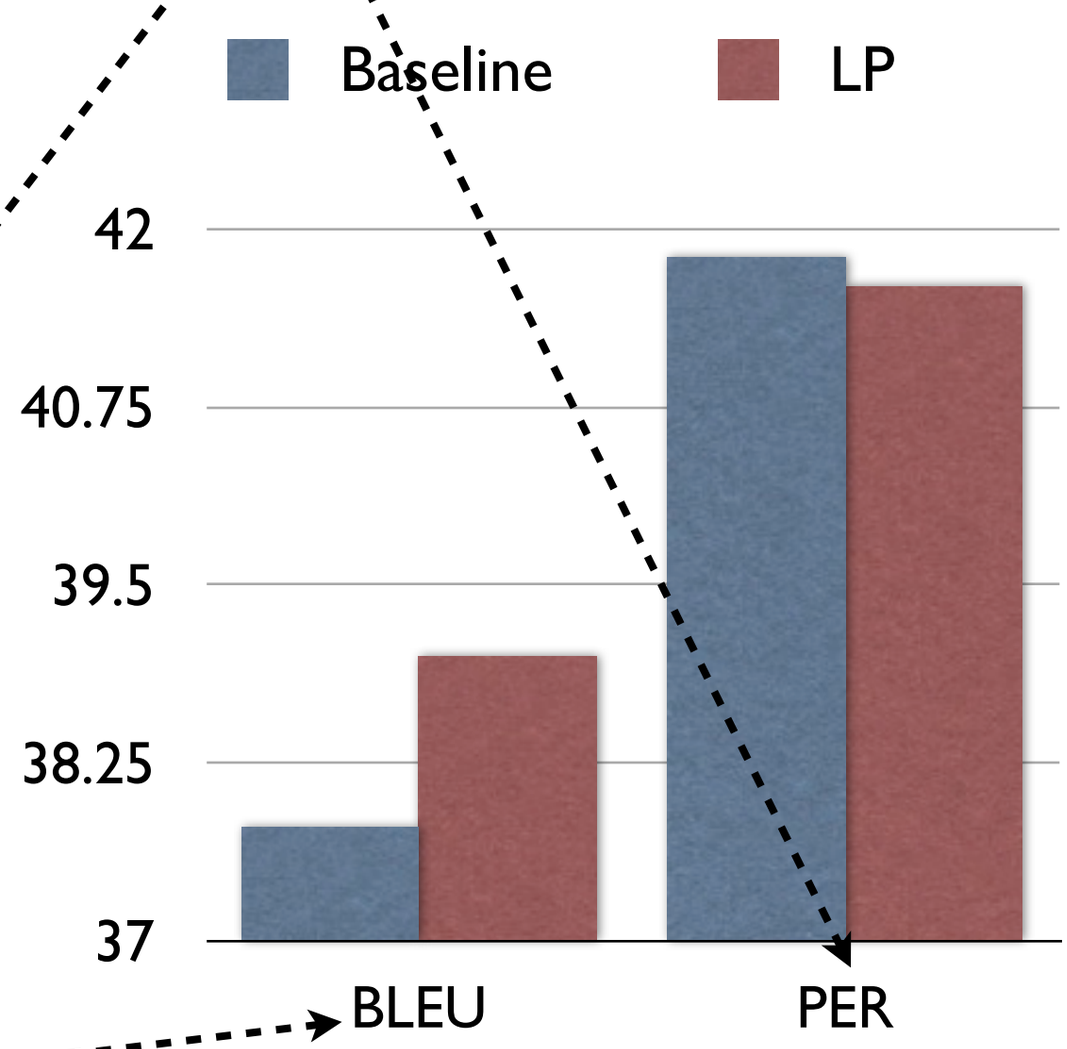

AE Results on eval set

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  - Phone Classification
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  - MultiLingual POS Tagging
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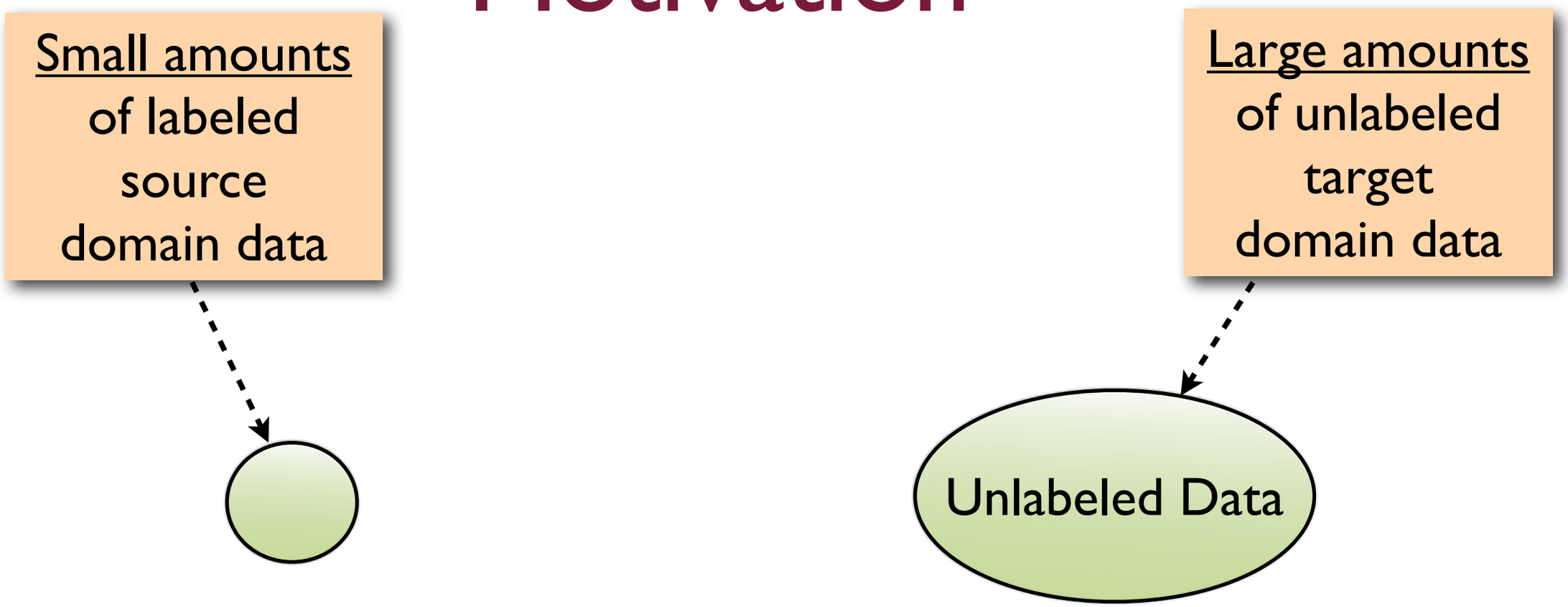
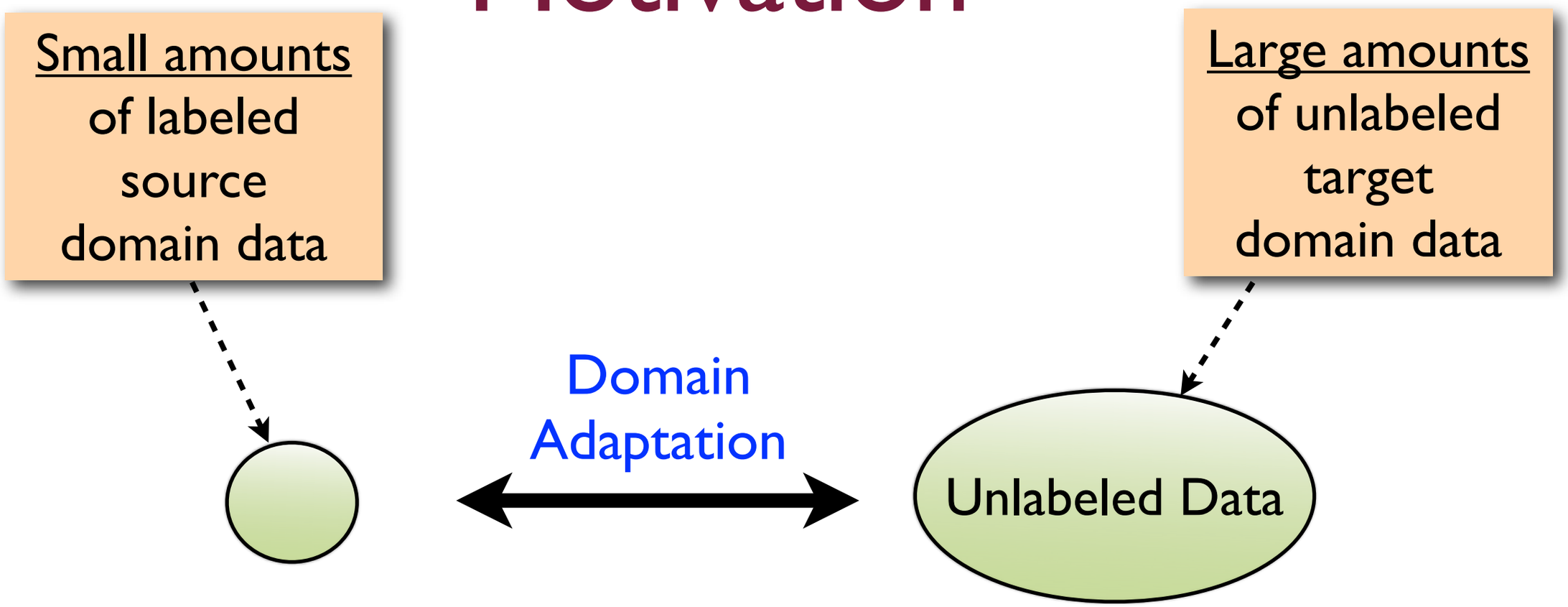
Motivation


Small amounts of labeled source domain data
Large amounts of unlabeled target domain data
Unlabeled Data

Motivation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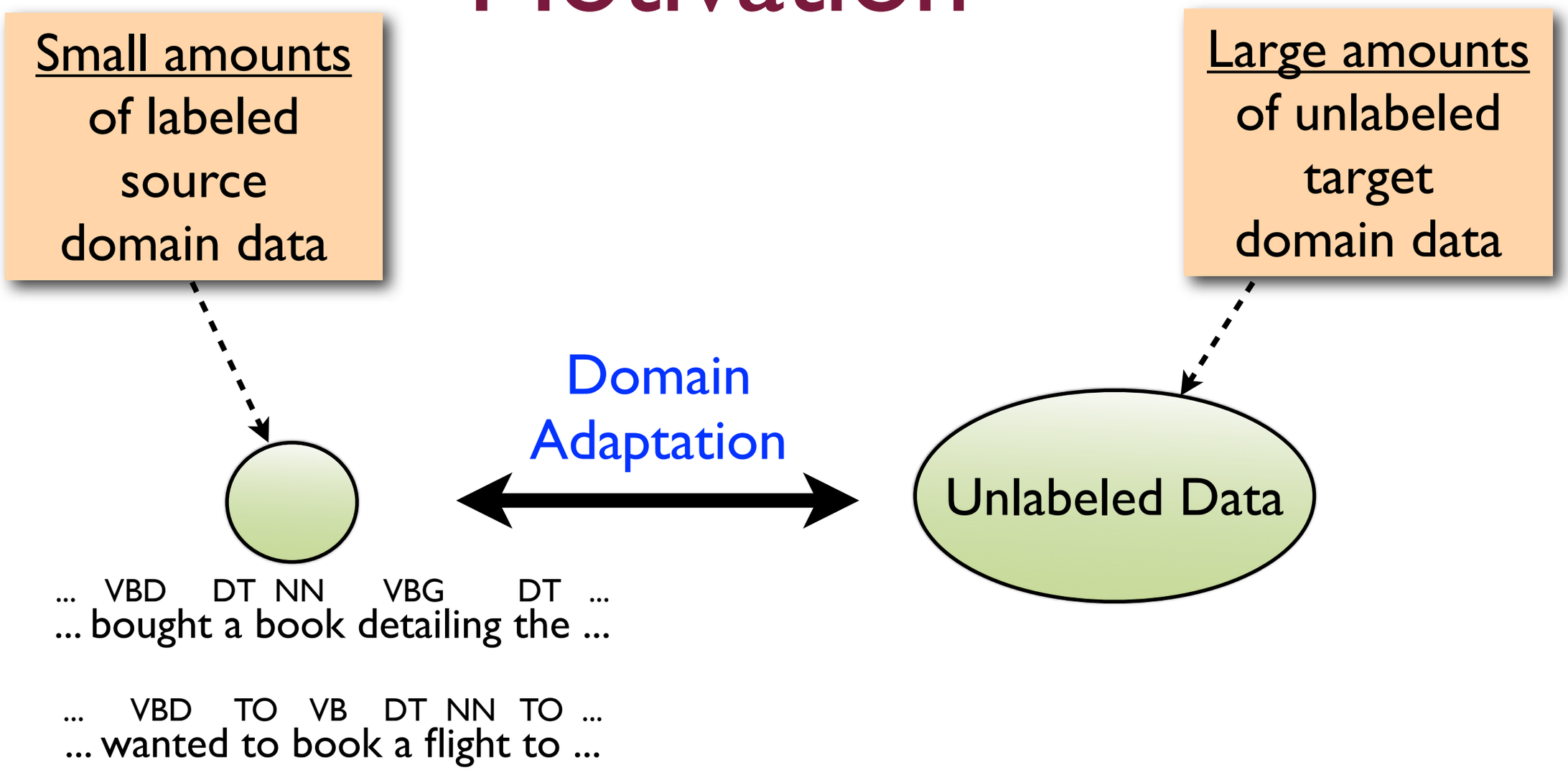
Domain Adaptation
Unlabeled Data

Motivation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Domain Adaptation
Unlabeled Data
... VBD DT NN VBG DT ...
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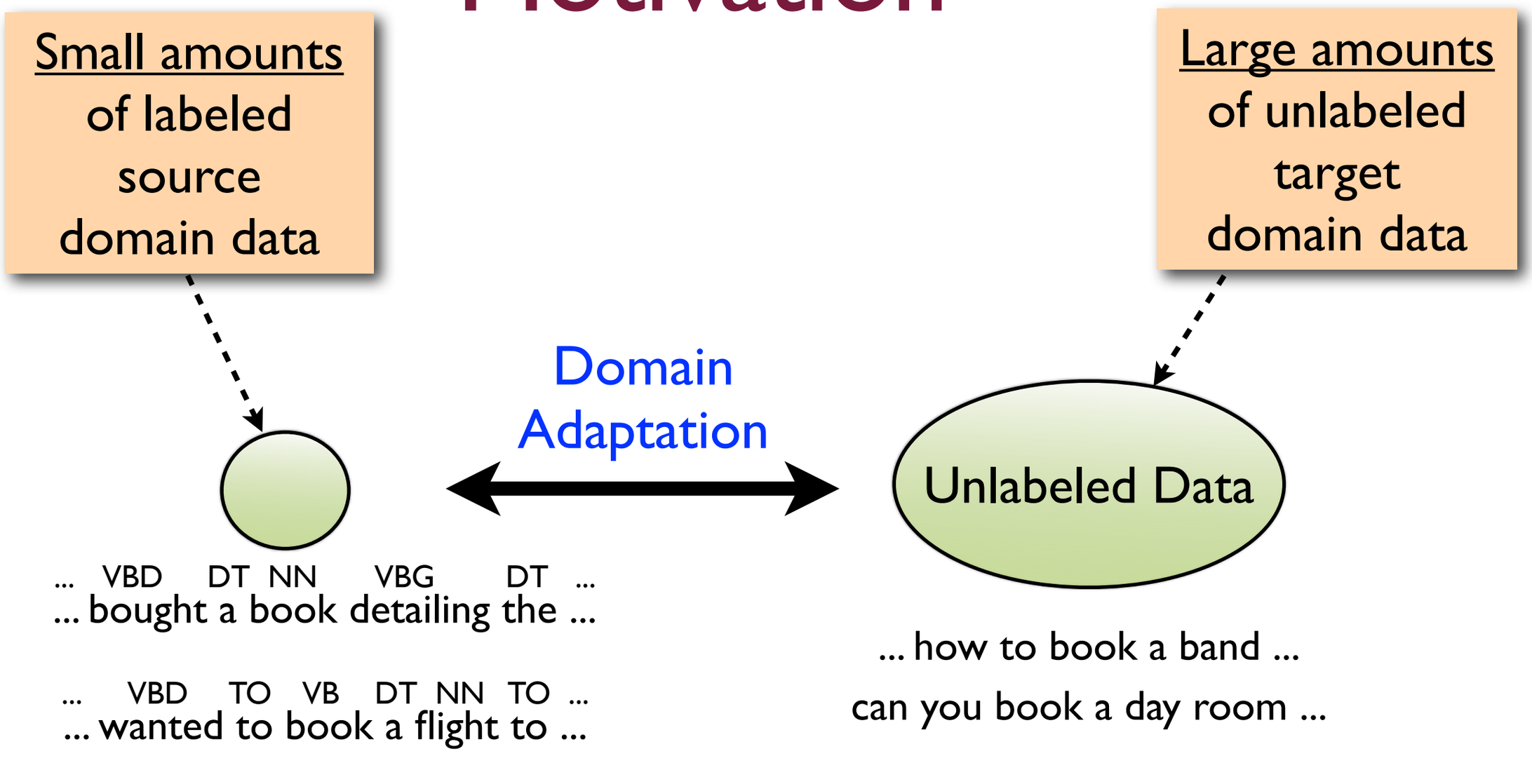
... VBD TO VB DT NN TO ...
... wanted to book a flight to ...
... DT NN VBZ PP DT ...
... the book is about the ...

Motivation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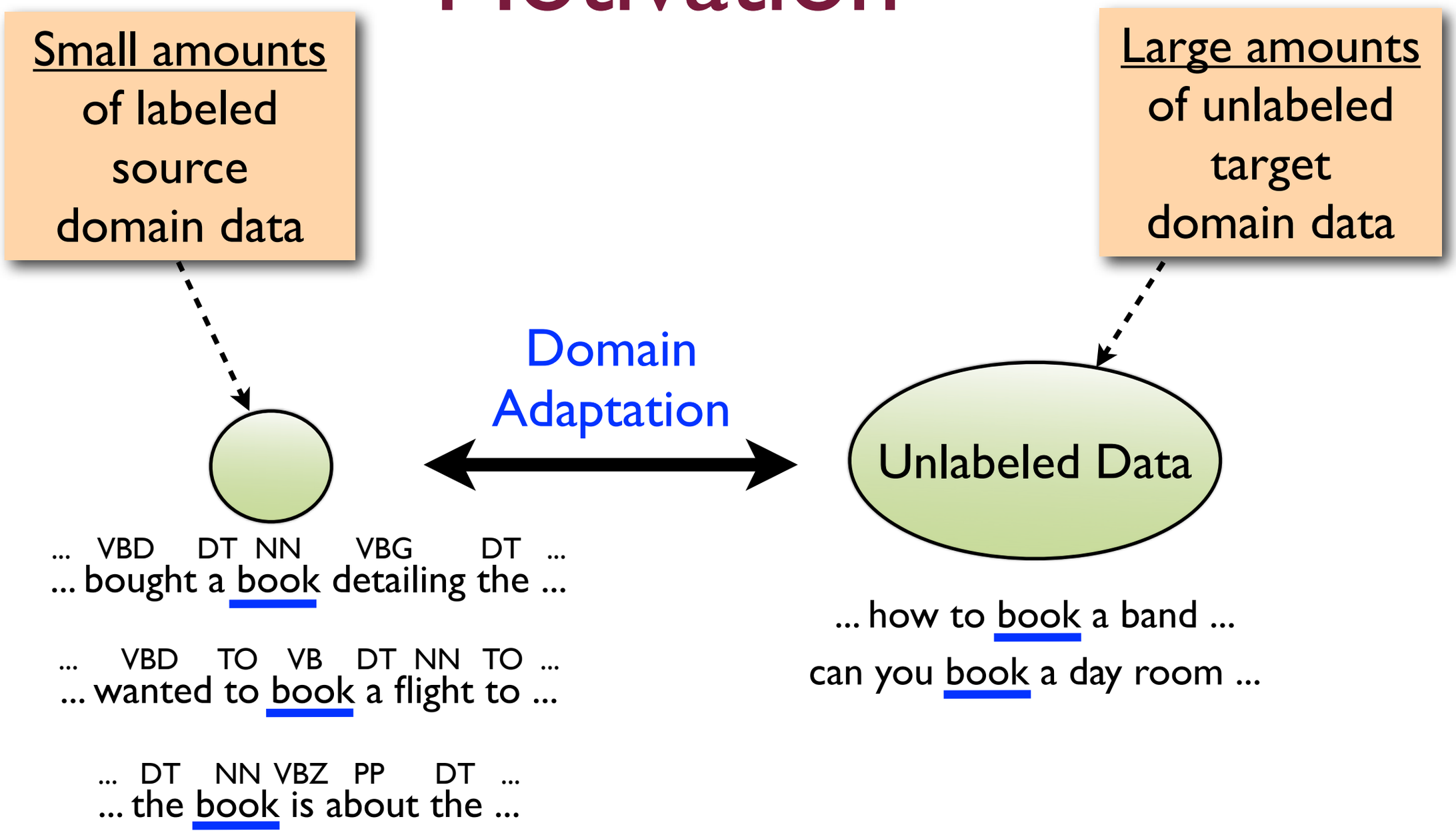
... how to book a band ...
can you book a day room ...

Motivation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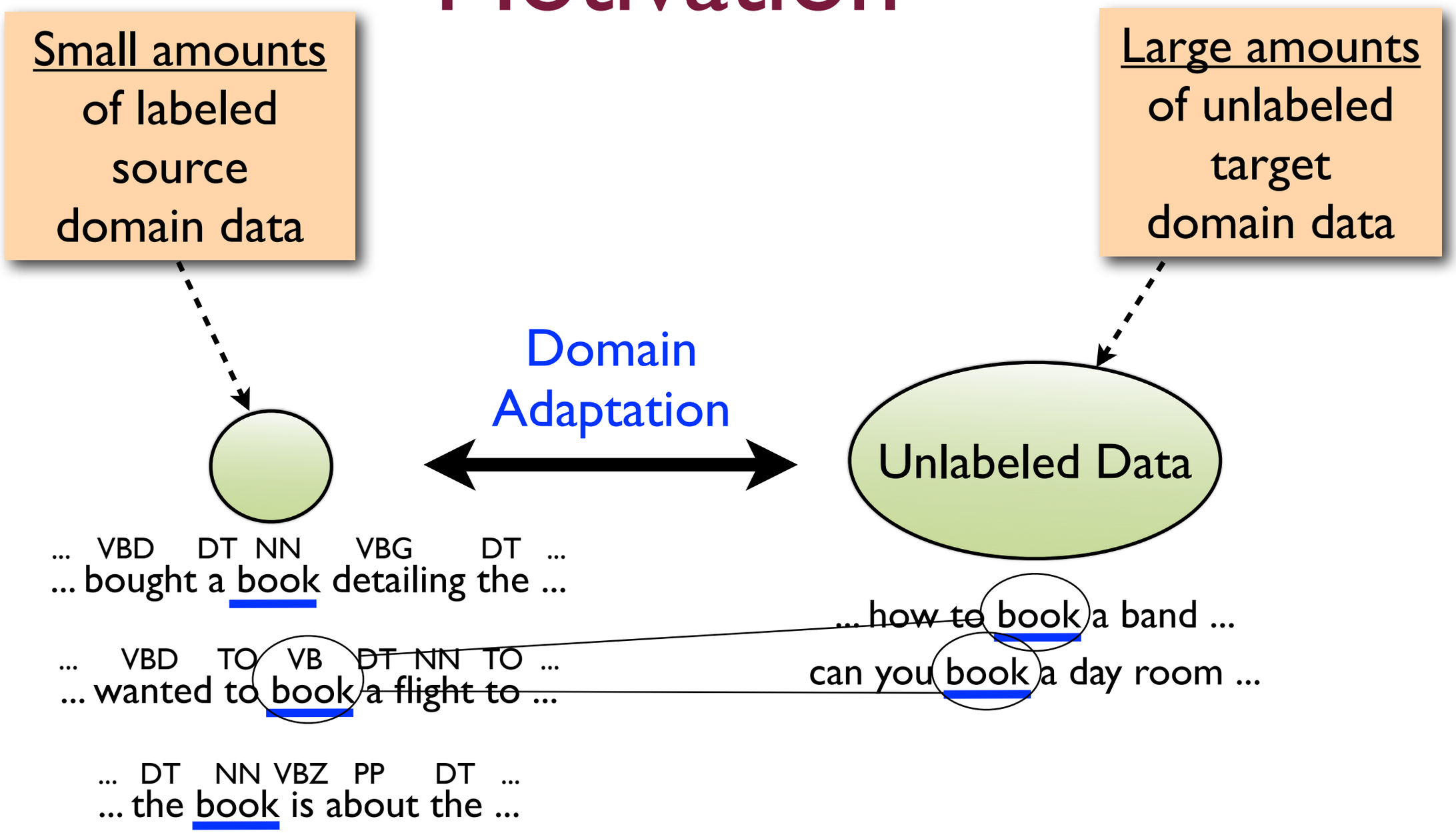
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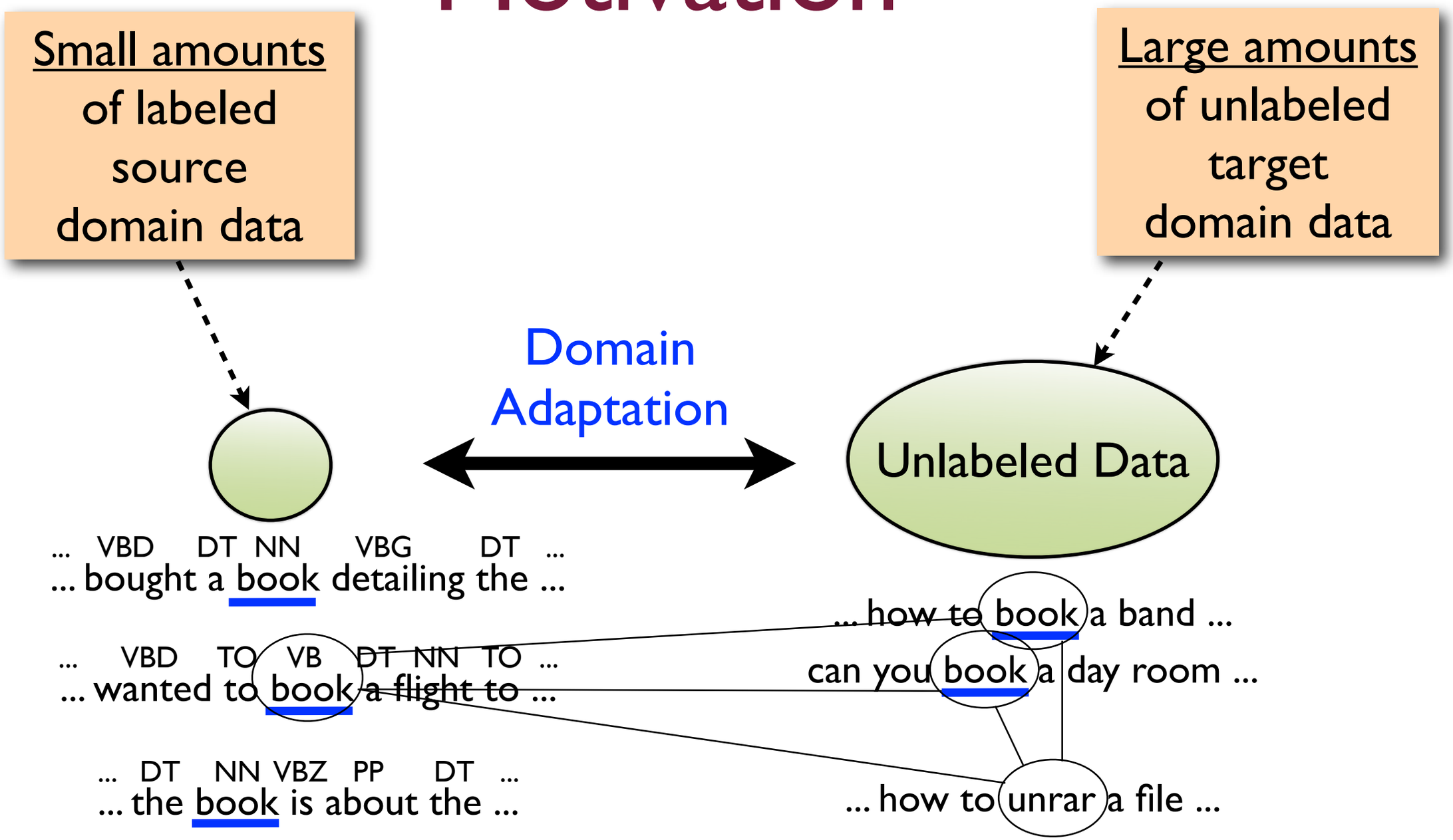
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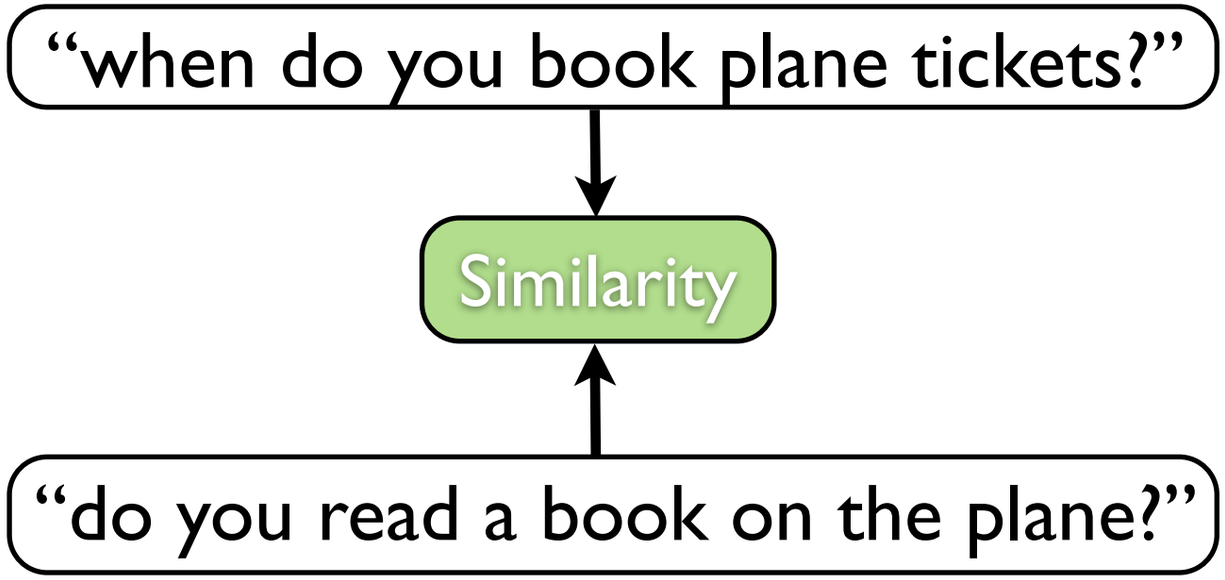
... how to book a band ...
can you book a day room ...
... how to unrar a file ...

Graph Construction (I)

"when do you book plane tickets?"

"do you read a book on the plane?"

Graph Construction (I)



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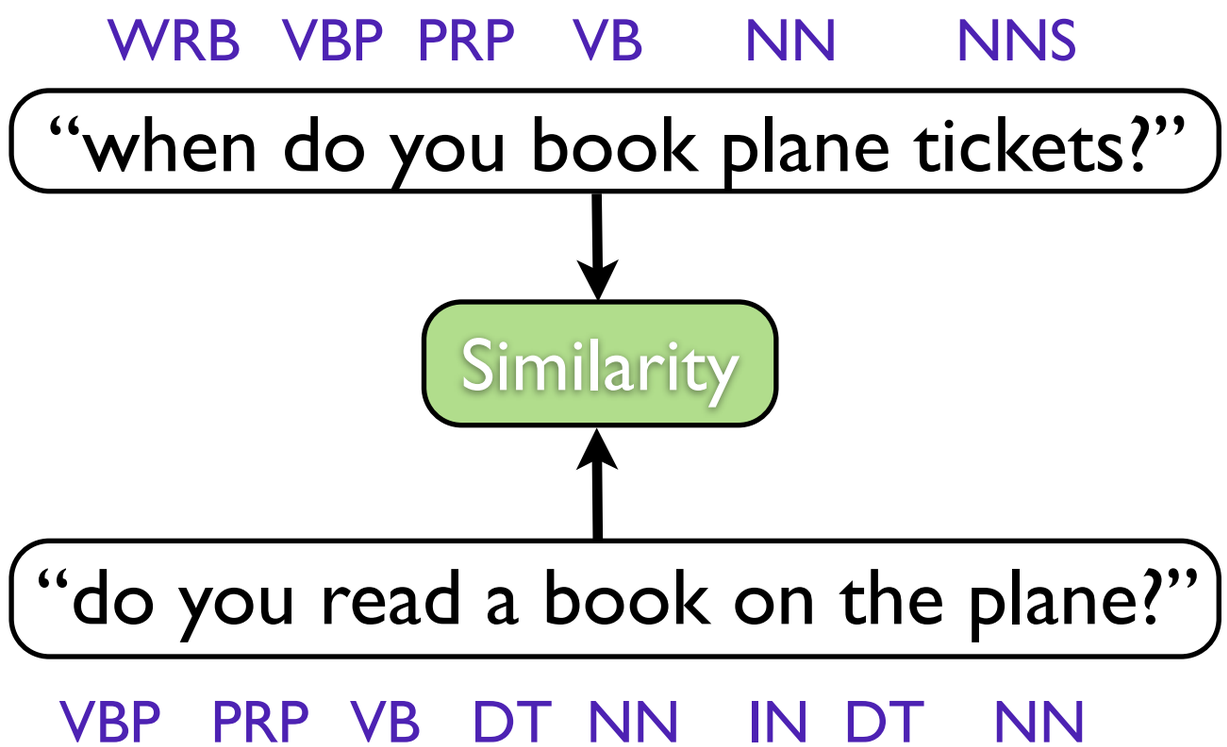

Graph Construction (II)

can you book a day room at hilton hawaiian village ?

what was the book that has no letter e ?

how much does it cost to book a band ?

how to get a book agent ?

Graph Construction (II)

can you book a day room at hilton hawaiian village ?

what was the book that has no letter e ?

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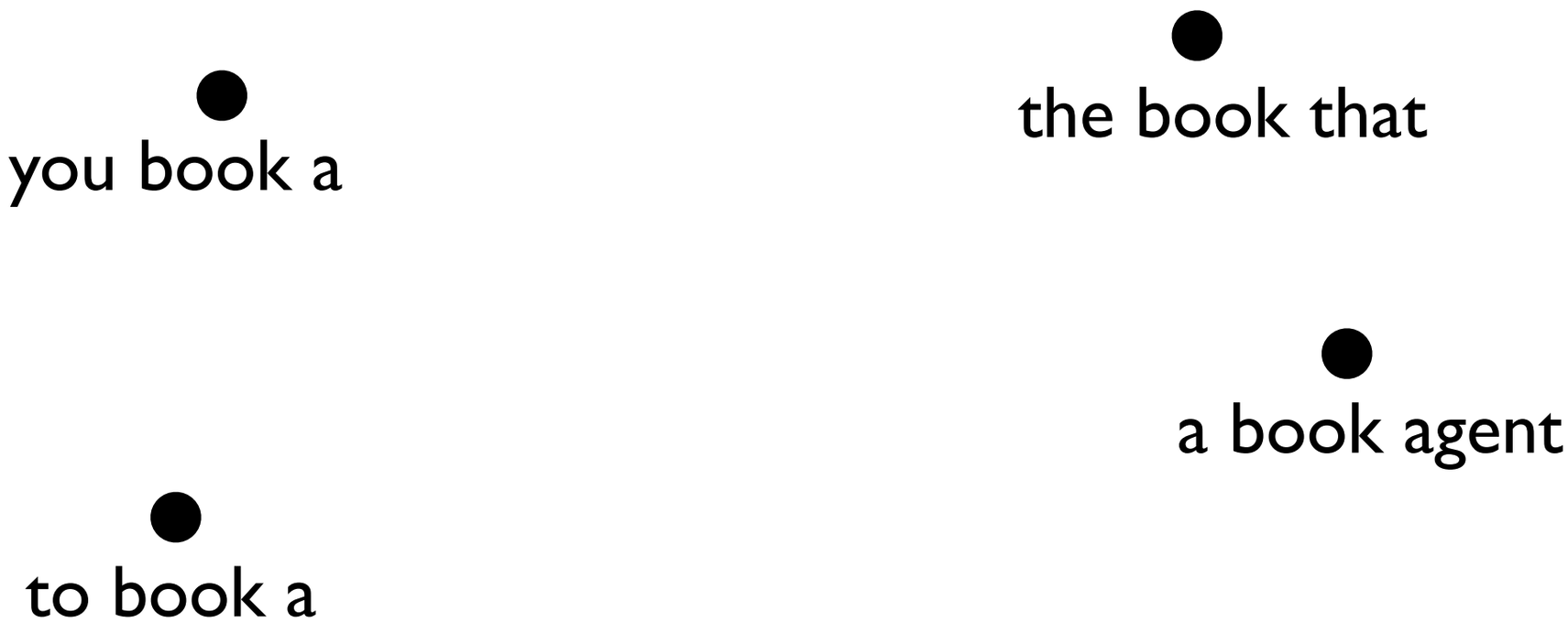
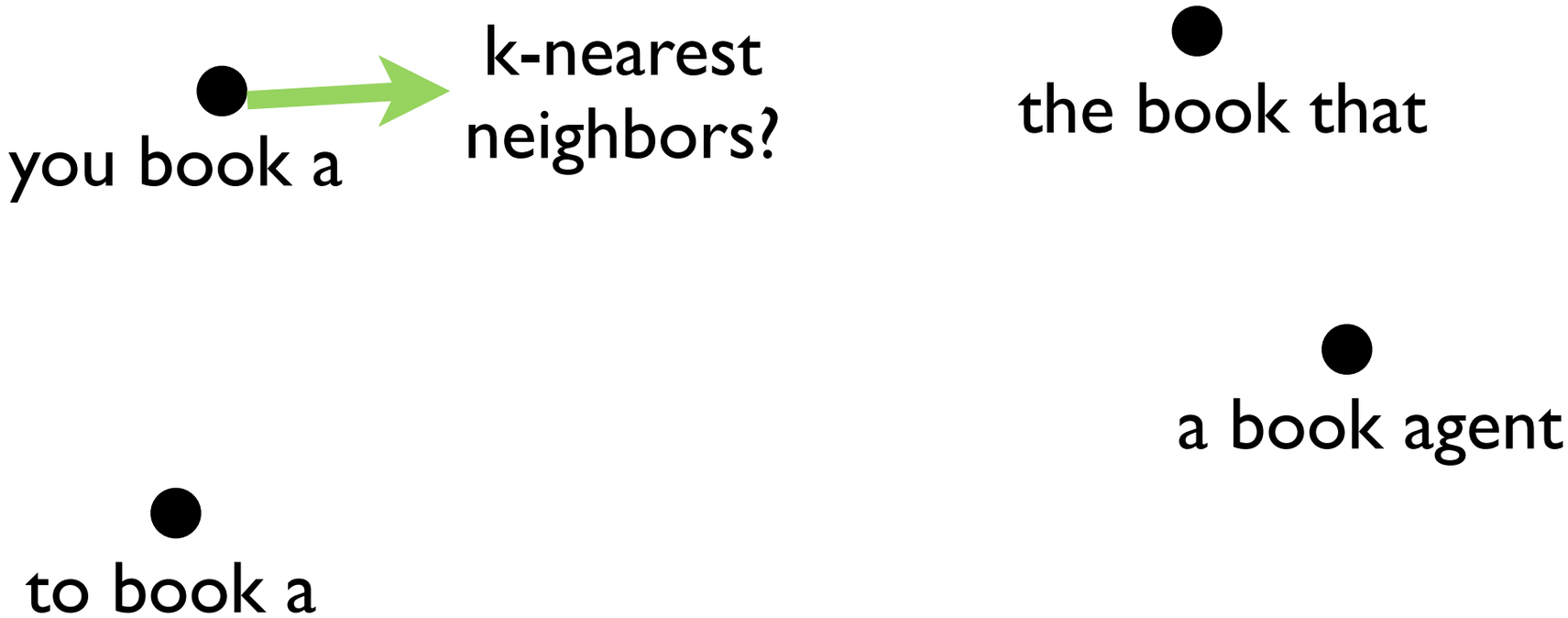
Graph Construction (II)



Graph Construction (II)



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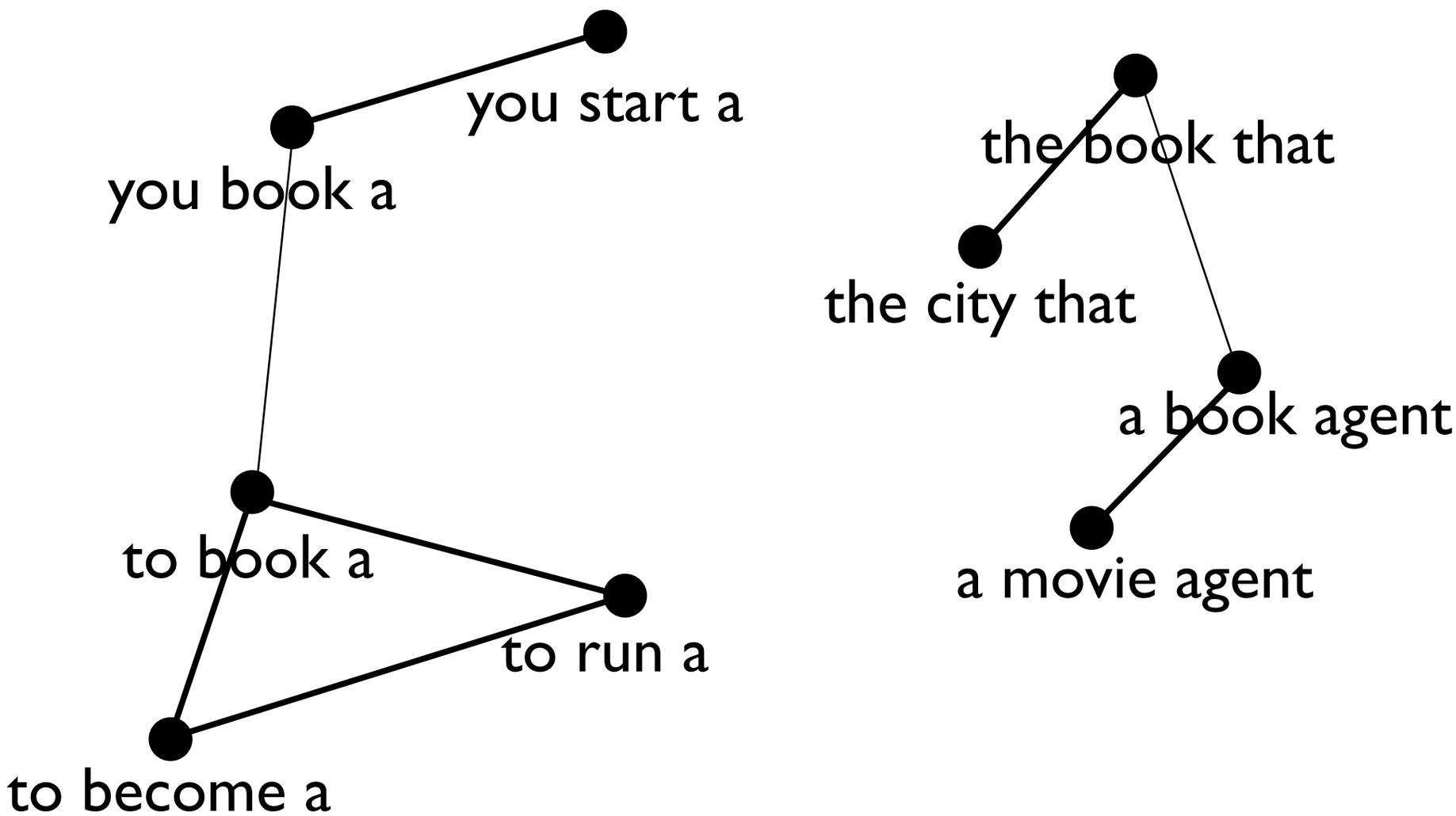

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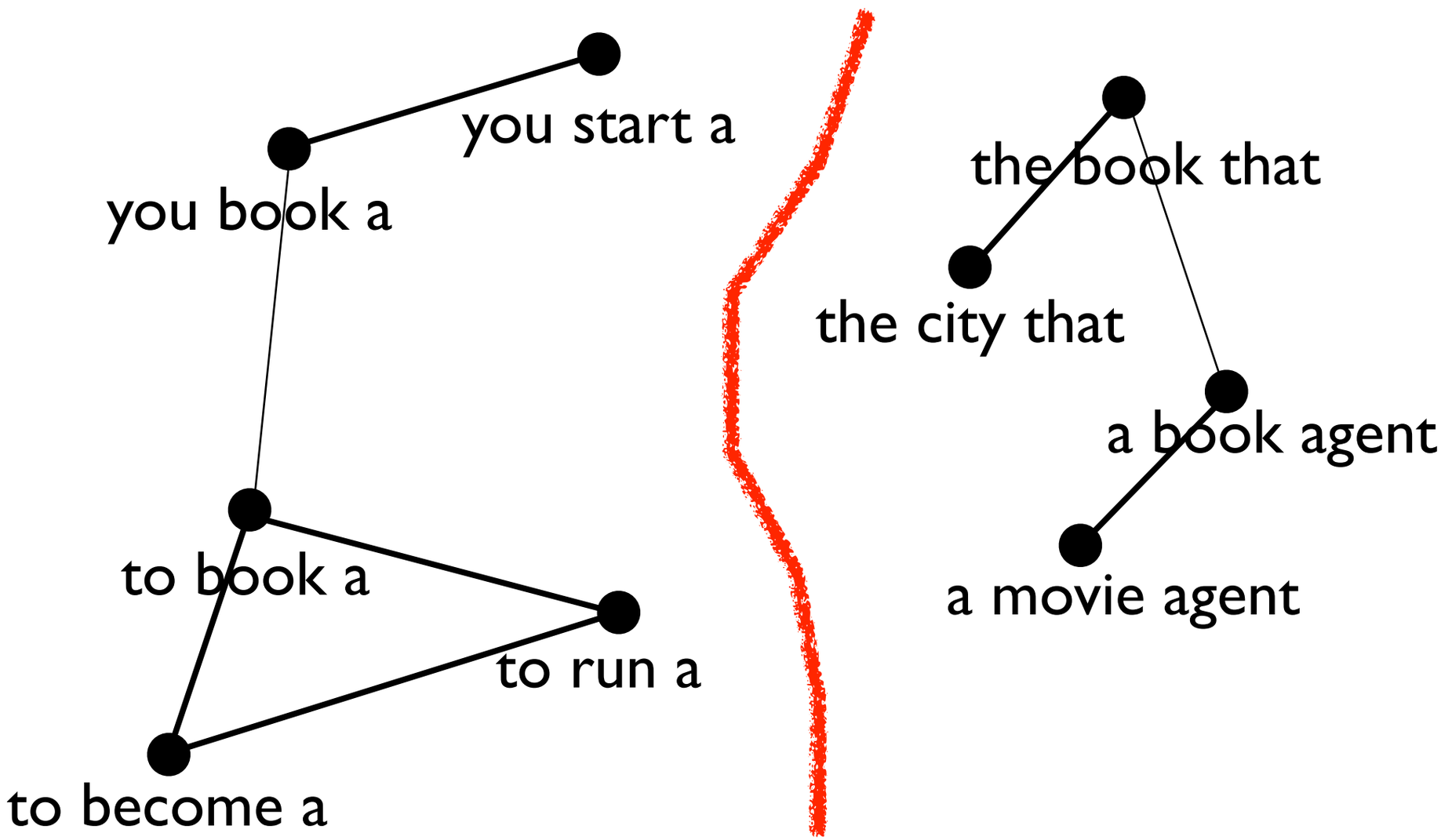

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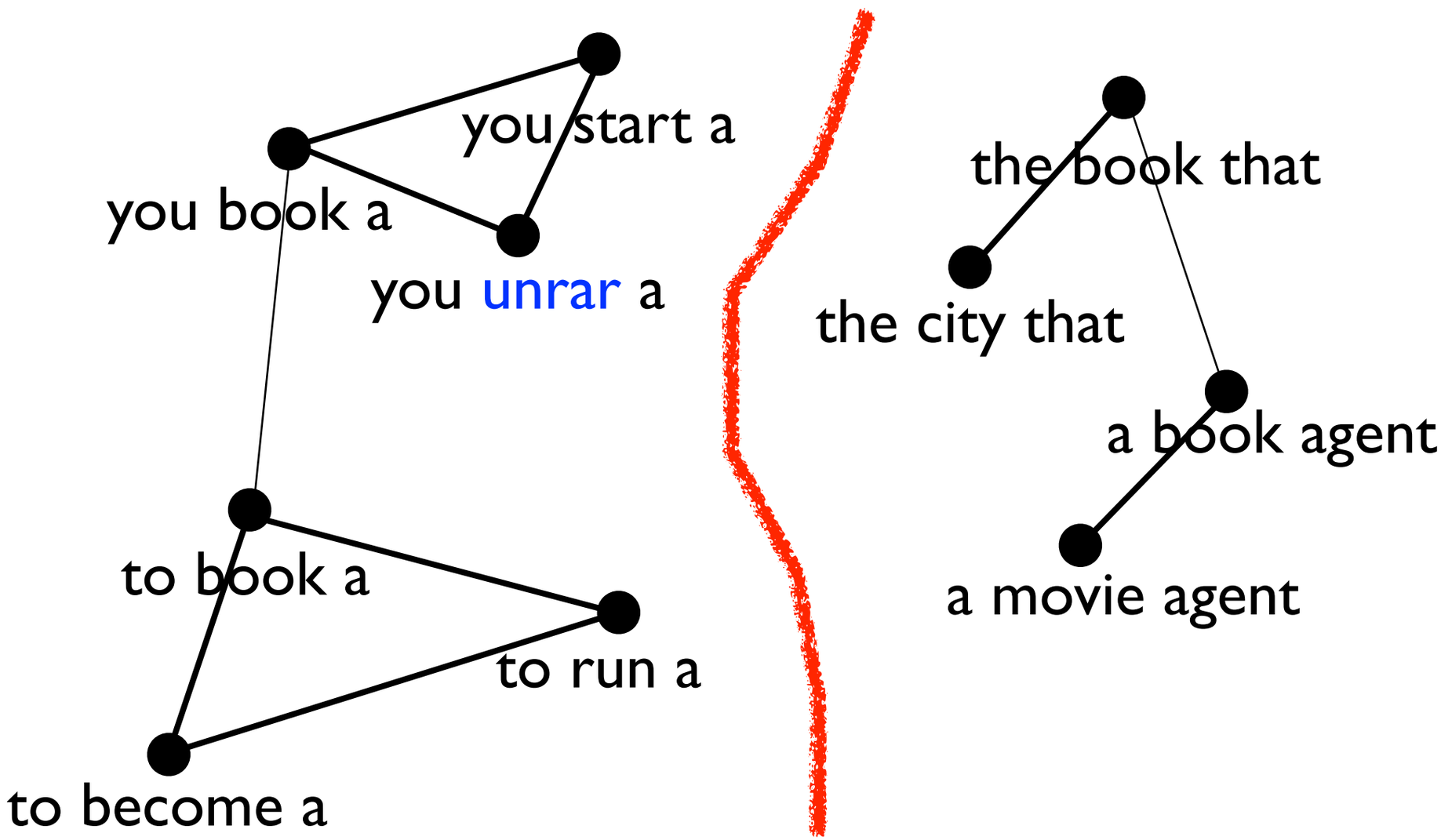

Graph Construction - Features

how much does it cost to book a band ?

Graph Construction - Features

how much does it cost to book a band ?

Graph Construction - Features

how much does it cost to book a band ?

Trigram + Context	cost to book a band

Graph Construction - Features

how much does it cost to book a band ?

Trigram + Context	cost to book a band
Left Context	cost to

Graph Construction - Features

how much does it cost to book a band ?

Trigram + Context	cost to book a band
Left Context	cost to
Right Context	a band

Graph Construction - Features

how much does it cost to book a band ?

Trigram + Context	cost to book a band
Left Context	cost to
Right Context	a band
Center Word	book

Graph Construction - Features

how much does it cost to book a band ?

Trigram + Context	cost to book a band
Left Context	cost to
Right Context	a band
Center Word	book
Trigram - Center Word	to _____ a
Left Word + Right Context	to _____ a band
Left Context + Right Word	cost to _____ a
Suffix	none

Graph Construction - Features

how much to book a flight to paris?

how much does it cost to book a band ?

Graph Construction - Features

how much to book a flight to paris?

how much does it cost to book a band ?

Graph Construction - Features

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Graph Construction - Features

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Graph Construction - Features

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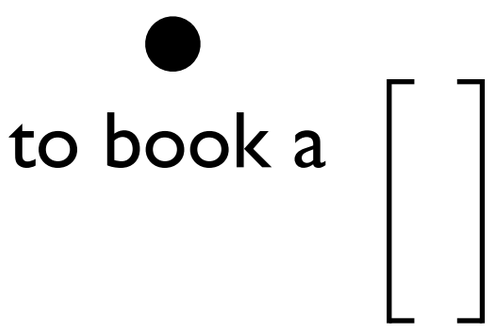
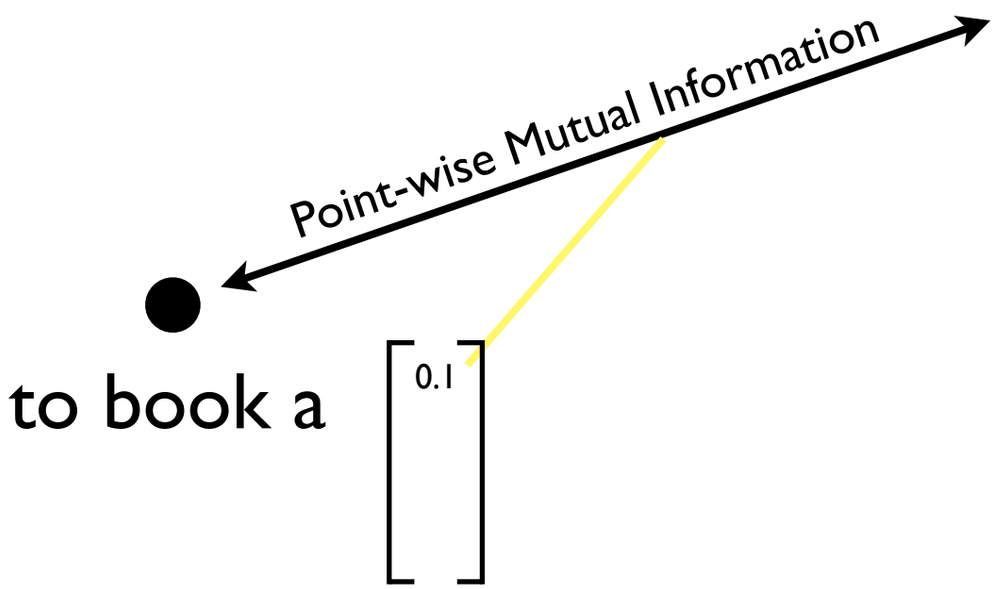


Trigram + Context
Left Context
Right Context
Center Word
Trigram - Center Word
Left Word + Right Context
Left Context + Right Word
Suffix

Graph Construction - Features



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Left Context
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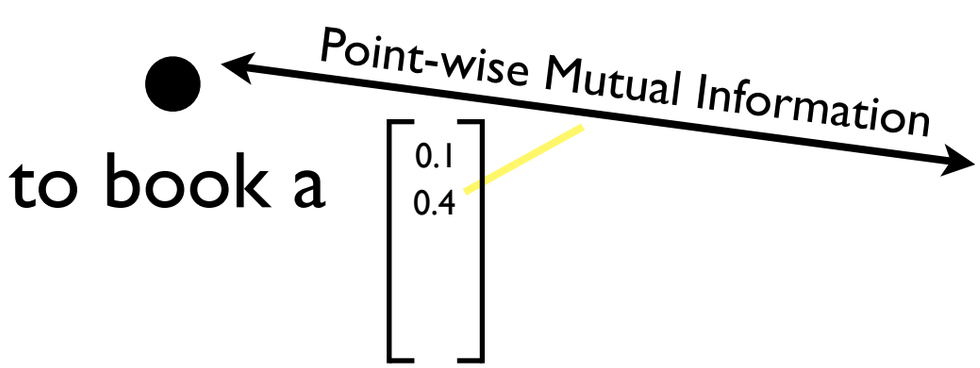
Graph Construction - Features



Trigram + Context
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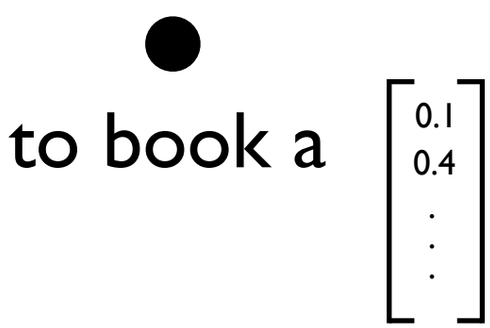
Graph Construction - Features



Trigram + Context
Left Context
Right Context
Center Word
Trigram - Center Word
Left Word + Right Context
Left Context + Right Word
Suffix

Similarity Function

to book a $\begin{bmatrix} 0.1 \\ 0.4 \\ \vdots \end{bmatrix}$

Cosine Similarity (,) = 0.56

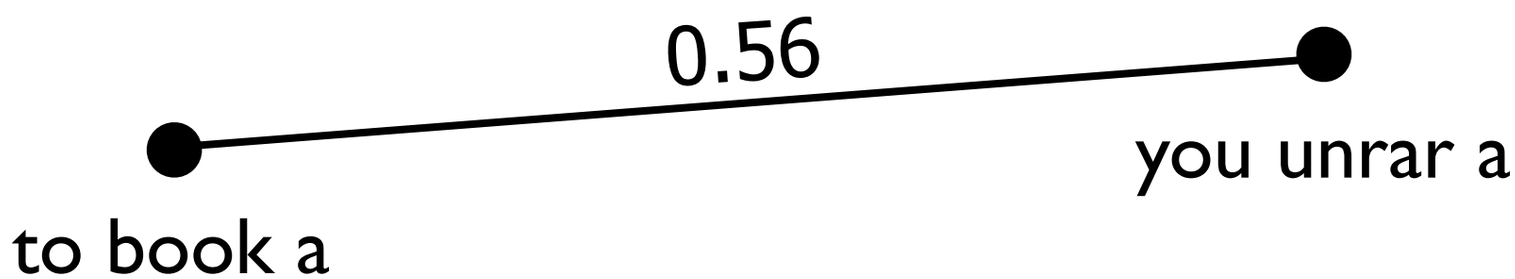
Similarity Function

to book a $\begin{bmatrix} 0.1 \\ 0.4 \\ \vdots \end{bmatrix}$

you unrar a

Cosine Similarity (,) = 0.56

Similarity Function



$$\text{Cosine Similarity}\left(\begin{bmatrix} 0.1 \\ 0.4 \\ \vdots \end{bmatrix}, \begin{bmatrix} 0.2 \\ 0.3 \\ \vdots \end{bmatrix} \right) = 0.56$$

Approach (I)

1. Train a CRF on labeled data
2. While not converged do:
   2.1. Posterior decode **unlabeled data** using CRF

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how to unrar a zipped file ?

how to get a book agent ?

how do you book a flight to multiple cities ?

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CRF

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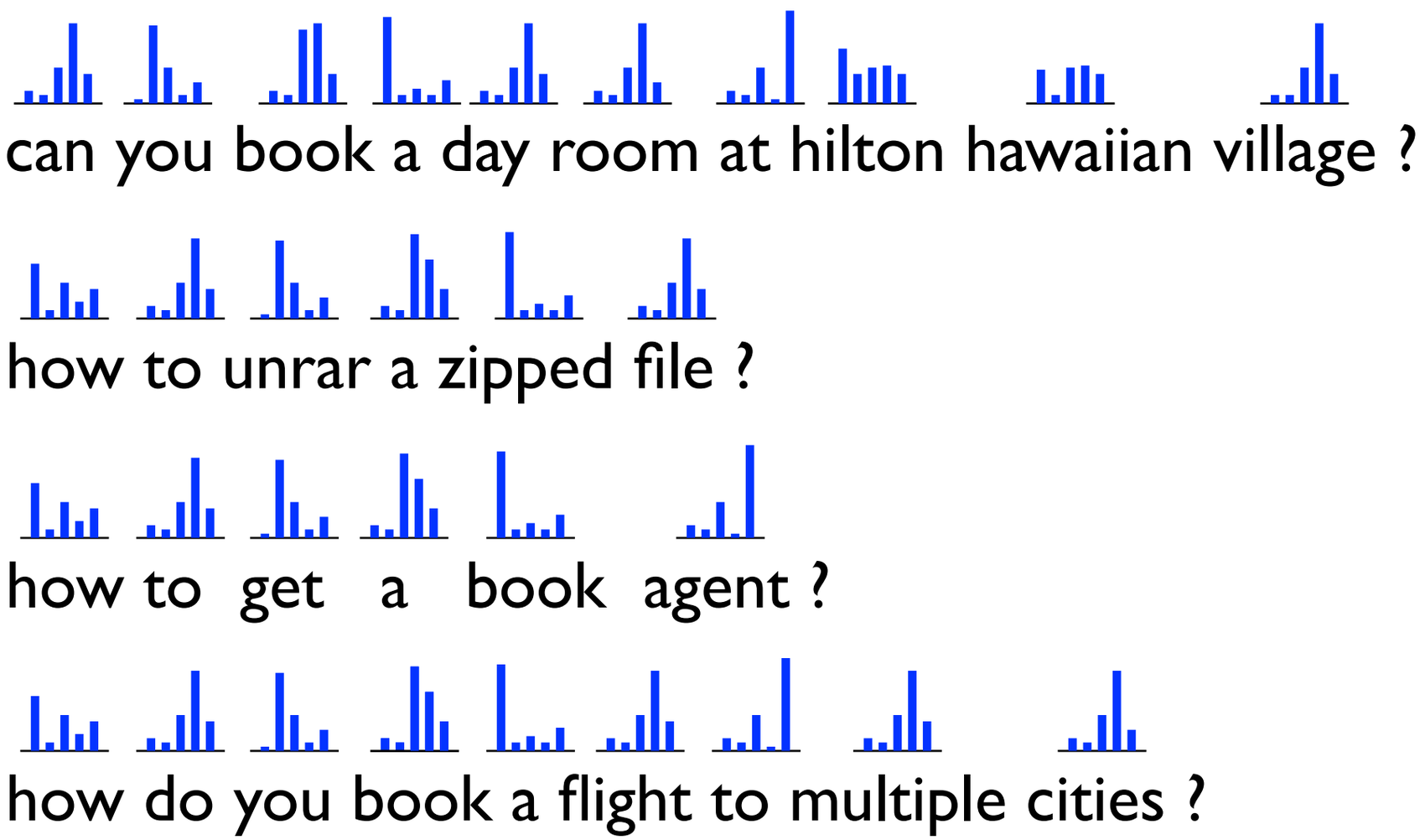

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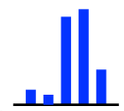
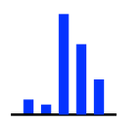


can you book a day room at hilton hawaiian village ?



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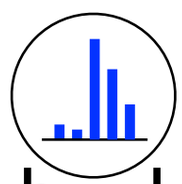
can you book a day room at hilton hawaiian village ?

- you book a



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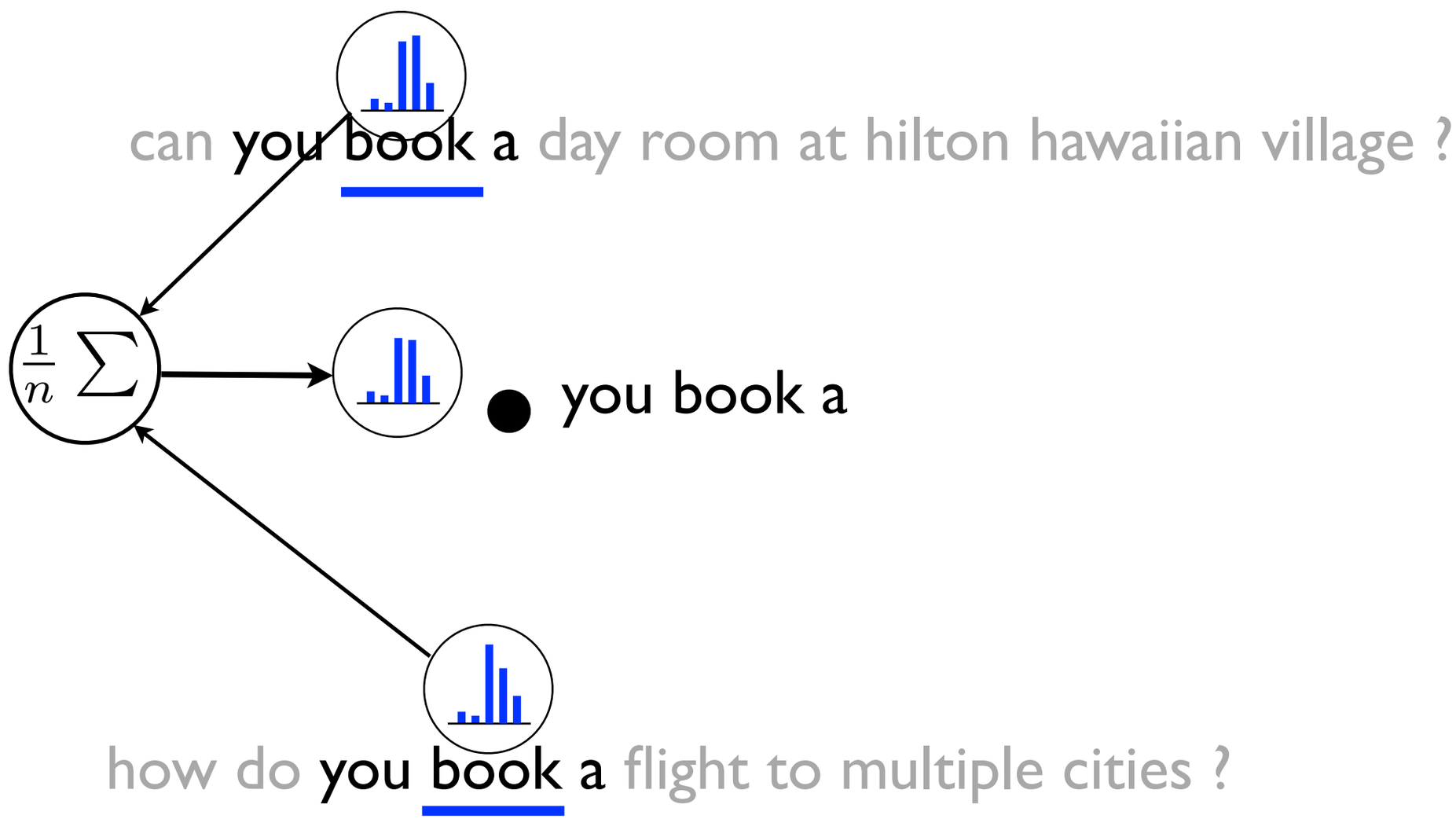

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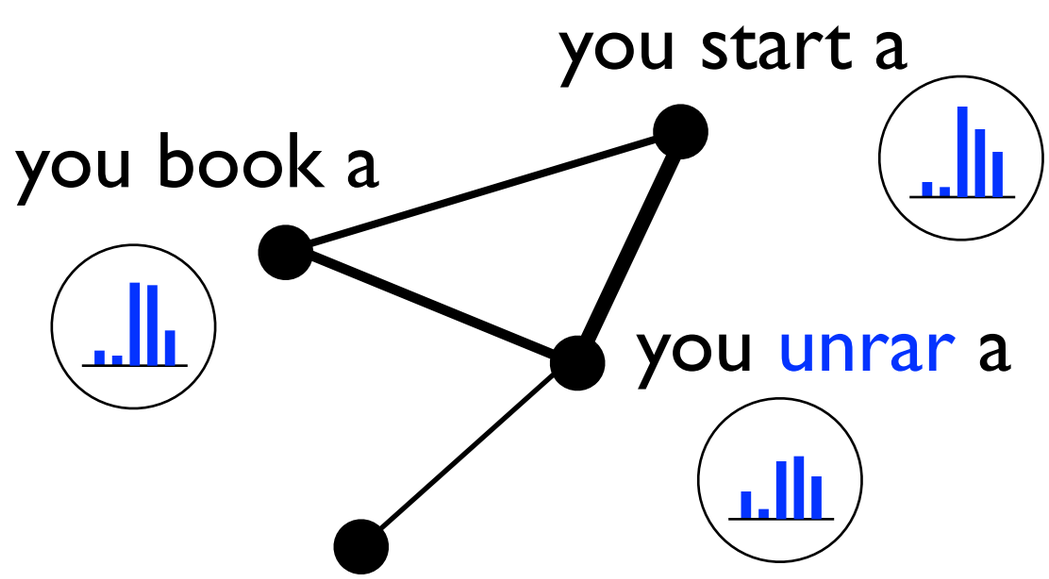

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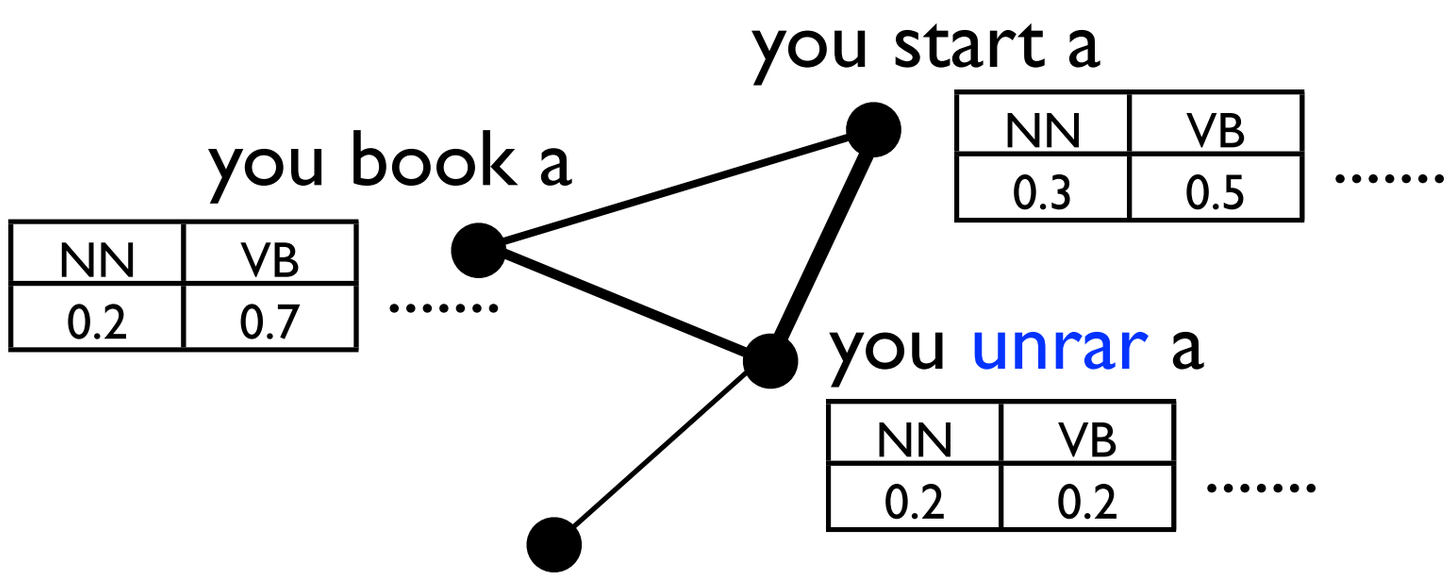

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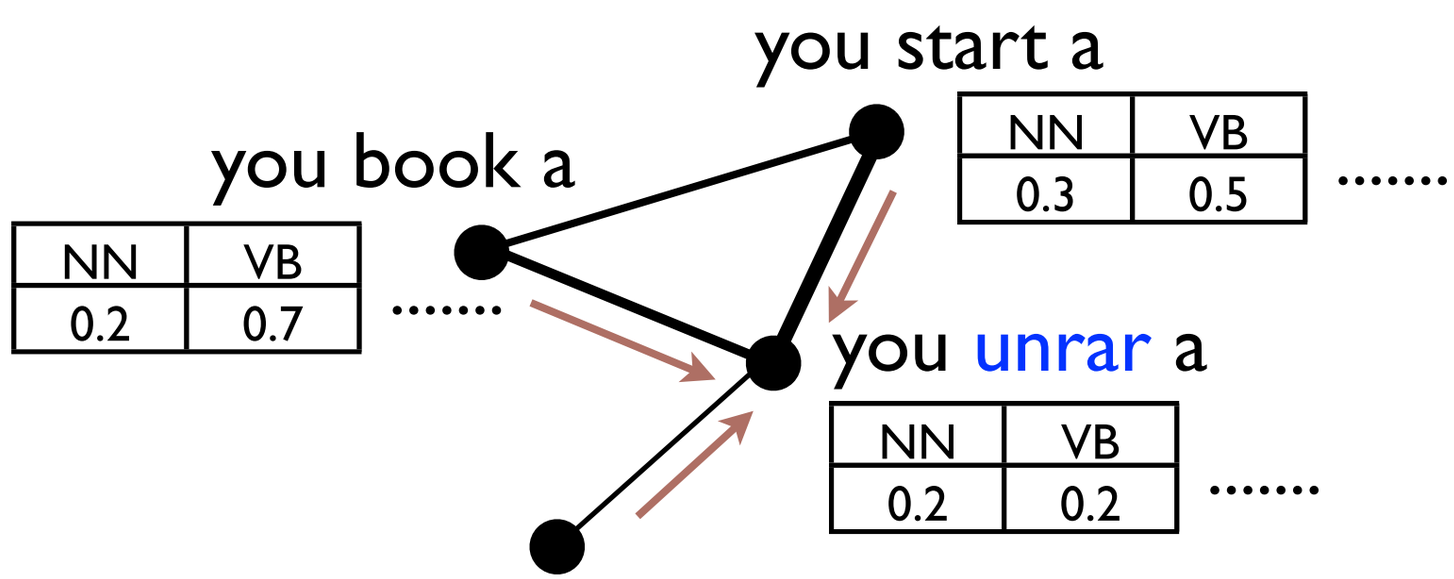

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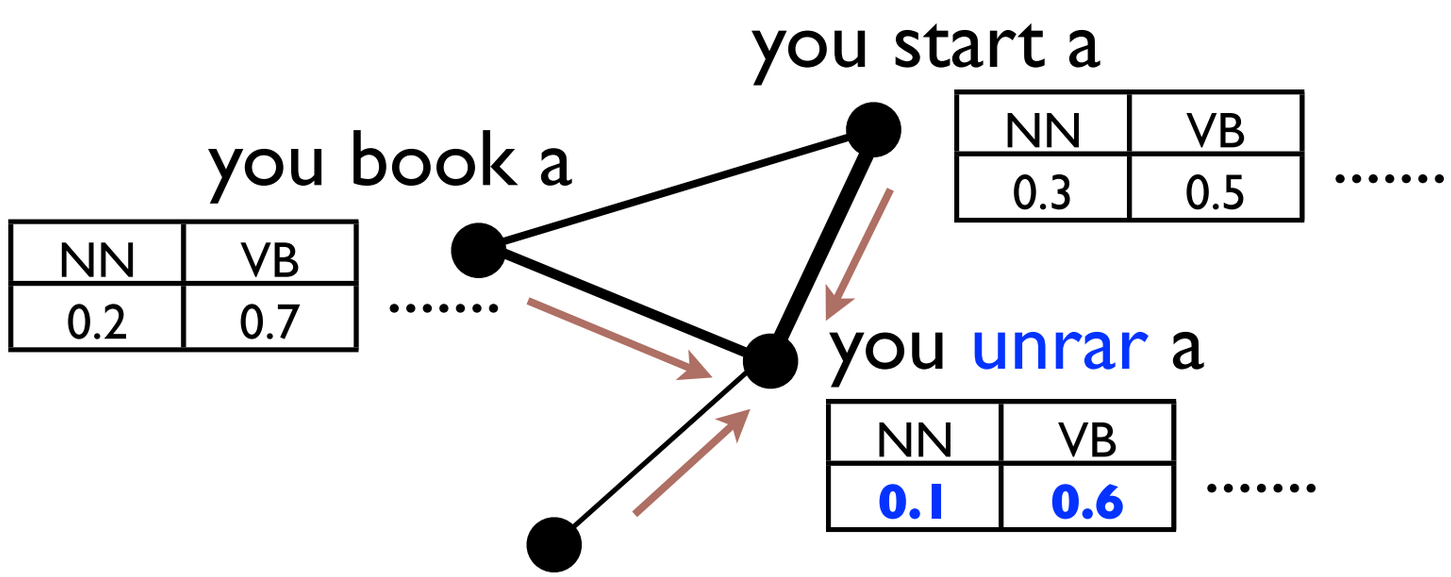

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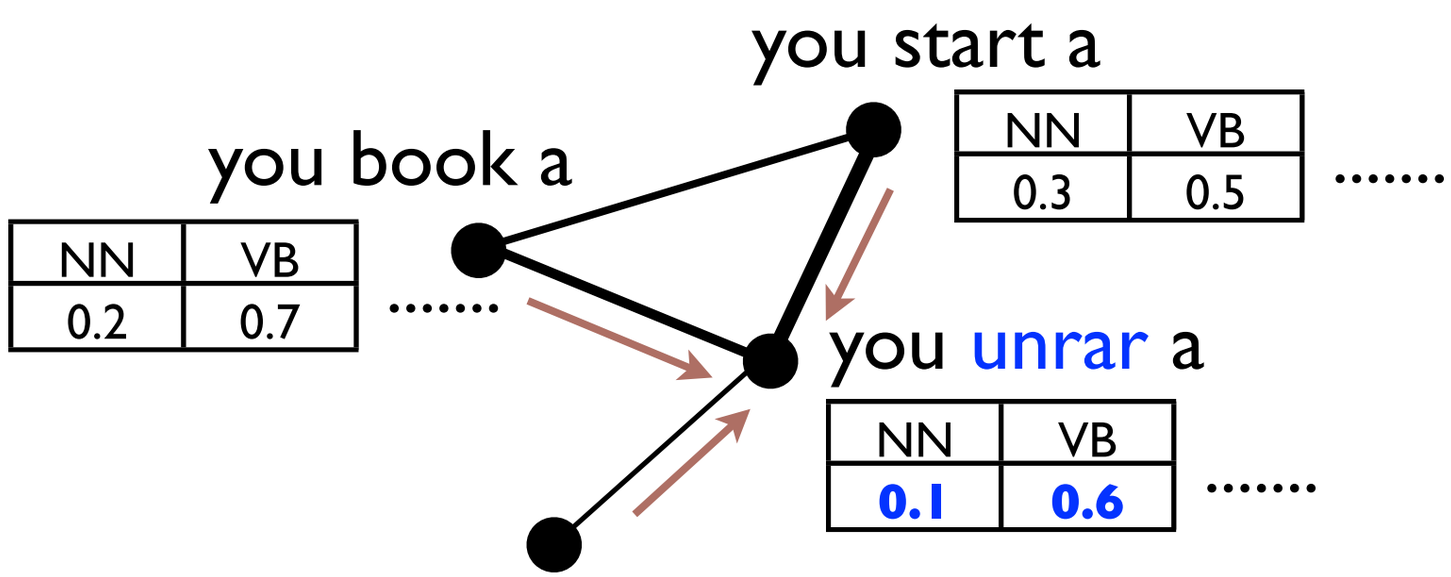

If two n-grams are similar according to the graph then their output distributions should be similar

Approach (IV)

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   2.4. Viterbi Decode

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Can you unrar a zipped file?

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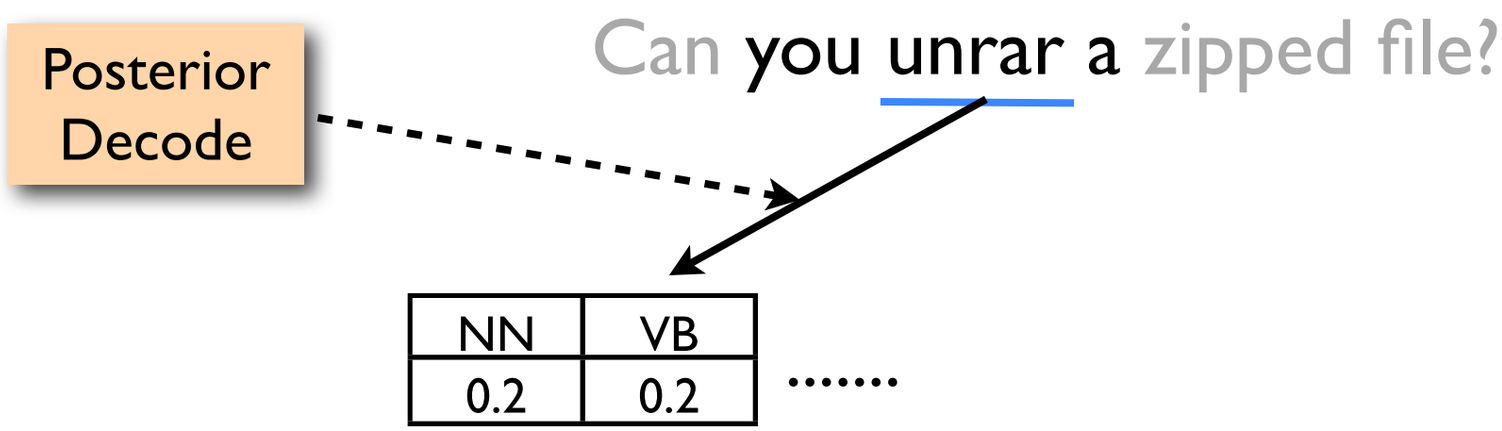

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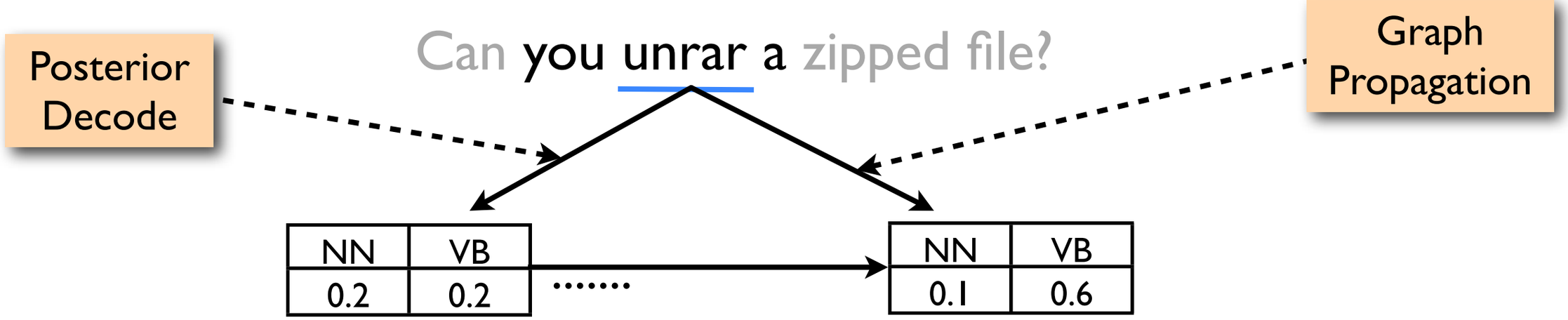

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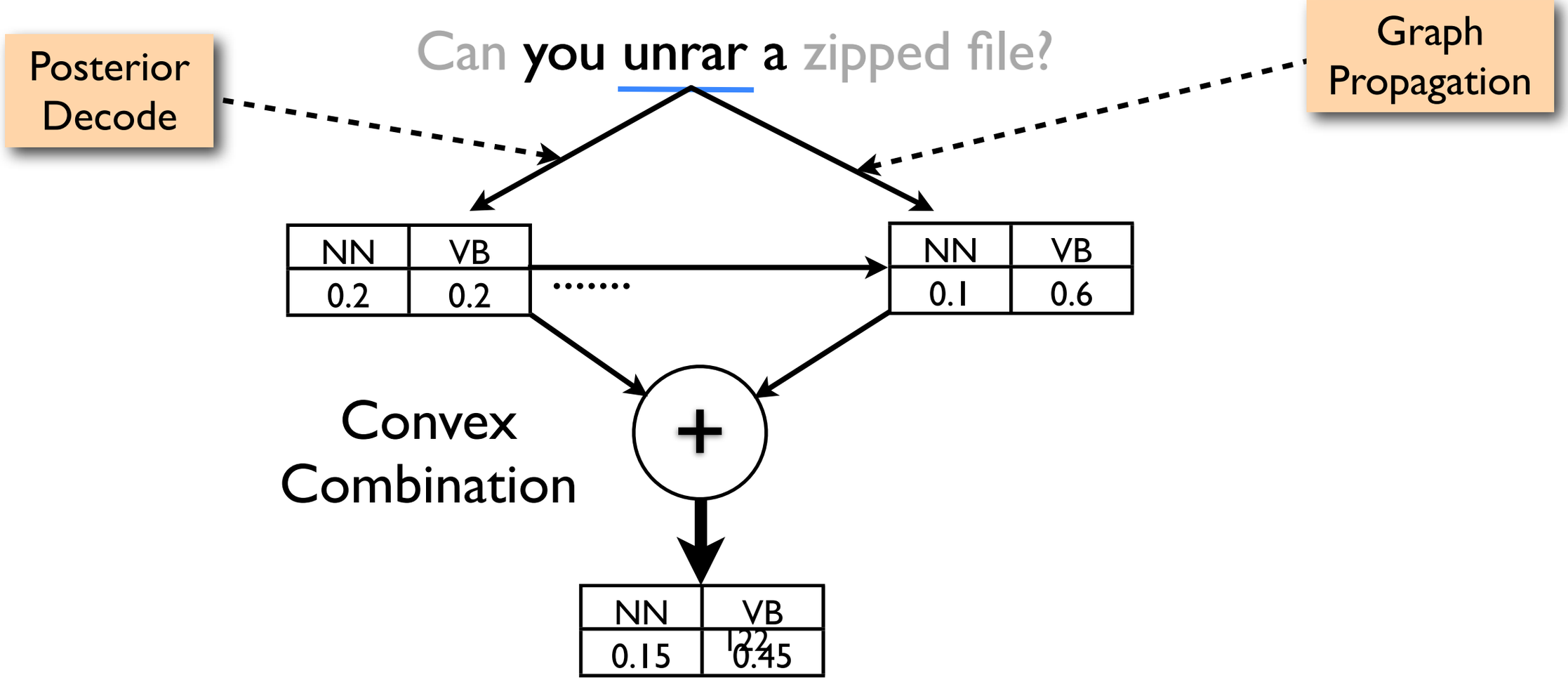

Approach (V)

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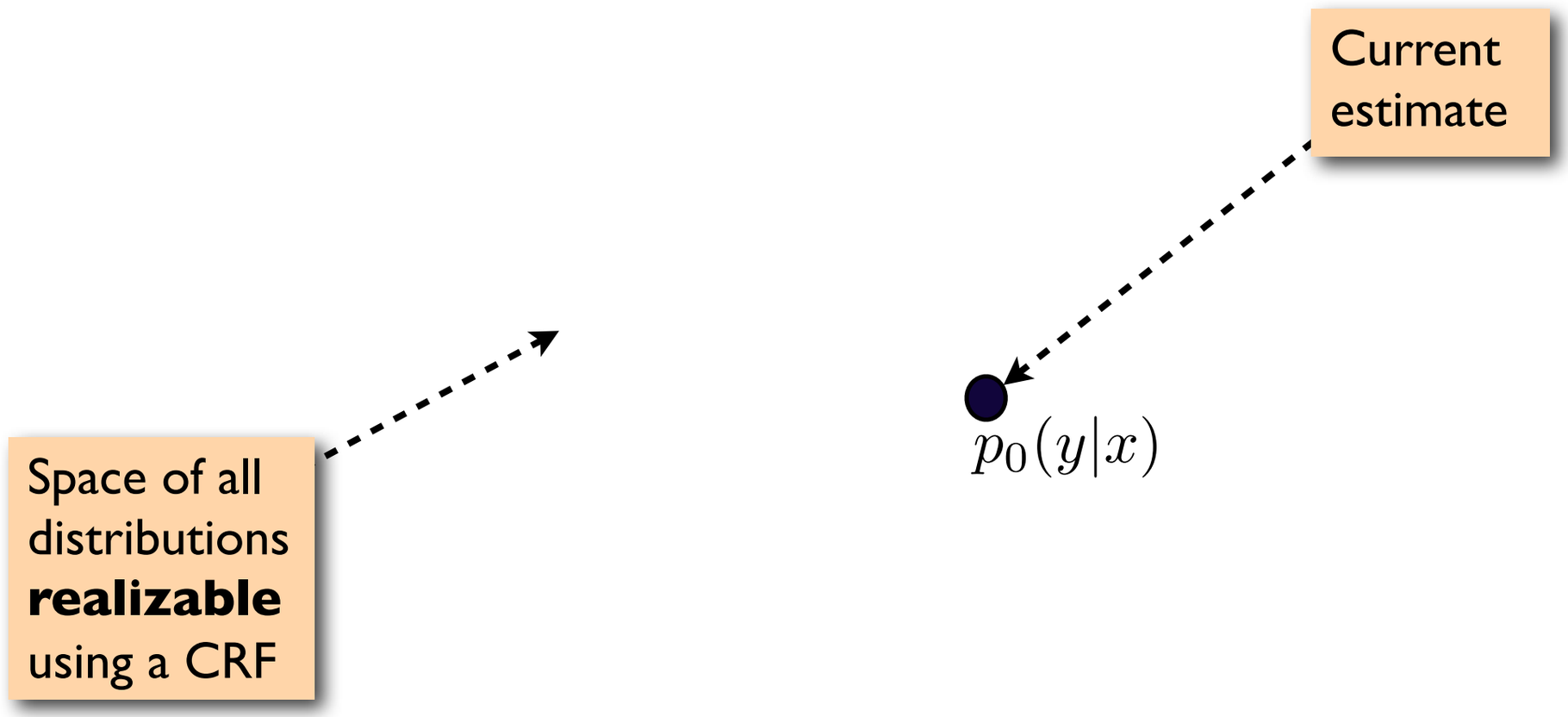
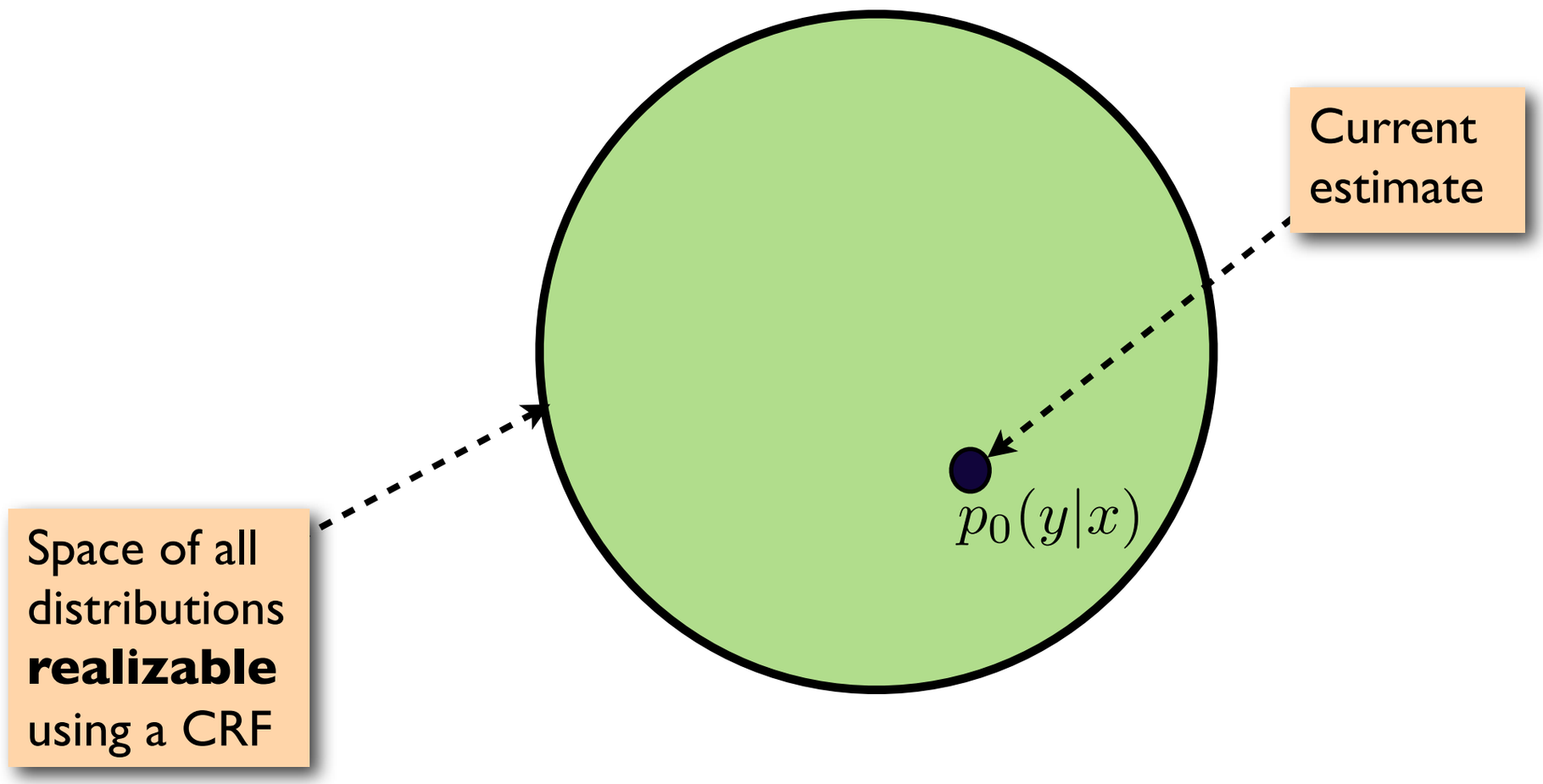
   2.2. Aggregate posteriors (token-to-type mapping)'
   2.3. Graph propagation
   2.4. Viterbi Decode
   2.5. Retrain CRF on labeled & **automatically labeled** unlabeled data

Viterbi Decoding : Intuition



Viterbi Decoding : Intuition



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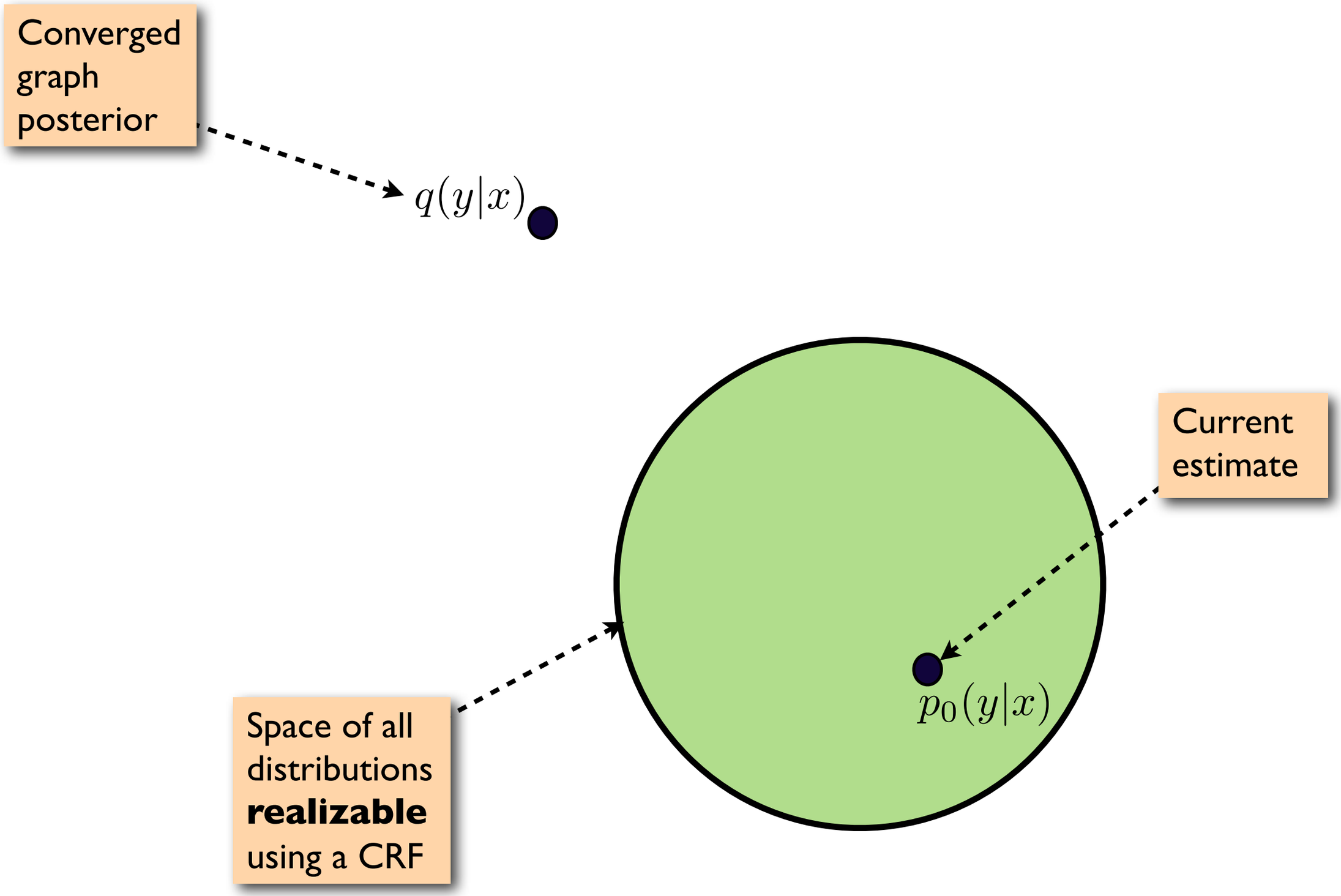

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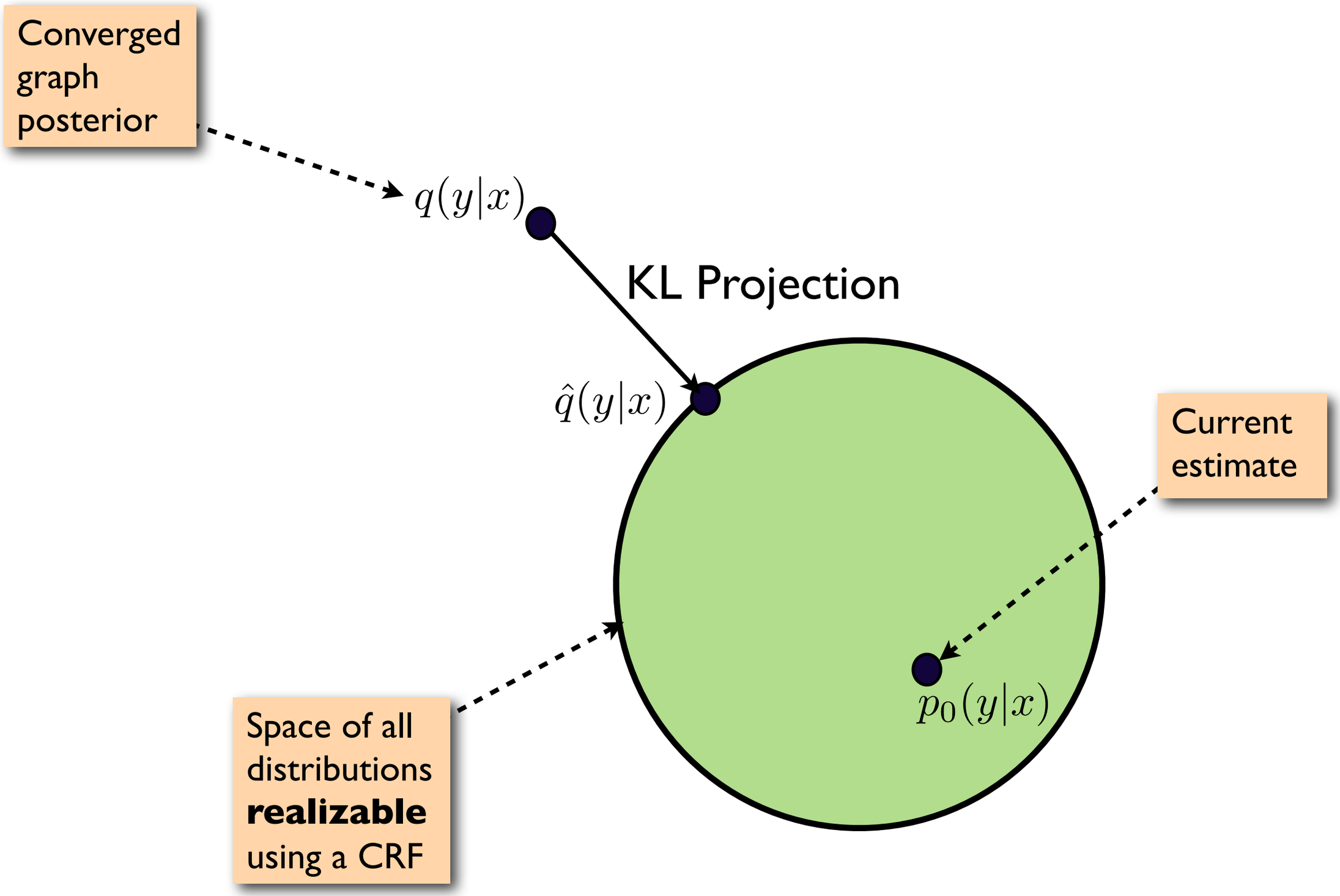

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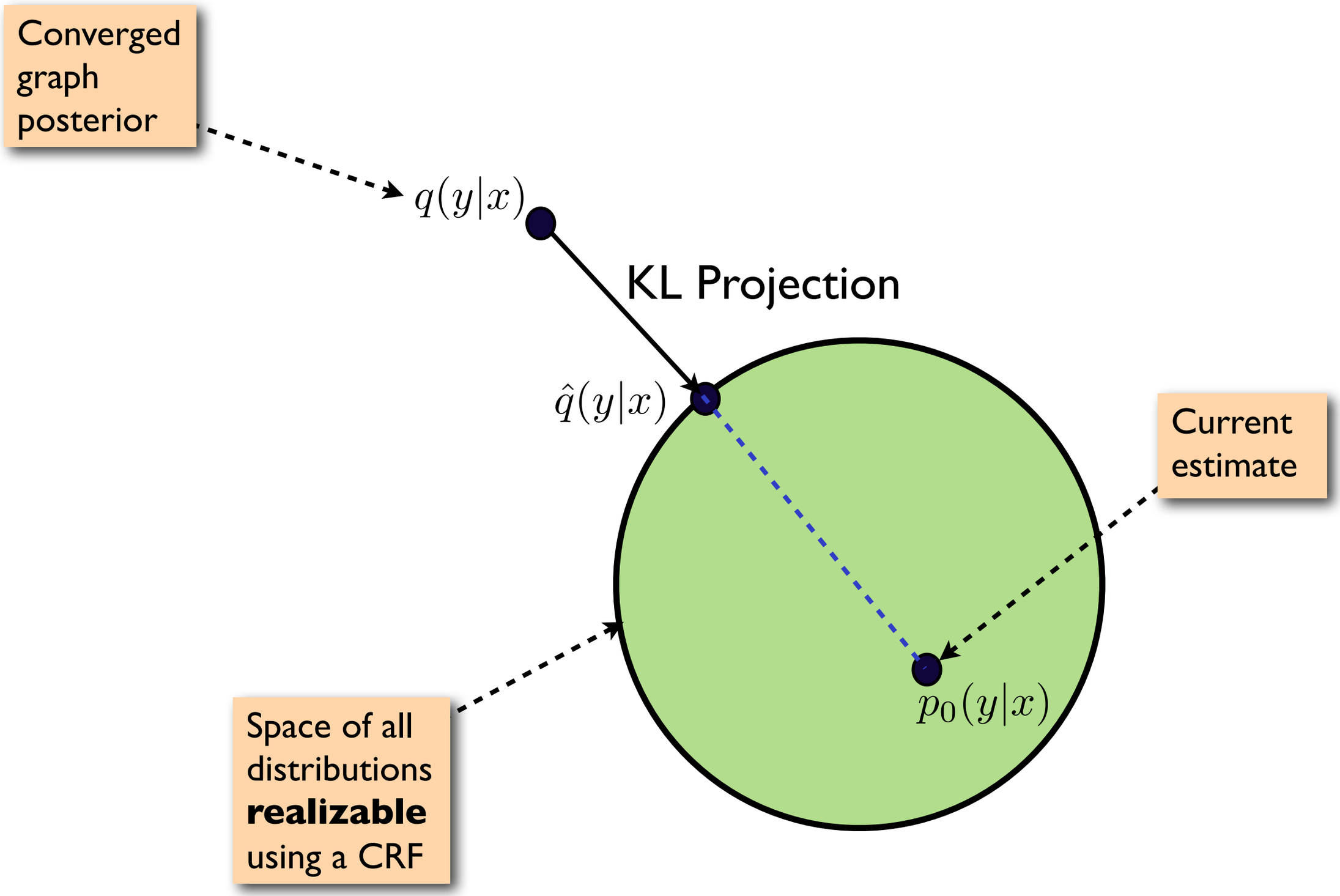
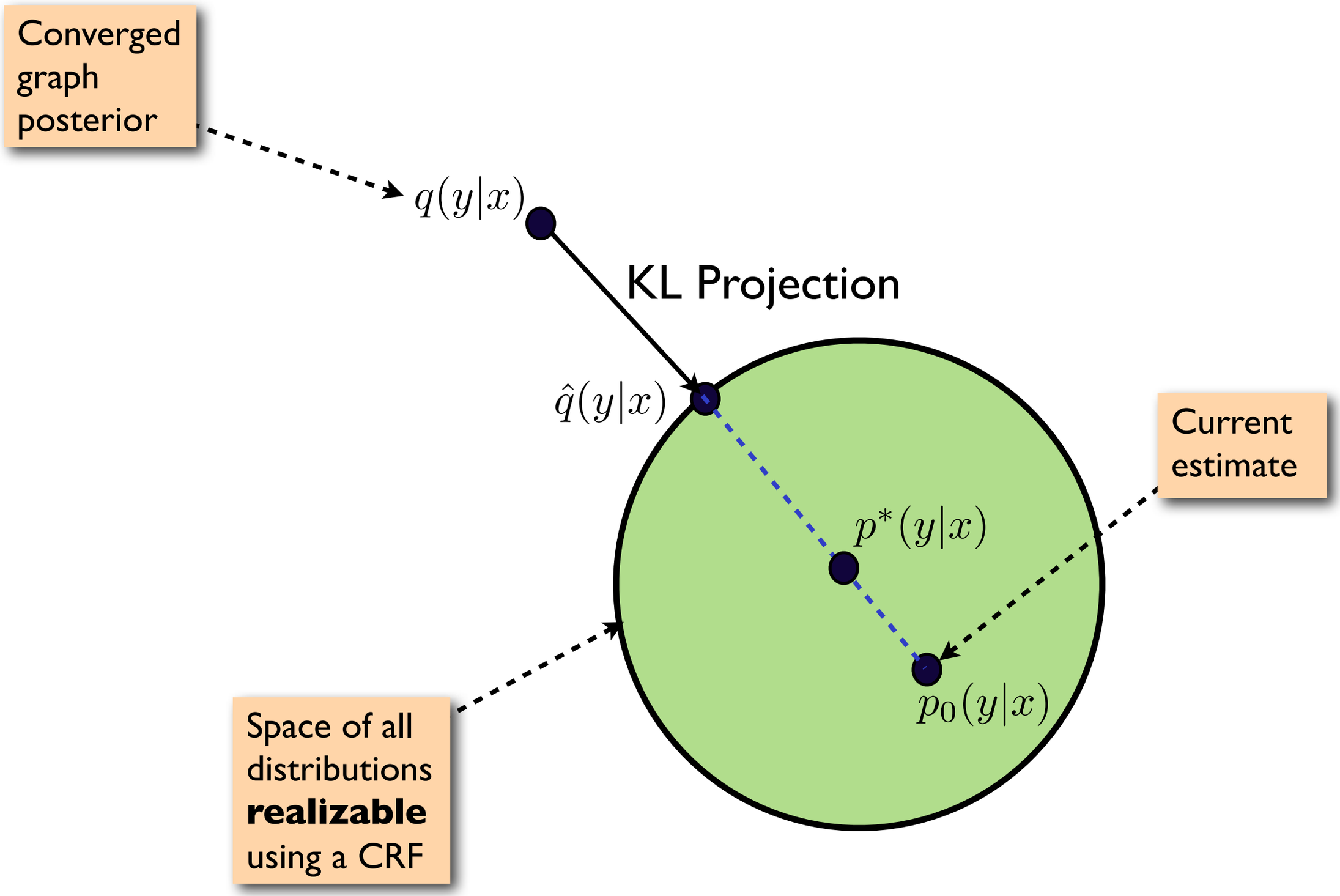


Viterbi Decoding : Intuition

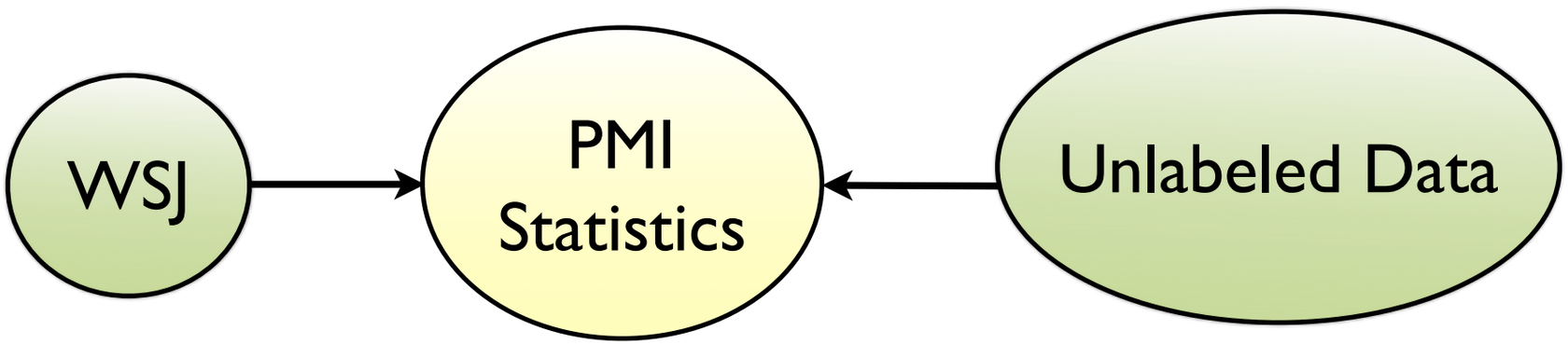


Corpora

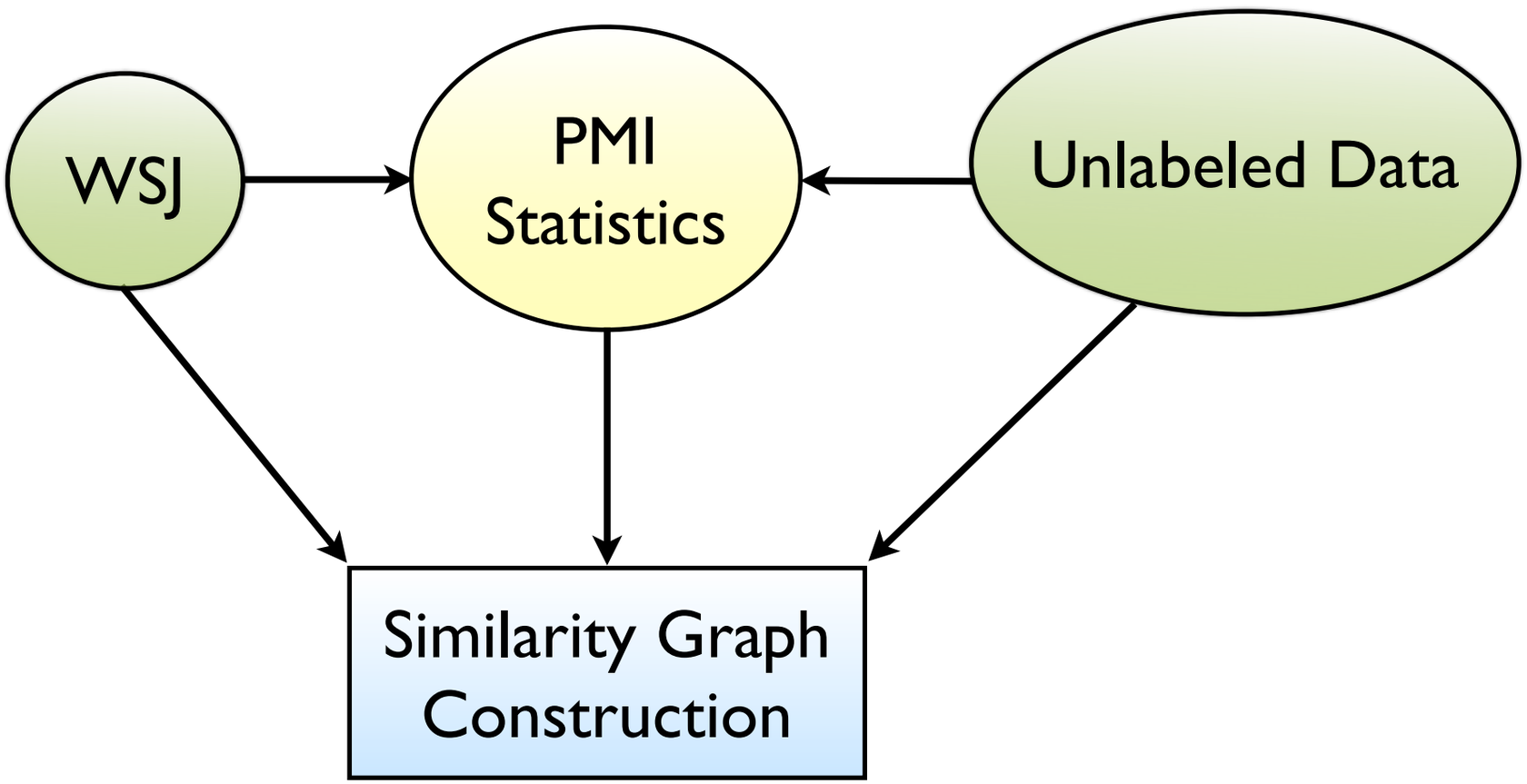
- Source Domain (labeled): Wall Street Journal (WSJ) section of the Penn Treebank.
- Target Domain:
  - QuestionBank: 4000 labeled sentences
  - Penn BioTreebank: 1061 labeled sentences

Graph Construction: Question Bank

Graph Construction: Question Bank



Graph Construction: Question Bank



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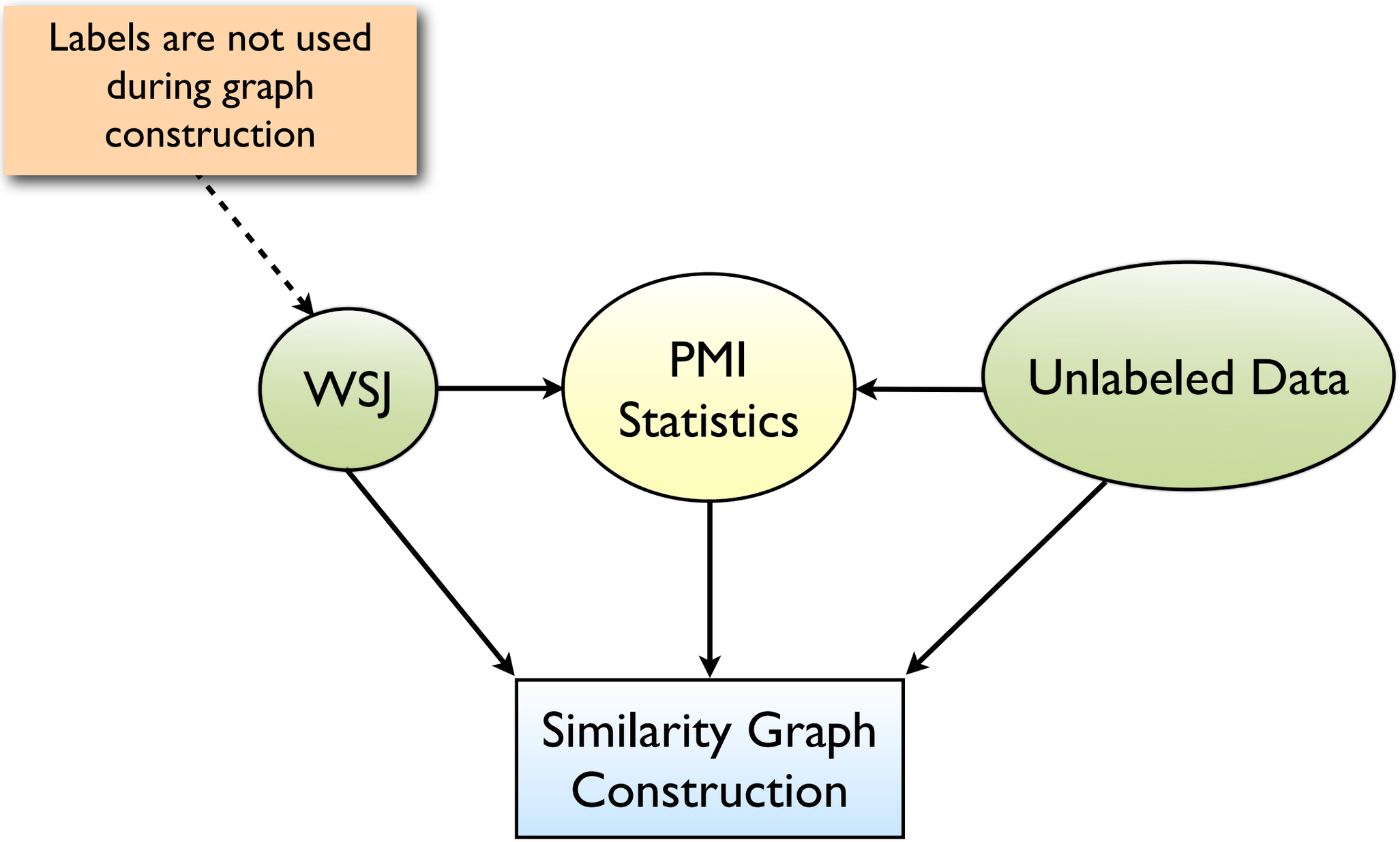

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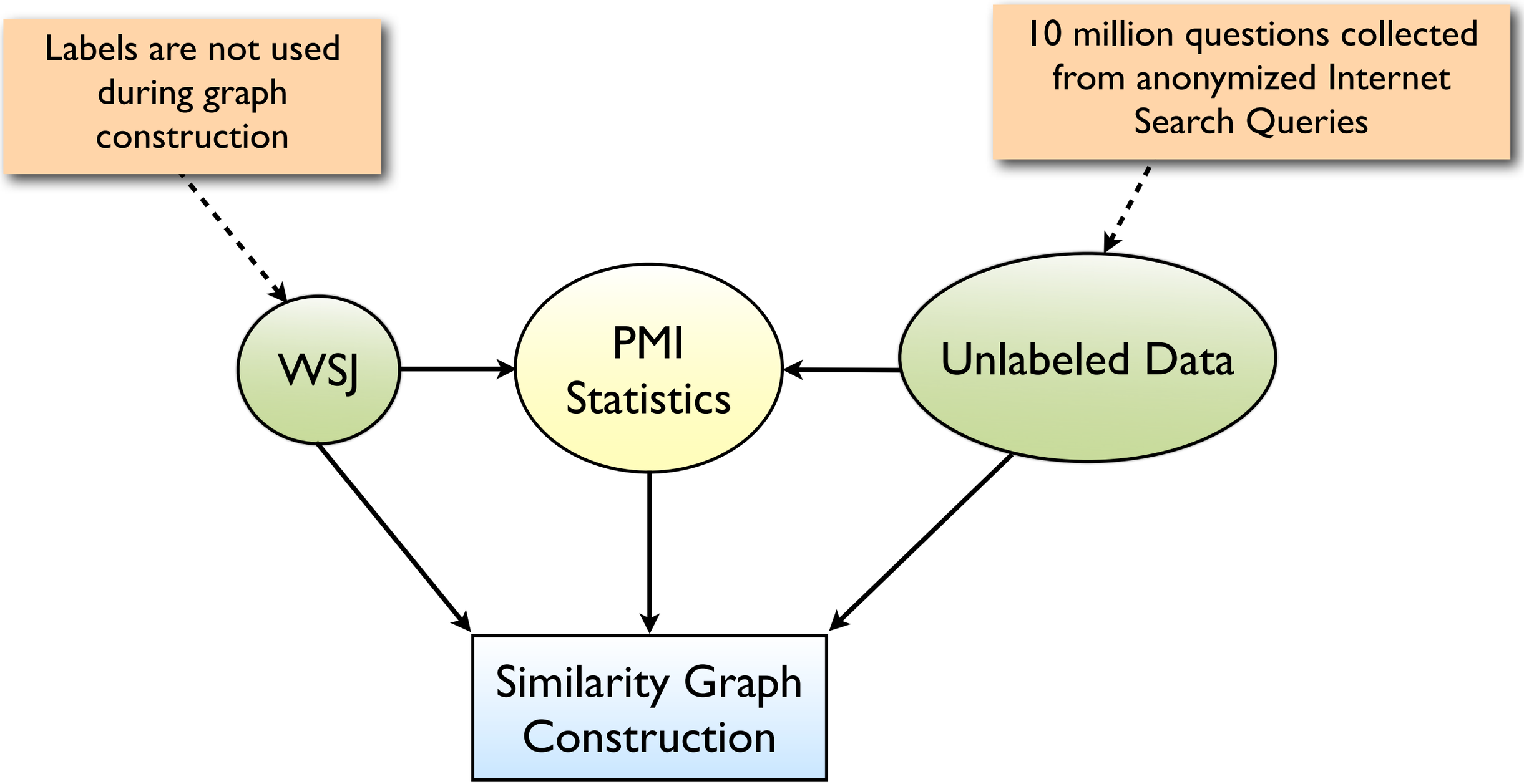
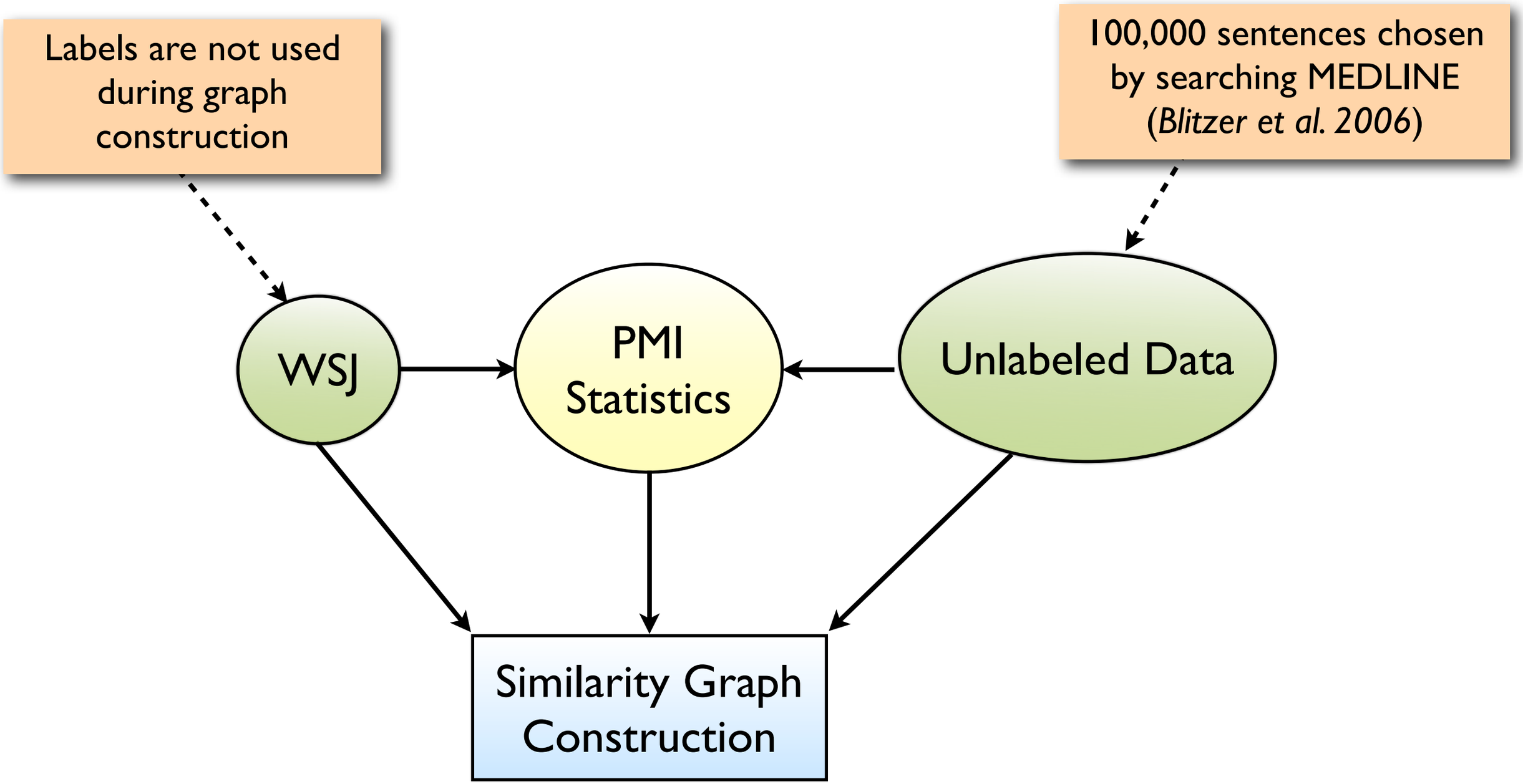


Graph Construction: Bio



Baseline (Supervised)

Not the same as features used using graph construction

- Features: word identity, suffixes, prefixes & special character detectors (dashes, digits, etc.).
- Achieves 97.17% accuracy on WSJ development set.

Results

	Questions	Bio
Baseline	83.8	86.2
Self-training	84.0	87.1
Semi-supervised CRF	**86.8**	**87.6**

Analysis

	Questions	Bio
percentage of unlabeled trigrams not connected to and any labeled trigram	12.4	46.8
average path length between an unlabeled trigram and its nearest labeled trigram	9.4	22.4

Analysis

Sparse Graph

	Questions	Bio
percentage of unlabeled trigrams not connected to and any labeled trigram	12.4	46.8
average path length between an unlabeled trigram and its nearest labeled trigram	9.4	22.4

Analysis

- Pros
  - Inductive
  - Produces a CRF (standard CRF inference infrastructure may be used)
- Issues
  - Graph construction
  - Graph is not integrated with CRF training

Outline

- Motivation
- Graph Construction
- Inference Methods
- Scalability
- Applications
  - Phone Classification
  - Text Categorization
  - Dialog Act Tagging
  - Statistical Machine Translation
  - POS Tagging
  - MultiLingual POS Tagging [Das & Petrov, ACL 2011]
- Conclusion & Future Work

Motivation

Motivation

- Supervised POS taggers for English have accuracies in the high 90's for most domains

Motivation

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- By comparison taggers in other languages are not as accurate

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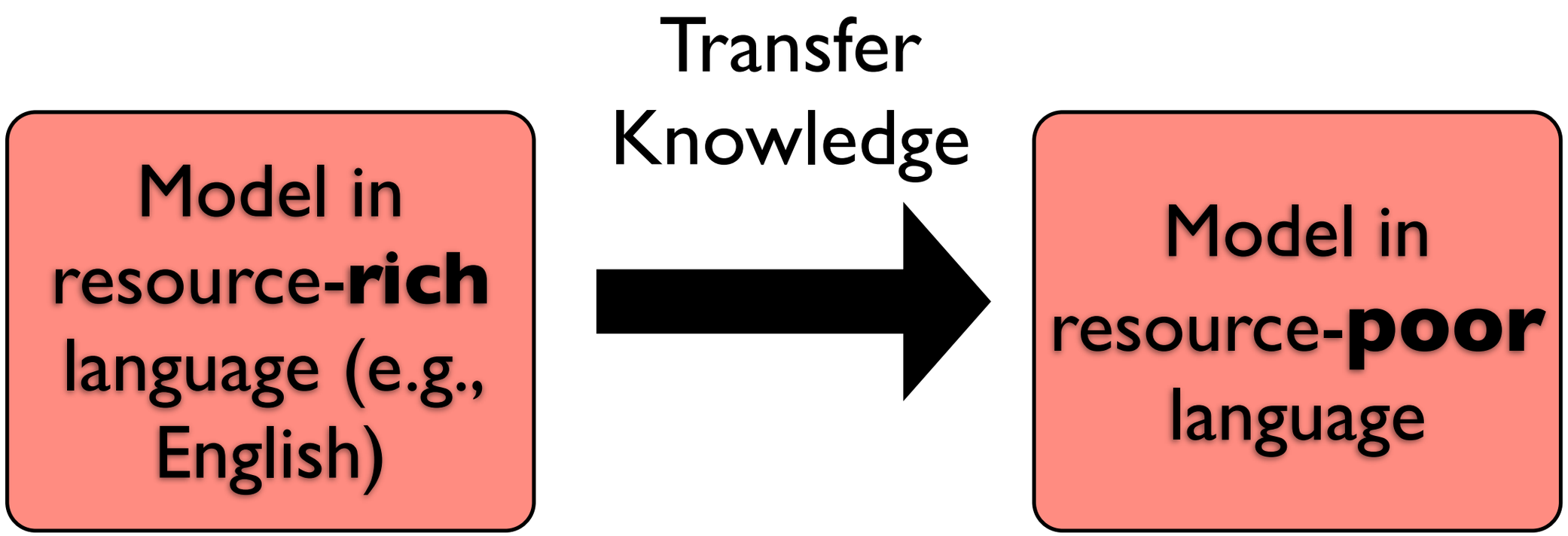


Cross-Lingual Projection

The food at Google is good .

Cross-Lingual Projection

96% Accuracy

DET	**NOUN**	**ADP**	**NOUN**	**VERB**	**ADJ**	**.**
The	food	at	Google	is	good	.

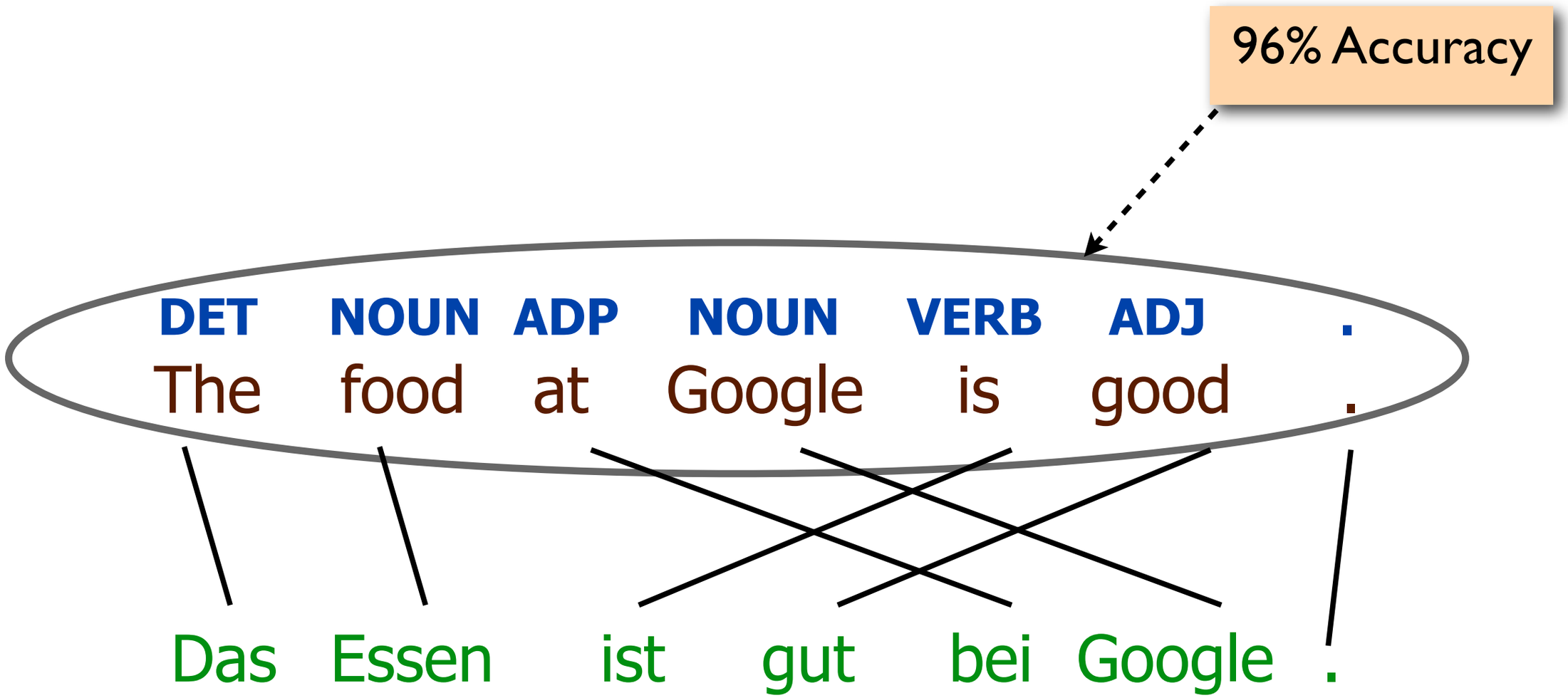
Cross-Lingual Projection

96% Accuracy

DET	**NOUN**	**ADP**	**NOUN**	**VERB**	**ADJ**	**.**
The	food	at	Google	is	good	.

Das Essen ist gut bei Google .

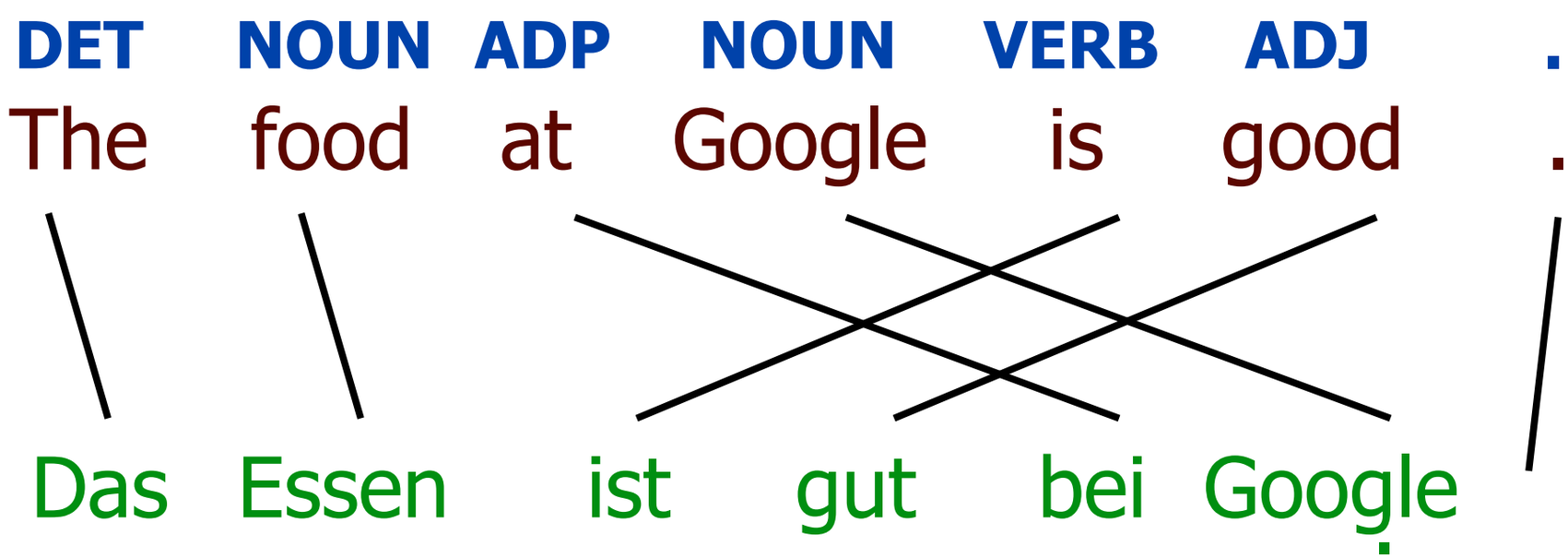
Cross-Lingual Projection



Automatic alignments from translation data

(available for more than 50 languages)

Cross-Lingual Projection



Cross-Lingual Projection

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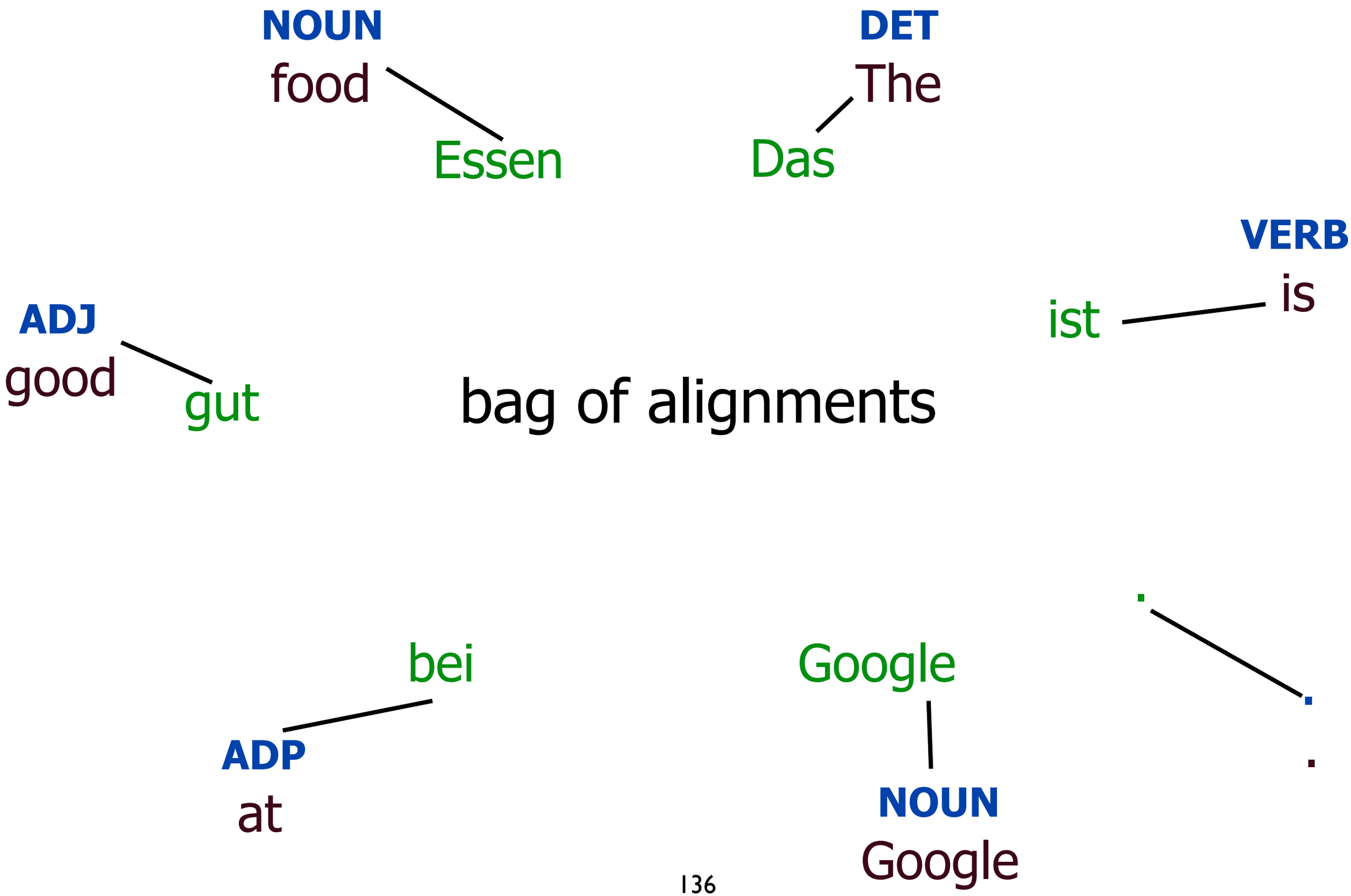
Cross-Lingual Projection



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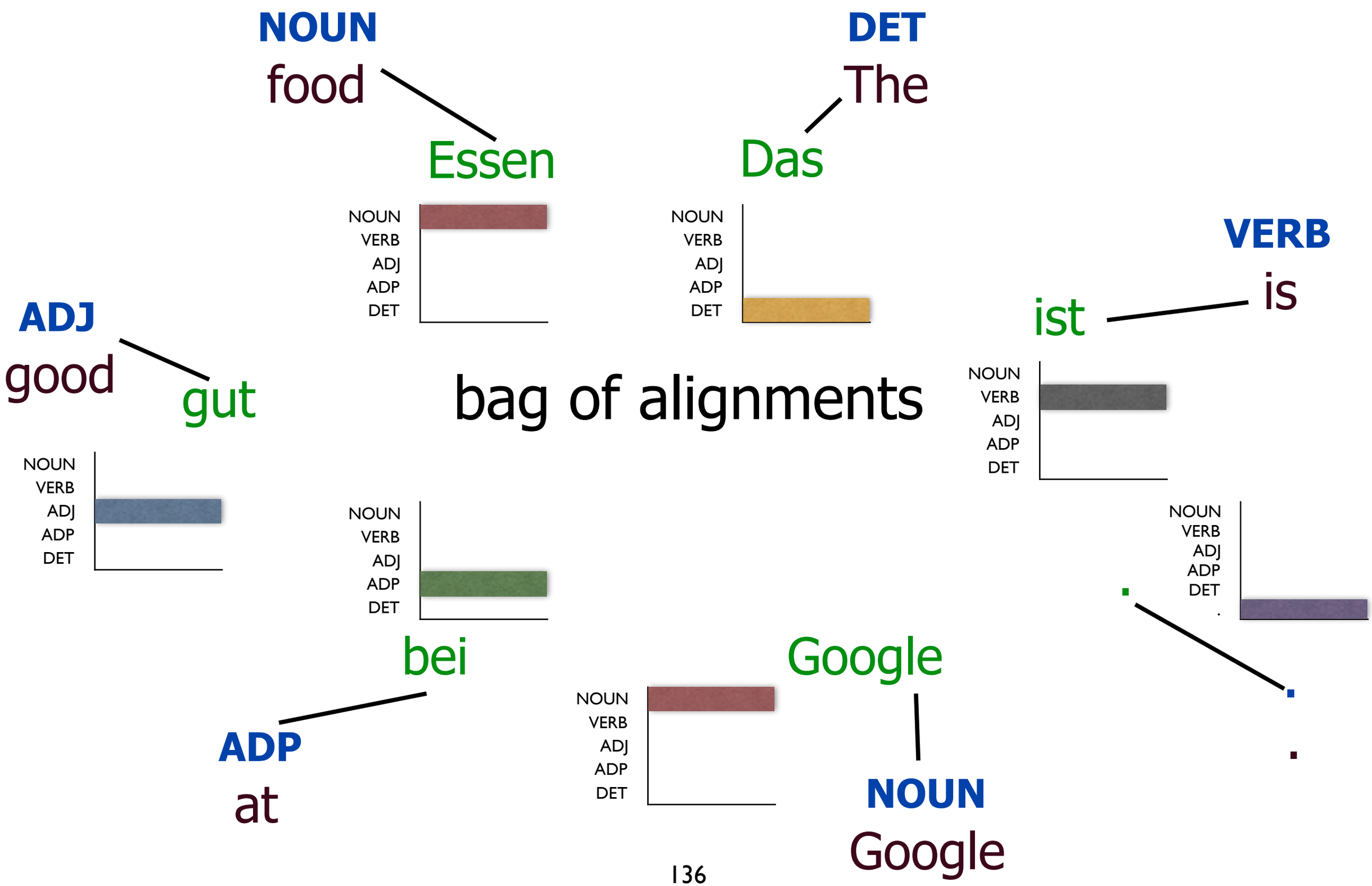

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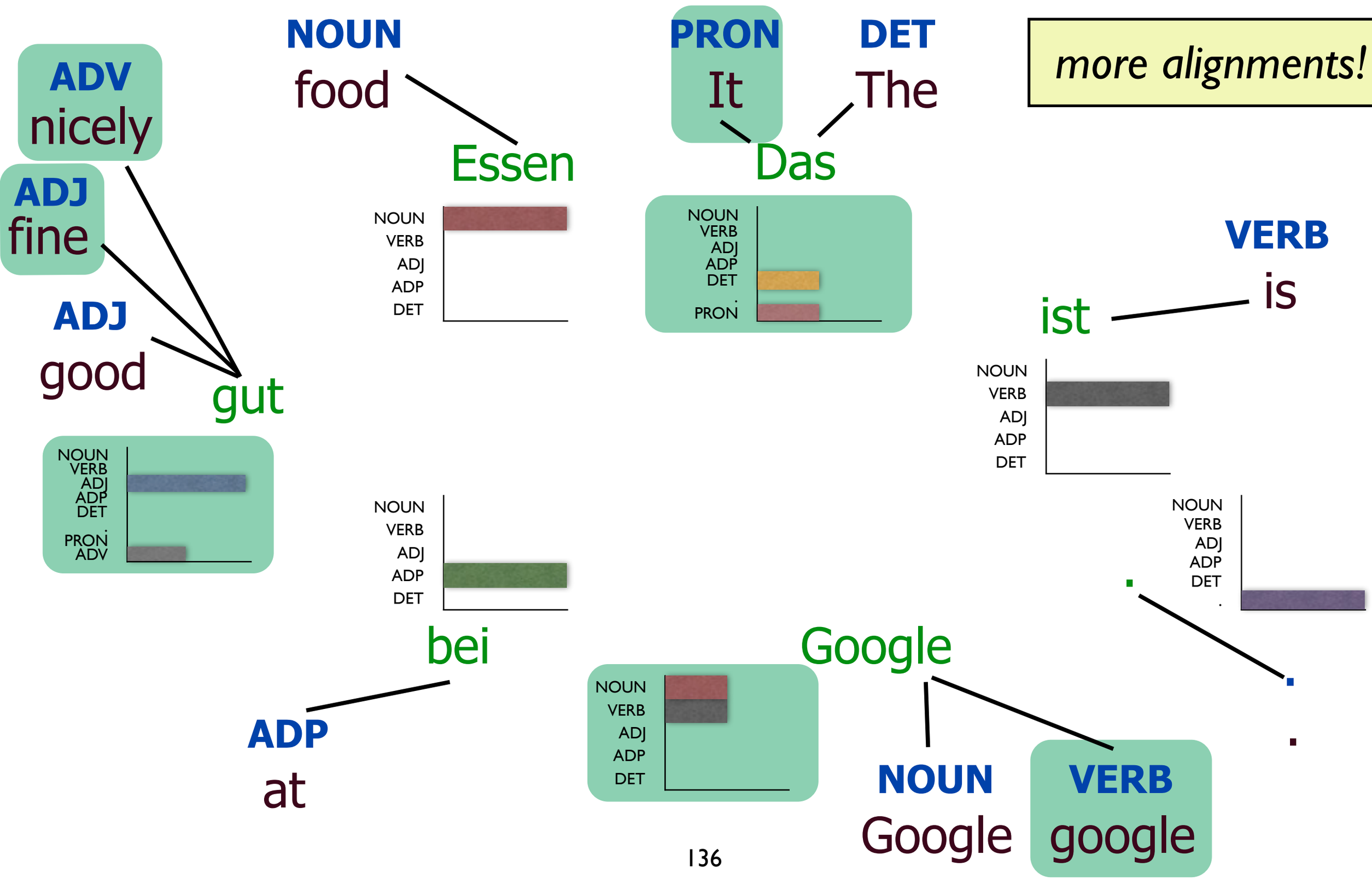

Cross-Lingual Projection Results

	Danish	Dutch	German	Greek	Italian	Portuguese	Spanish	Swedish	Average
Feature-HMM	69.1	65.1	81.3	71.8	68.1	78.4	80.2	70.1	73.0

Feature-HMM [Berg-Kirkpatrick, NAACL 2010]

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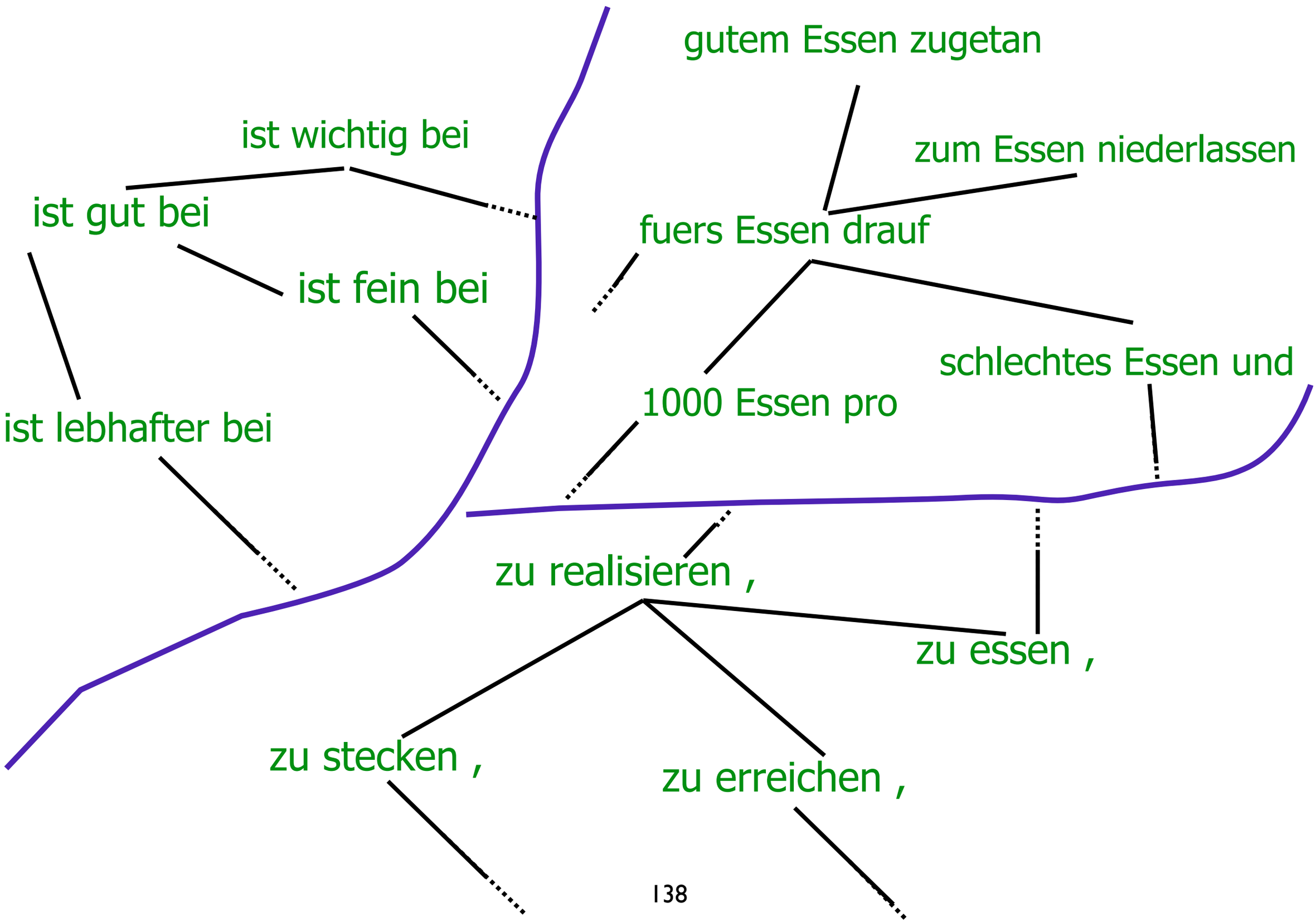
Graph Regularization



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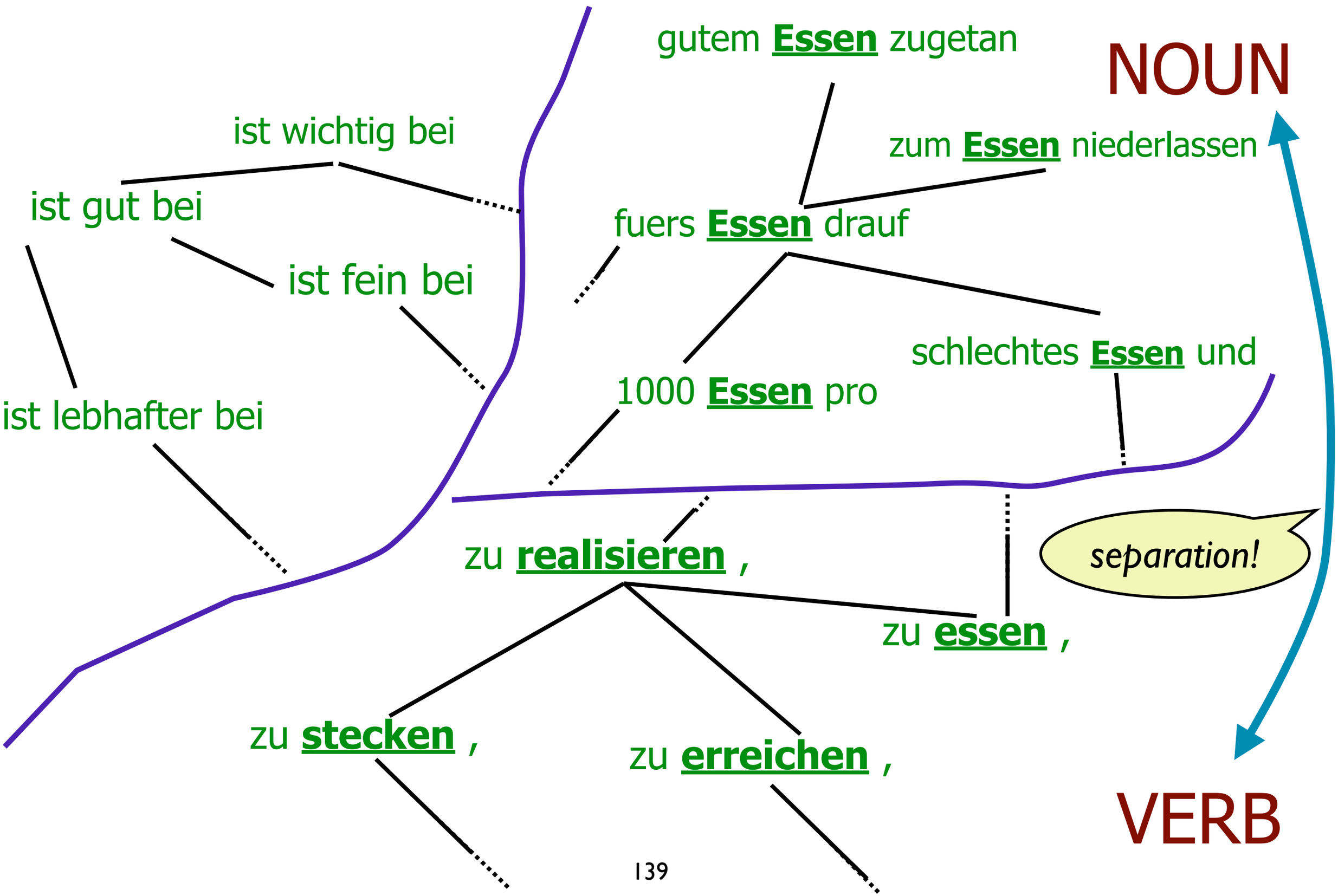

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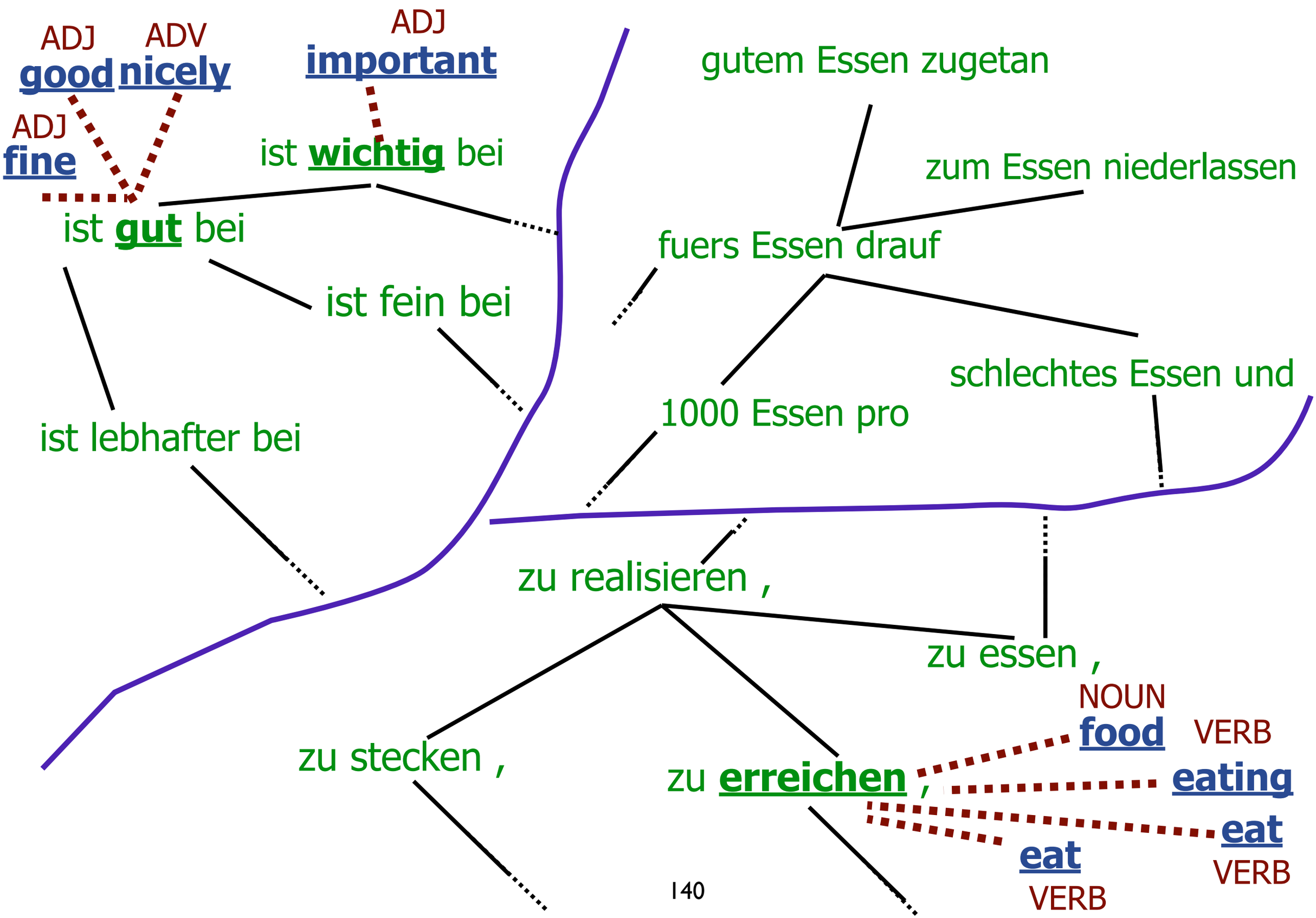

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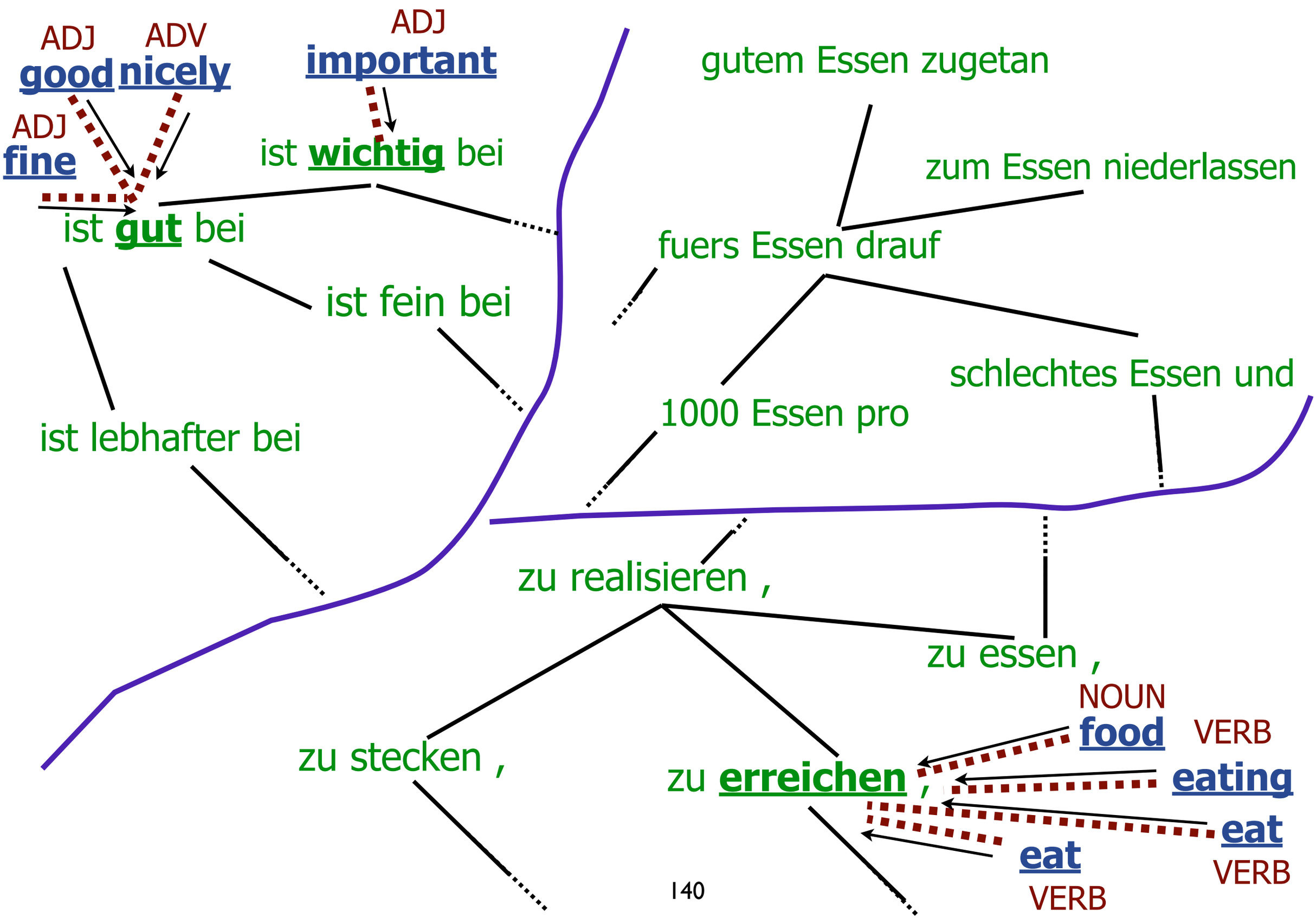

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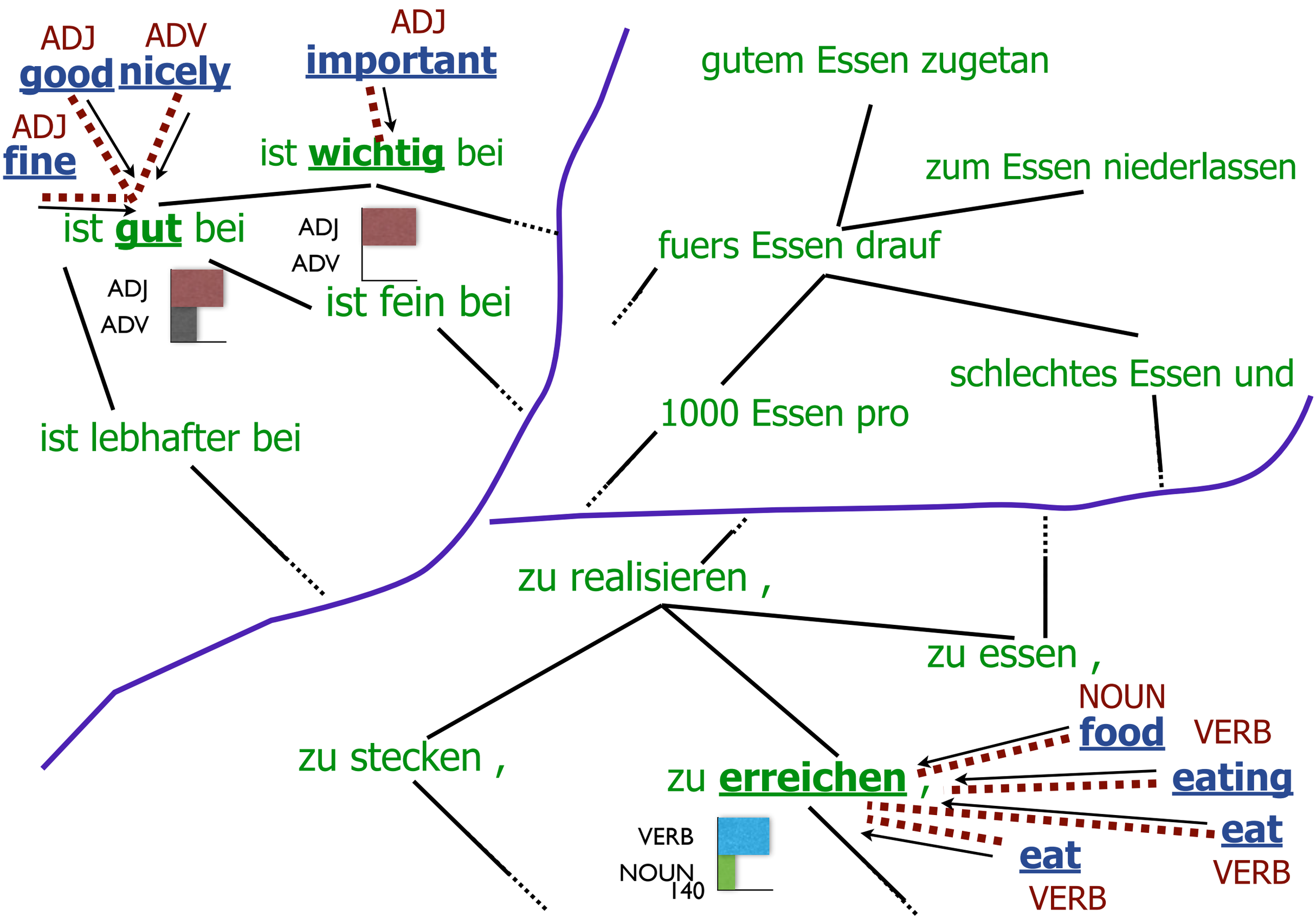

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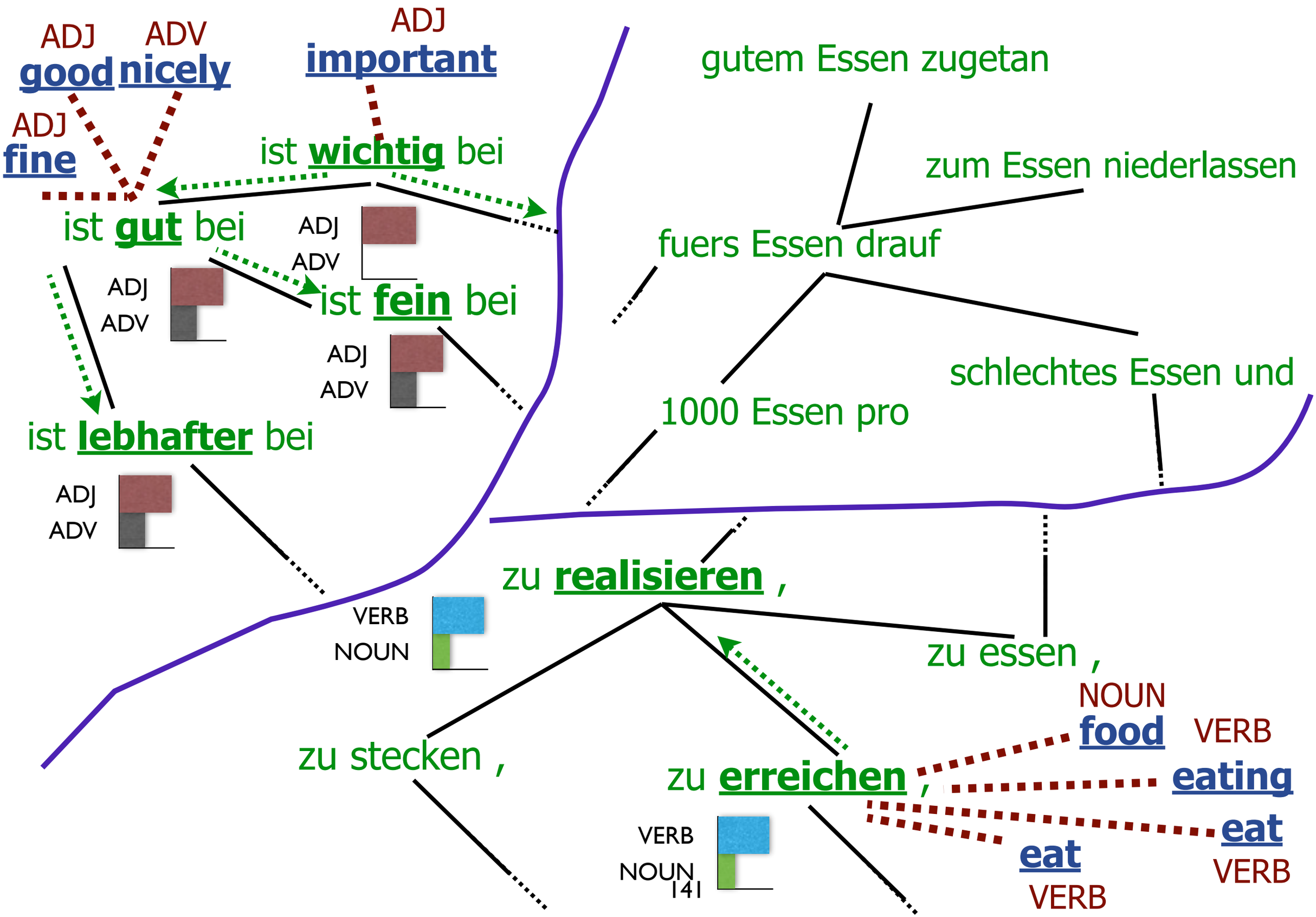

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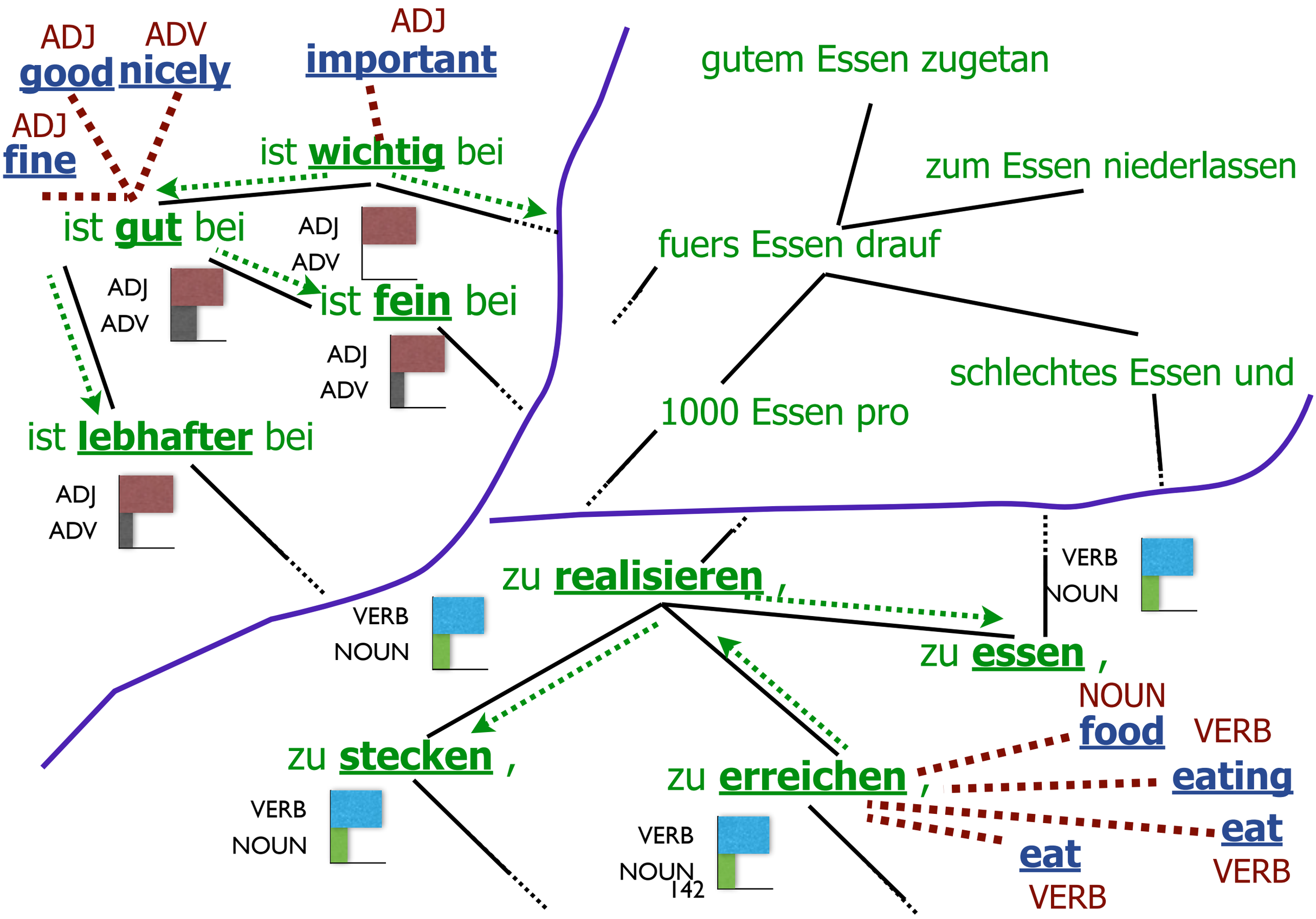

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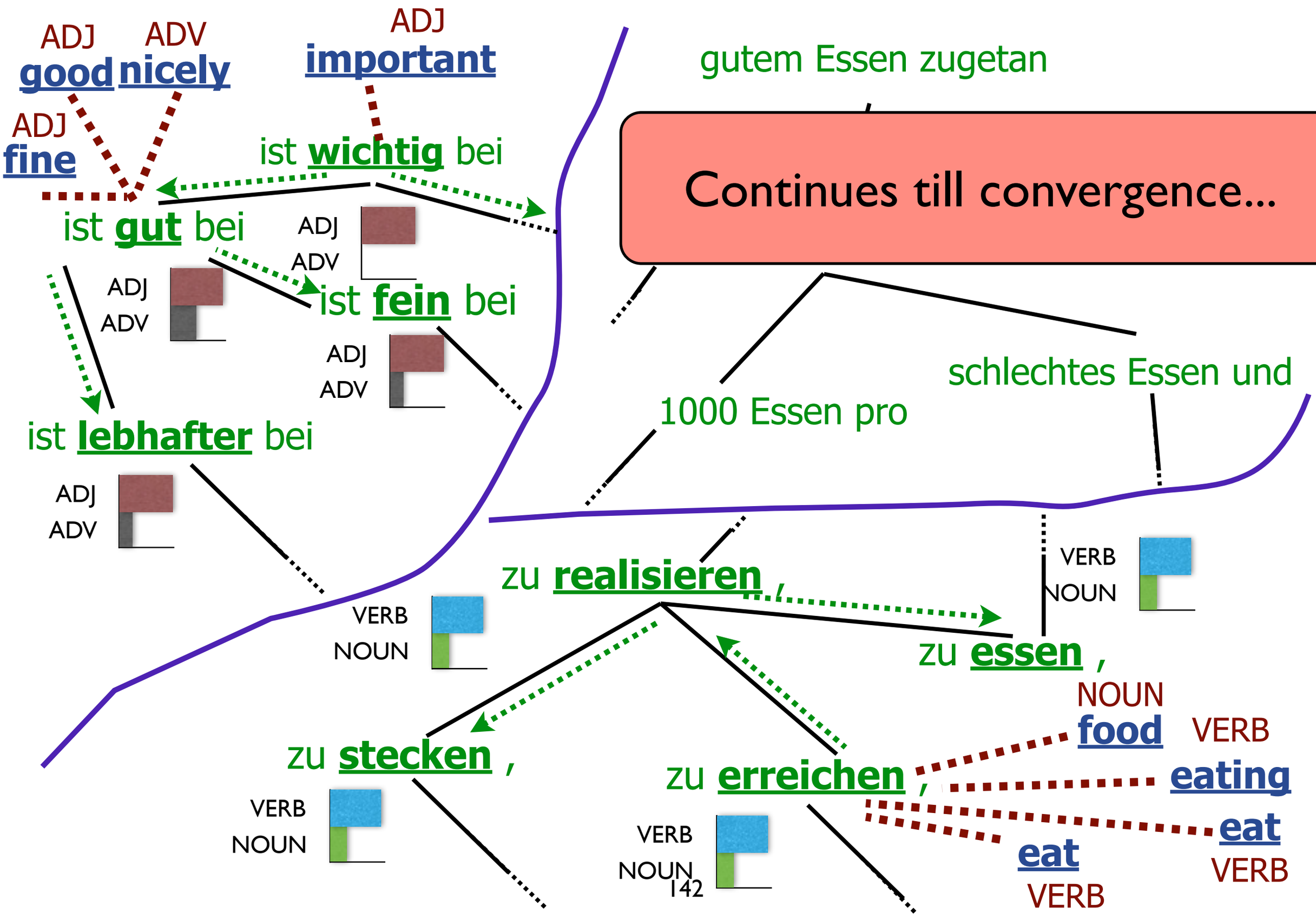

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Oracle (Supervised)	96.9	94.9	98.2	97.8	95.8	97.2	96.8	94.8	96.6

Outline

- Motivation
- Graph Construction
- Inference Methods
- Scalability
- Applications
- Conclusion & Future Work

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- Manifold Regularization
  - for generalization to unseen data (induction)

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- Effective in practice

Open Challenges

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- Using side information

Acknowledgments

- National Science Foundation (NSF) IIS-0447972
- DARPA HRO1107-1-0029, FA8750-09-C-0179
- Google Research Award
- Dipanjan Das (Google), Fernando Pereira (Google), Matan Orbach (Technion), Noah Smith (CMU)

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Thank You!

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